第二章:

大气环流的外部强迫(III)

–Simple energy balance climate model

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2022.10.06

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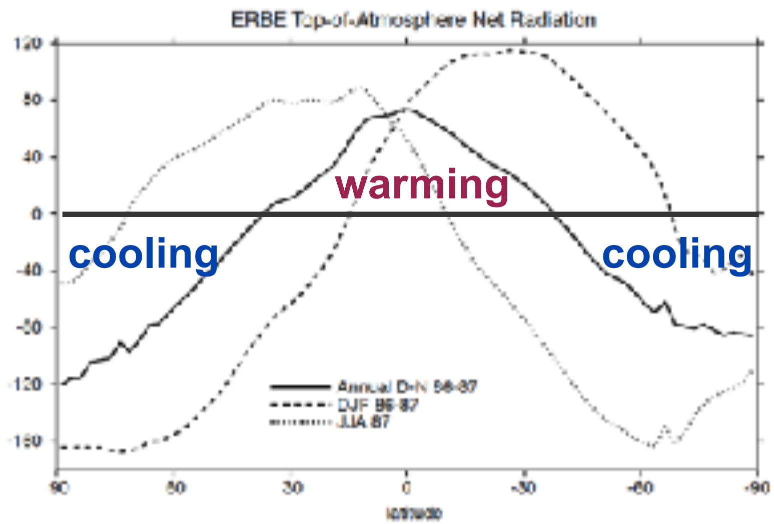
大气环流的外部强迫（III）

Reference reading:

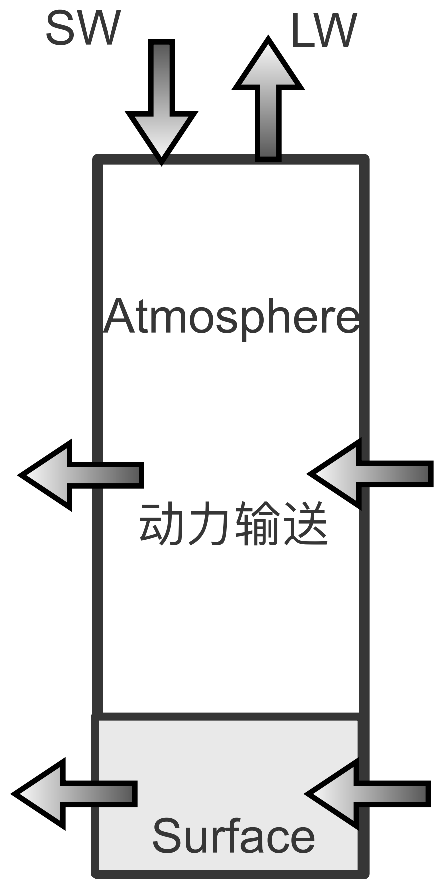
Lindzen 2005, Chapter 2

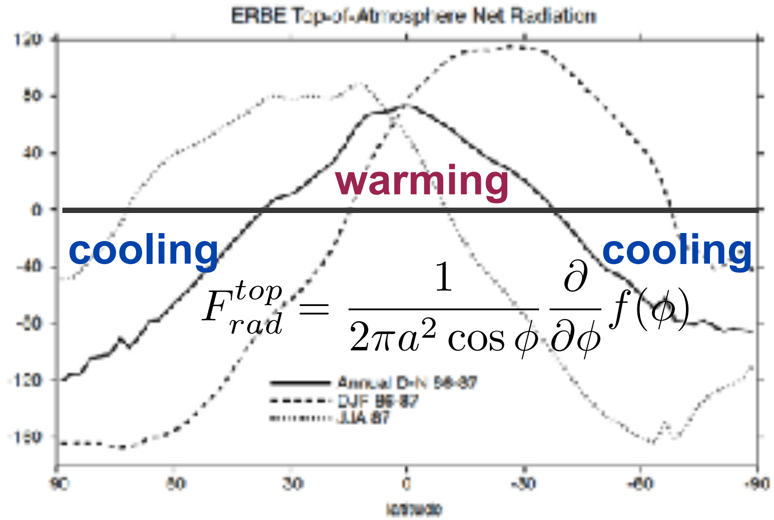
2022.10.06

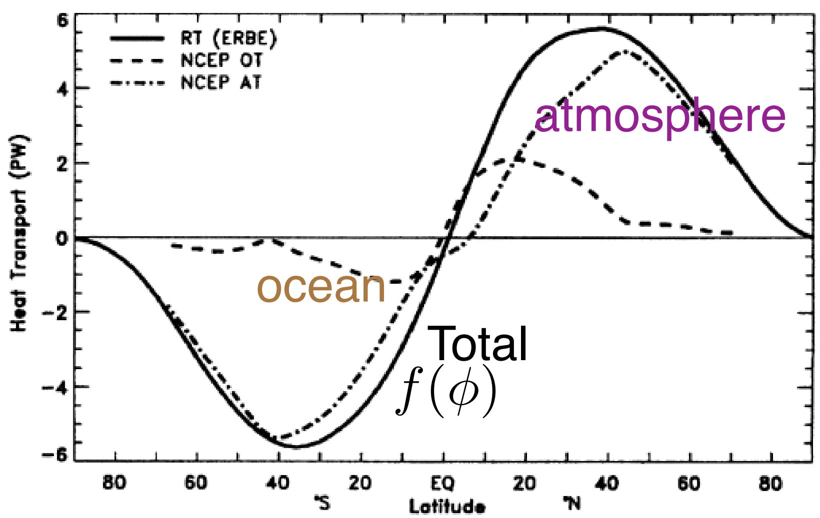


$$F_{rad}^{top} = \frac{1}{2\pi a^2 \cos\phi} \frac{\partial}{\partial\phi} f(\phi)$$

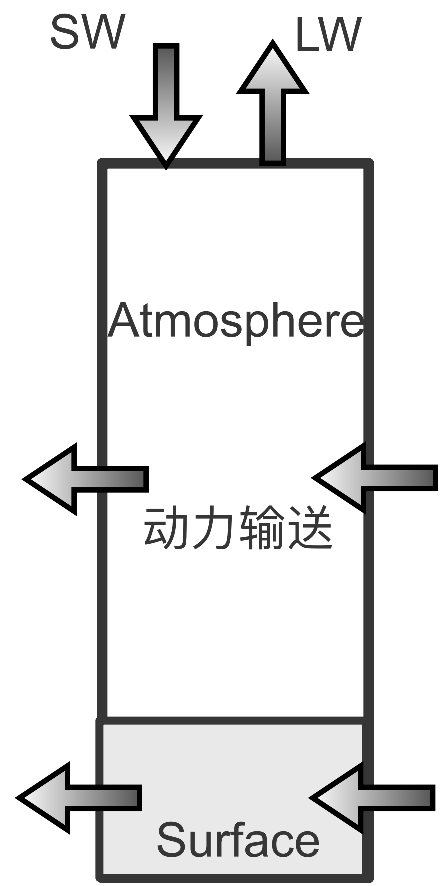
$f(\phi)$ — meridional energy transport by atmosphere and oceans







Wunsch (2005), J. Climate



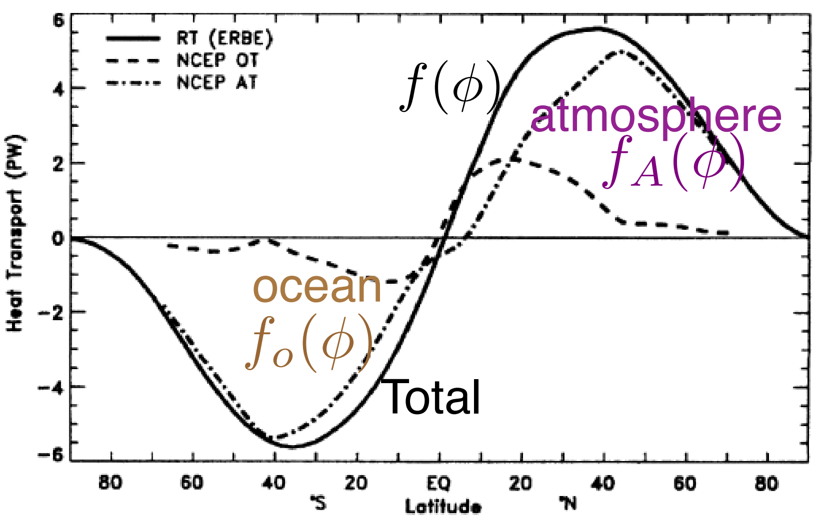
$$F_{rad}^{top} = \frac{1}{2\pi a^2 \cos\phi} \frac{\partial}{\partial\phi} f(\phi)$$

