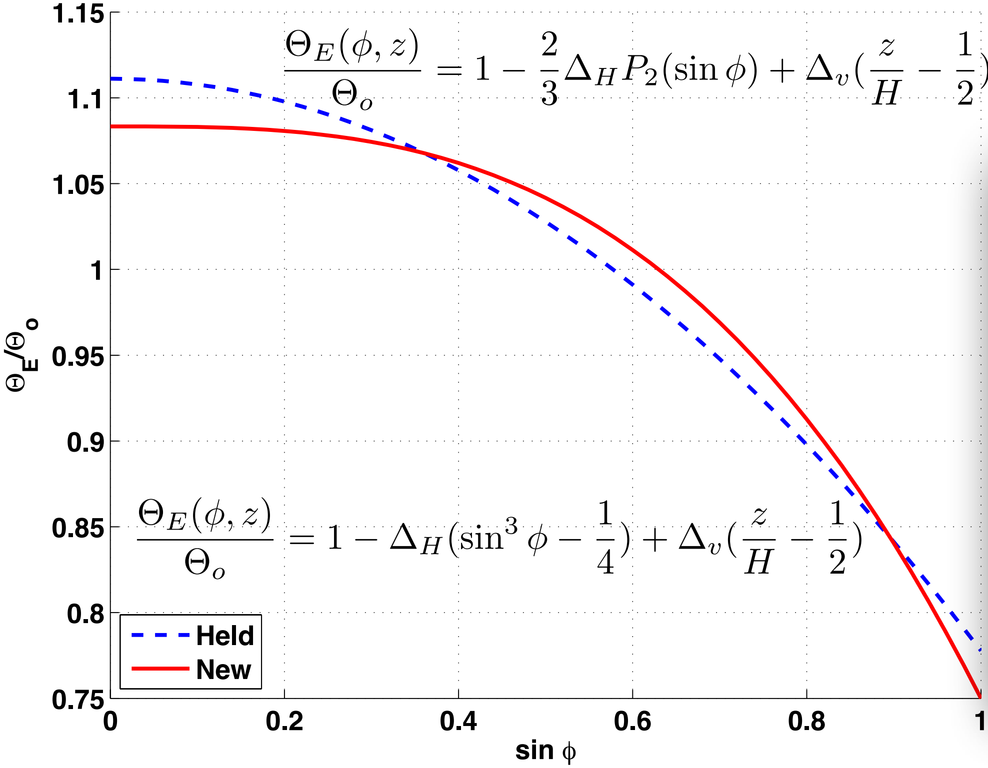
Assignments 3

Question#1: Hadley cell under different forcing



Held-Hou(1980) 讨论了当外部强迫的经向分布呈二次勒让德多项式，即

$$\frac{\Theta_E(\phi,z)}{\Theta_o}=1-\frac{2}{3}\Delta_H P_2(\sin\phi)+\Delta_v\left(\frac{z}{H}-\frac{1}{2}\right)$$

的情况下，哈德莱环流内的风场、温度场、环流的空间范围等将怎样随纬度和外力强迫的强度而变化。

如果将外力强迫的空间分布改为

$$\frac{\Theta_E(\phi,z)}{\Theta_o}=1-\Delta_H\left(\sin^3\phi-\frac{1}{4}\right)+\Delta_v\left(\frac{z}{H}-\frac{1}{2}\right).$$

1. 请推导出哈德莱环流内的高空风场和垂直平均位温场 $\frac{\bar{\Theta}}{\Theta_o}$ 将如何随纬度分布；
2. 同样利用小角度假设，请推导出环流的空间范围 ϕ_H 的表达式。如果设 $r=\frac{gH}{\Omega^2 a^2}$，请分别画出当 $\Delta_H=1/3$ 和 $\Delta_H=1/6$ 时，与Held-Hou的情况相比，ϕ_H 怎样随 r 而变化。
3. 选做题目：在此情况下，近地面风场的分布有怎样变化。