Atmosphere:

$$F_{rad}^{top} - F_{rad}^{sfc} + F_{LH} + F_{SH} = \frac{1}{2\pi a^2 \cos\phi} \frac{\partial}{\partial\phi} f_A(\phi)$$

Ocean:

$$F_{rad}^{sfc} - F_{LH} - F_{SH} = \frac{1}{2\pi a^2 \cos\phi} \frac{\partial}{\partial\phi} f_o(\phi)$$



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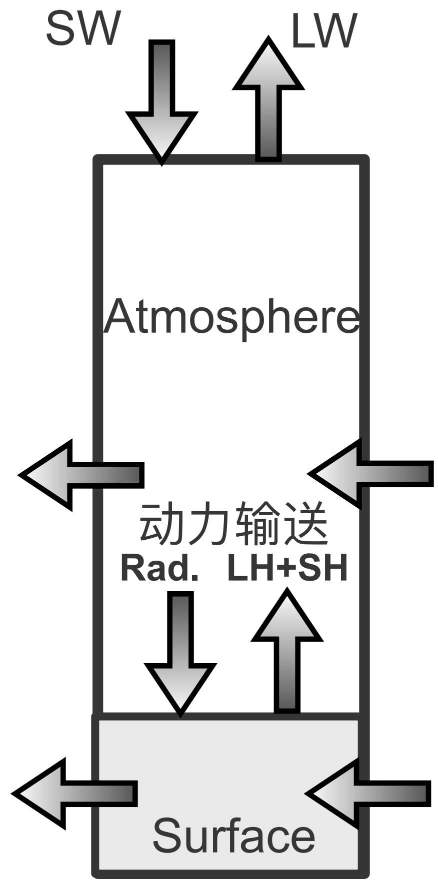
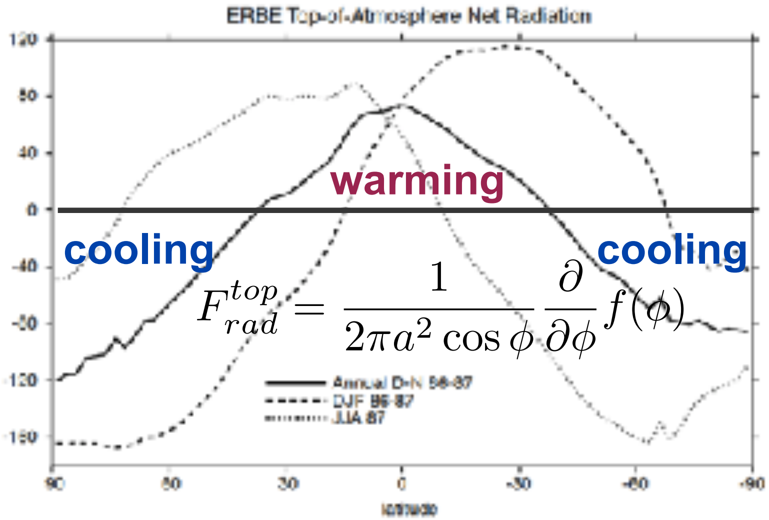
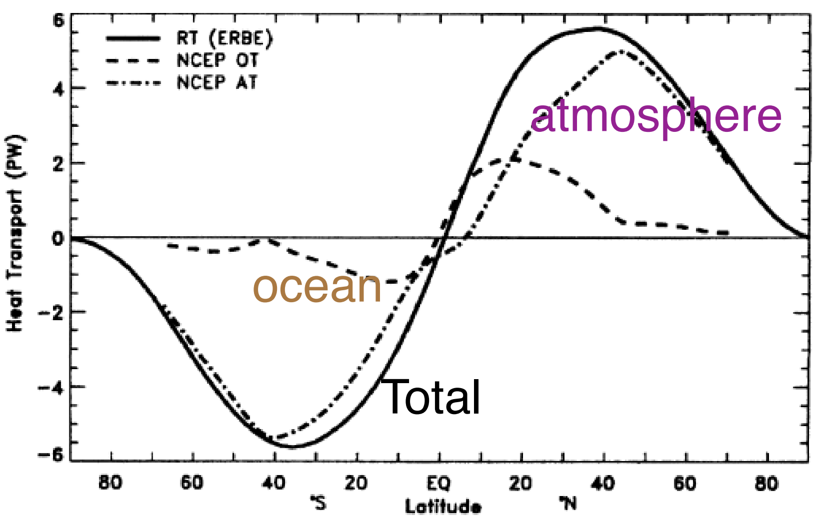


Simple energy balance climate models



$$F_{rad}^{top} = \frac{1}{2\pi a^2 \cos\phi} \frac{\partial}{\partial \phi} f(\phi)$$



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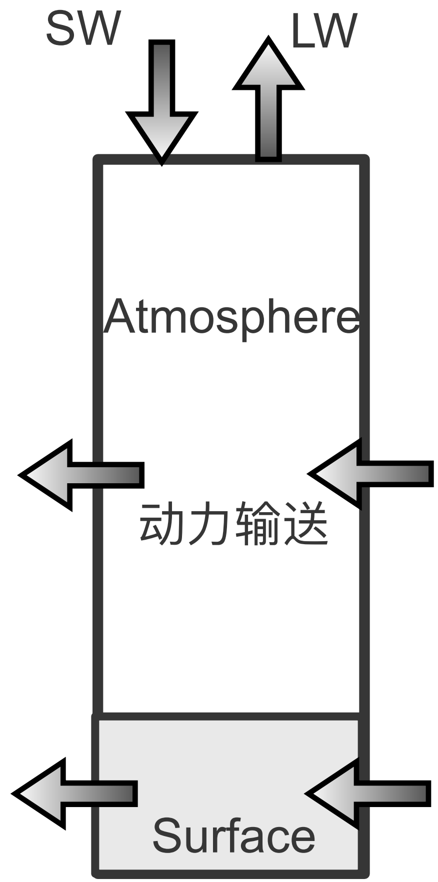
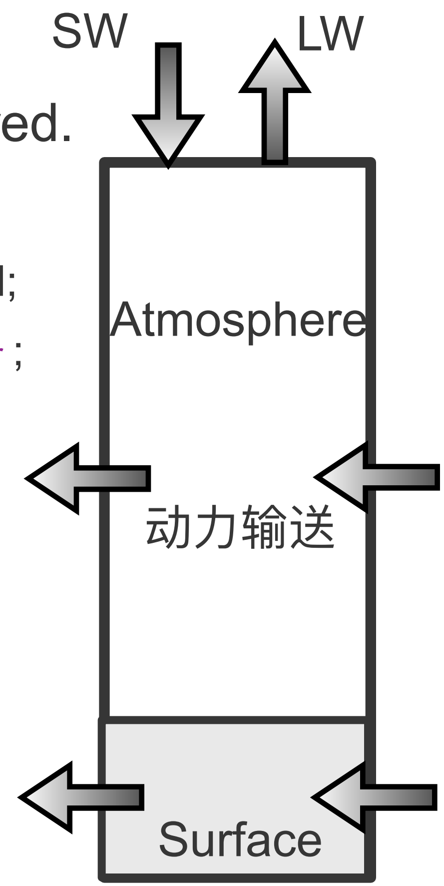


Simple energy balance climate models

- Simplest models in which the interactions between ***radiation*** and ***dynamic heat transport*** can be considered.
  - Assumptions are made below:
    - One-dimensional, only latitude dependences are considered;
    - Global energy budgets are assumed to be expressed in T_{sur} ;
    - Only annual mean conditions are considered;



$$\mathcal{C}\frac{\partial T(x,t)}{\partial t} = \text{solar radiation} - \text{infrared cooling}$$

$$-\text{divergence of heat flux}$$

$x = \sin\phi$, where ϕ is latitude.

$$\mathcal{C}\frac{\partial T(x,t)}{\partial t} = F_{rad}^{top} - \frac{1}{2\pi a^2}\frac{\partial}{\partial x} f(x)$$

Simple energy balance climate models

$$\mathcal{C}\frac{\partial T(x,t)}{\partial t} = \text{solar radiation} - \text{infrared cooling}$$
$$-\text{divergence of heat flux}$$

$x = \sin\phi$, where ϕ is latitude.