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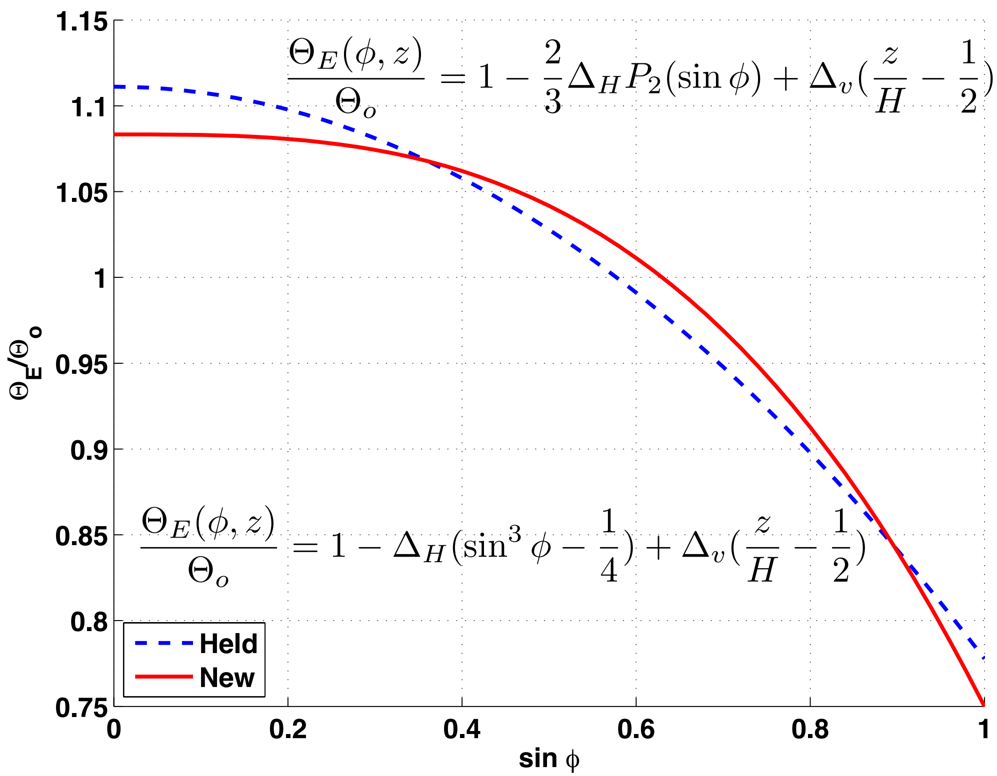

- Angular momentum:

$$[u] = \Omega a \frac{\sin^2\phi}{\cos\phi} \equiv U_M$$

- Thermal wind relation:

$$f[u(H) - u(0)] + \frac{\tan\phi}{a}[u^2(H) - u^2(0)] = -\frac{gH}{a\Theta_o}\frac{\partial\tilde{\Theta}}{\partial\phi}$$

$$\frac{\tilde{\Theta}(0) - \tilde{\Theta}(\phi)}{\Theta_o} = \frac{\Omega^2 a^2}{2gH}\frac{\sin^4\phi}{\cos^2\phi}$$

Need to know $\frac{\tilde{\Theta}(0)}{\Theta_o}$

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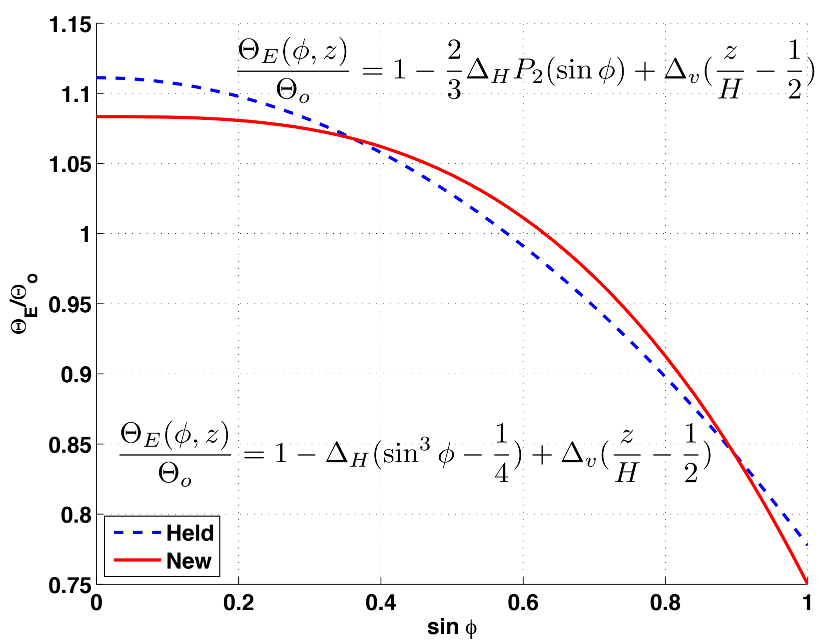

- Temperature should be continuous at the edge:

$$\tilde{\Theta}(\phi_H) = \tilde{\Theta}_E(\phi_H)$$

- Hadley cell does not produce net heating but just carry heat poleward over the

$$\int_0^{\phi_H} \tilde{\Theta}\cos\phi d\phi = \int_0^{\phi_H} \tilde{\Theta}_E \cos\phi d\phi$$

Assume small ϕ, $\sin\phi \sim \phi$

$$\phi_H = \frac{15}{8}\frac{gH\Delta_H}{\Omega^2 a^2}$$

$$\frac{\tilde{\Theta}(0)}{\Theta_o} \approx \frac{\tilde{\Theta}_E(0)}{\Theta_o} - \Delta_H\phi_H^3 + \frac{\Omega^2 a^2}{2gH}\phi_H^4 = \frac{\tilde{\Theta}_E(0)}{\Theta_o} - \frac{1}{16}\Delta_H\phi_H^3$$

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根据小角度假设（$\sin\phi\sim\phi, \cos\phi\sim 1$）

垂直平均之后的外力强迫为：

$$\frac{\tilde{\Theta}_E(\phi)}{\Theta_o} = 1 - \Delta_H\left(\sin^3\phi - \frac{1}{4}\right) \rightarrow \frac{\tilde{\Theta}_E}{\Theta_o} = \frac{\tilde{\Theta}_E(0)}{\Theta_o} - \Delta_H\phi^3$$

垂直平均位温场随纬度分布为：

$$\frac{\tilde{\Theta}(0) - \tilde{\Theta}(\phi)}{\Theta_o} = \frac{\Omega^2 a^2}{2gH}\frac{\sin^4\phi}{\cos^2\phi} \rightarrow \frac{\tilde{\Theta}}{\Theta_o} = \frac{\tilde{\Theta}(0)}{\Theta_o} - \frac{\Omega^2 a^2}{2gH}\phi^4$$

$\tilde{\Theta}(\phi_H) = \tilde{\Theta}_E(\phi_H)$ 连续条件：

$$\frac{\tilde{\Theta}(0)}{\Theta_o} - \frac{\Omega^2 a^2}{2gH}\phi_H^4 = \frac{\tilde{\Theta}_E(0)}{\Theta_o} - \Delta_H\phi_H^3 \rightarrow \frac{\tilde{\Theta}(0)}{\Theta_o} - \frac{\tilde{\Theta}_E(0)}{\Theta_o} = \frac{\Omega^2 a^2}{2gH}\phi_H^4 - \Delta_H\phi_H^3$$

$\int_0^{\phi_H}\tilde{\Theta}\cos\phi d\phi = \int_0^{\phi_H}\tilde{\Theta}_E\cos\phi d\phi$ 守恒条件：

$$\int_0^{\phi_H}\left(\frac{\tilde{\Theta}(0)}{\Theta_o} - \frac{\Omega^2 a^2}{2gH}\phi^4\right)d\phi = \int_0^{\phi_H}\left(\frac{\tilde{\Theta}_E(0)}{\Theta_o} - \Delta_H\phi^3\right)d\phi \rightarrow \frac{\tilde{\Theta}(0)}{\Theta_o} - \frac{\tilde{\Theta}_E(0)}{\Theta_o} = \frac{\Omega^2 a^2}{10gH}\phi_H^4 - \frac{1}{4}\Delta_H\phi_H^3$$

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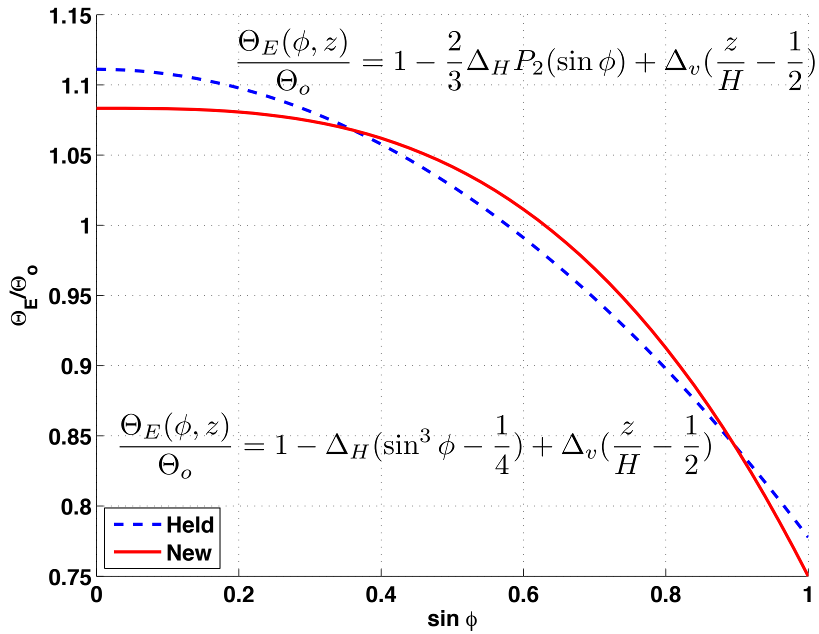

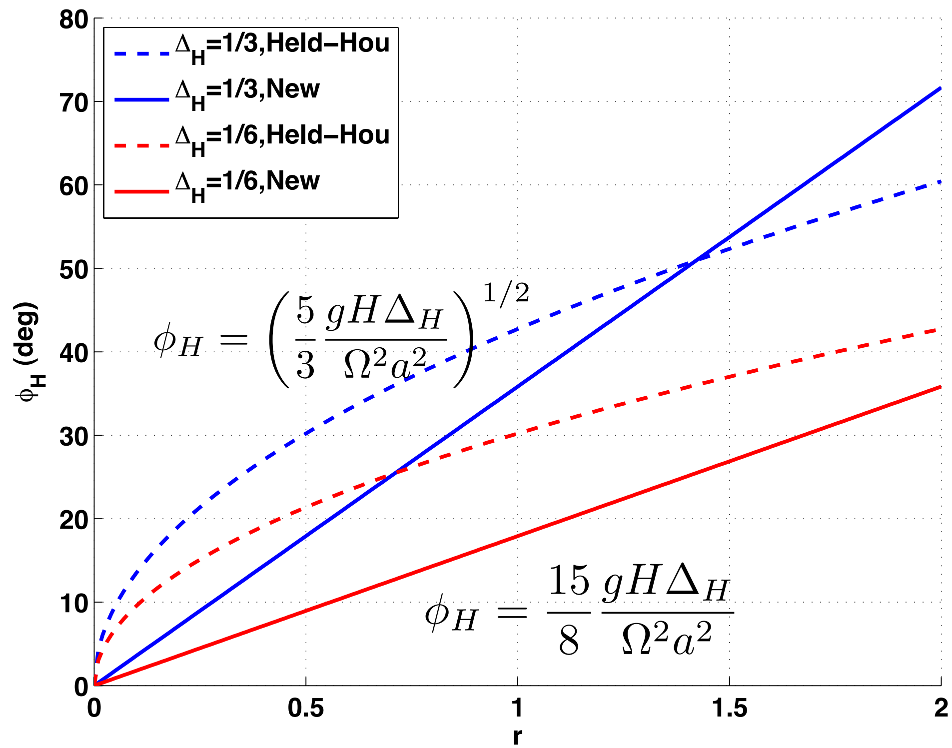

Held-Hou model

-Surface winds

- From the zonal momentum equation

$$\frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}\left(\cos^2\phi\int_0^H uvdz\right) = -Cu(0)$$

$$\frac{1}{\Theta_o}\int_0^H v\Theta dz \approx V\Delta_V$$

$$\frac{1}{H}\int_0^H \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(v\Theta\cos\phi)dz = \frac{\tilde{\Theta}_E - \tilde{\Theta}}{\tau}$$

Then, mass flux V can be solved. Similarly, we have the momentum flux,

$$\int_0^H uvdz \approx VU_m$$

$$\left.\begin{aligned}
0 &= -\nabla\cdot(\mathbf{v}u) + fv + \frac{uv\tan\theta}{a} + \frac{\partial}{\partial z}\left(\nu\frac{\partial u}{\partial z}\right) \\
0 &= -\nabla\cdot(\mathbf{v}v) - fu - \frac{u^2\tan\theta}{a} - \frac{1}{a}\frac{\partial\Phi}{\partial\theta} + \frac{\partial}{\partial z}\left(\nu\frac{\partial v}{\partial z}\right) \\
0 &= -\nabla\cdot(\mathbf{v}\Theta) - (\Theta - \Theta_E)\tau^{-1} + \frac{\partial}{\partial z}\left(\nu\frac{\partial\Theta}{\partial z}\right) \\
0 &= -\nabla\cdot\mathbf{v} \\
\frac{\partial\Phi}{\partial z} &= g\Theta/\Theta_0
\end{aligned}\right\}, \quad (1)$$

with boundary conditions

$$\left.\begin{aligned}
&\text{at } z = H: \quad w = 0; \quad \frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = \frac{\partial\Theta}{\partial z} = 0 \\
&\text{at } z = 0: \quad w = 0; \quad \frac{\partial\Theta}{\partial z} = 0; \\
&\qquad \nu\frac{\partial u}{\partial z} = Cu; \quad \nu\frac{\partial v}{\partial z} = Cv
\end{aligned}\right\}. \quad (1a)$$

-Surface Winds

- From thermodynamic equation:

$$\frac{1}{H\Theta_0}\int_0^H \frac{1}{acos\phi}\frac{\partial}{\partial\phi}(v\Theta cos\phi)dz = \frac{\bar{\Theta}_e - \bar{\Theta}}{\tau\Theta_0} = \frac{1}{\tau}(\frac{1}{16}(\frac{15}{8})^3\Delta_H^4 r^3 - \Delta_H\phi^3 + \frac{1}{2}\frac{\Omega^2a^2}{gH}\phi^4)$$

$$\frac{1}{\Theta_0}\int_0^H (v\Theta)dz = \int_0^\phi \frac{Ha}{\tau}(\frac{1}{16}(\frac{15}{8})^3\Delta_H^4 r^3 - \Delta_H\phi^3 + \frac{1}{2}\frac{\Omega^2a^2}{gH}\phi^4)\,d\phi$$

$$\frac{1}{\Theta_0}\int_0^H (v\Theta)dz = \frac{Har^4\Delta_H^5}{\tau}\frac{1}{16}(\frac{15}{8})^4[\frac{\phi}{\phi_H} - 4(\frac{\phi}{\phi_H})^4 + 3(\frac{\phi}{\phi_H})^5]$$

Then, mass flux V can be solved.

$$\int_0^H uvdz \approx VU_m$$

$$Cu(0) = -\frac{\Omega}{\Delta v}\frac{Har^5\Delta_H^6}{\tau}\frac{1}{16}(\frac{15}{8})^5[3(\frac{\phi}{\phi_H})^2 - 24(\frac{\phi}{\phi_H})^5 + 21(\frac{\phi}{\phi_H})^6]$$

Held-Hou model

-Surface Winds

$$Cu(0) = -\frac{\Omega}{\Delta v}\frac{Har^5\Delta_H^6}{\tau}\frac{1}{16}(\frac{15}{8})^5[3(\frac{\phi}{\phi_H})^2 - 24(\frac{\phi}{\phi_H})^5 + 21(\frac{\phi}{\phi_H})^6]$$