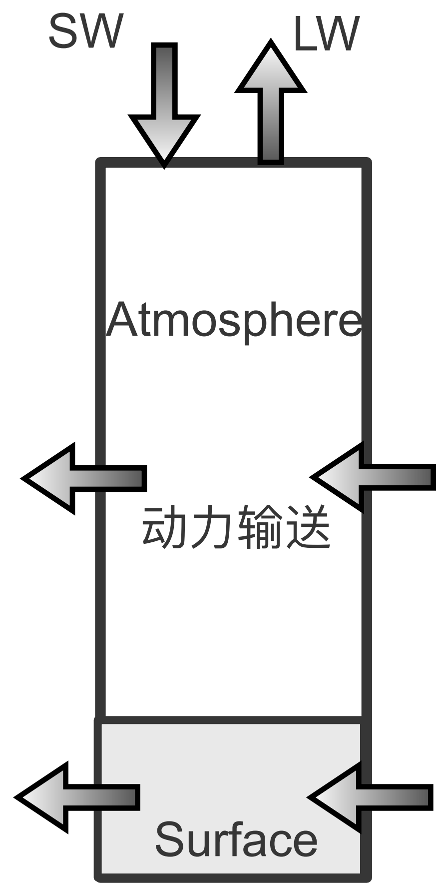
$$\text{solar radiation} = Qs(x)\mathcal{A}(T)$$

$s(x)$ – latitudinal distribution of SW, whose integral from the equator to pole is unity

$$\mathcal{C}\frac{\partial T(x,t)}{\partial t} = Qs(x)\mathcal{A}(T) - I(T) + F(T)$$

In equilibrium,

$$Qs(x)\mathcal{A}(T) - I(T) + F(T) = 0$$



Simple energy balance climate models

In equilibrium,

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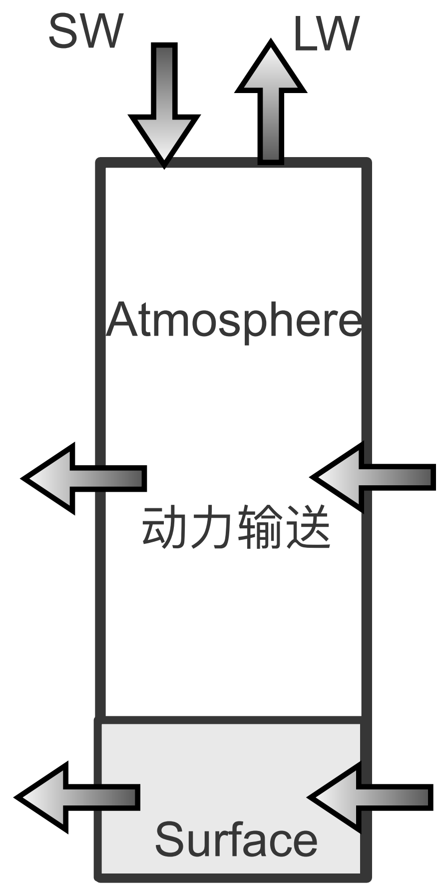

The snow line case:

- Made assumptions below:
  - Planetary albedo is assumed to depend primarily on snow /ice cover;

$$\begin{aligned}\mathcal{A}(T) &= \quad \alpha, \quad \text{for } T < T_{snow} \\ \text{or} &= \quad \beta, \quad \text{for } T > T_{snow}\end{aligned}$$

Simple energy balance climate models

In equilibrium,

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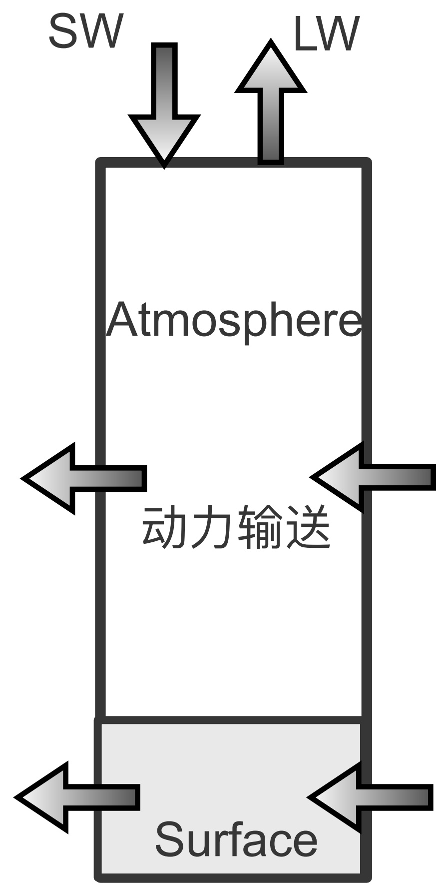


The snow line case:

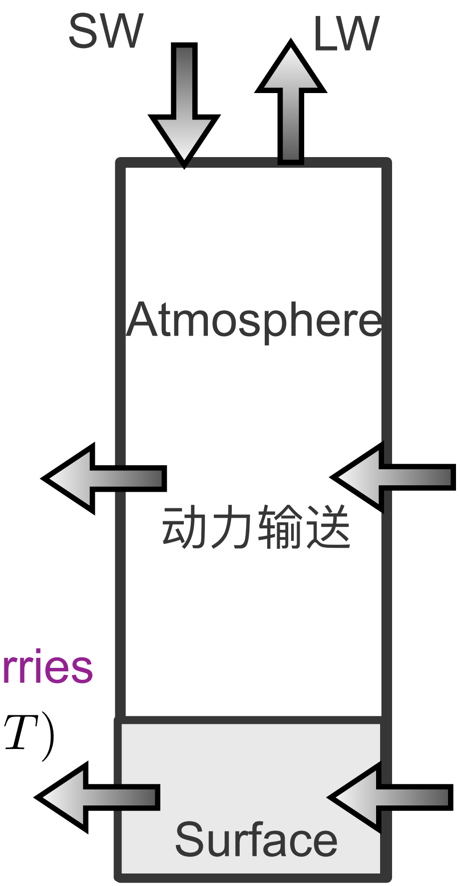
- Made assumptions below:
  - Planetary albedo is assumed to depend primarily on snow /ice cover;
  - The infrared cooling $I = A + BT$

$$\begin{aligned} \mathcal{A}(T) &= \alpha, \quad \text{for } T < T_{snow} \\ \text{or} &= \beta, \quad \text{for } T > T_{snow} \end{aligned}$$

Simple energy balance climate models

In equilibrium,

$$Qs(x)\mathcal{A}(T) - I(T) + F(T) = 0$$



The snow line case:

- Made assumptions below:
  - Planetary albedo is assumed to depend primarily on snow /ice cover;
  - The infrared cooling $I = A + BT$
  - The primary feature of the heat transport is that it carries heat from warmer to colder regions. $F(T) = C(\bar{T} - T)$

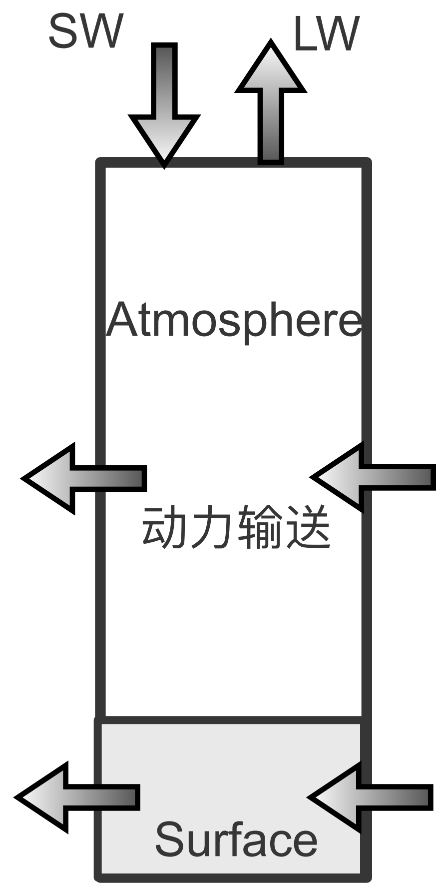
$$\begin{aligned} \mathcal{A}(T) &= \quad \alpha, \quad \text{for } T < T_{snow} \\ \text{or} &= \quad \beta, \quad \text{for } T > T_{snow} \end{aligned}$$

Note: *F(T)* is the **divergence** of heat flux

Simple energy balance climate models

In equilibrium,

$$Qs(x)\mathcal{A}(T) - I(T) + F(T) = 0$$



- One-dimensional, only latitude dependences are considered;
- Global energy budgets are assumed to be expressed in T_{sur} ;
- Planetary albedo is assumed to depend primarily on snow /ice cover;
- Only annual mean conditions are considered;
- The primary feature of the heat transport is that it carries heat from warmer to colder regions.

Budyko, M.I. (1969). The effect of solar radiation variations on the climate of the earth. *Tellus* **21**, 611-619.

Simple energy balance climate models

In equilibrium,

$$Qs(x)\mathcal{A}(T) - I(T) + F(T) = 0$$

The snow line case: infrared cooling $I = A + BT$

$$F(T) = C(\bar{T} - T)$$

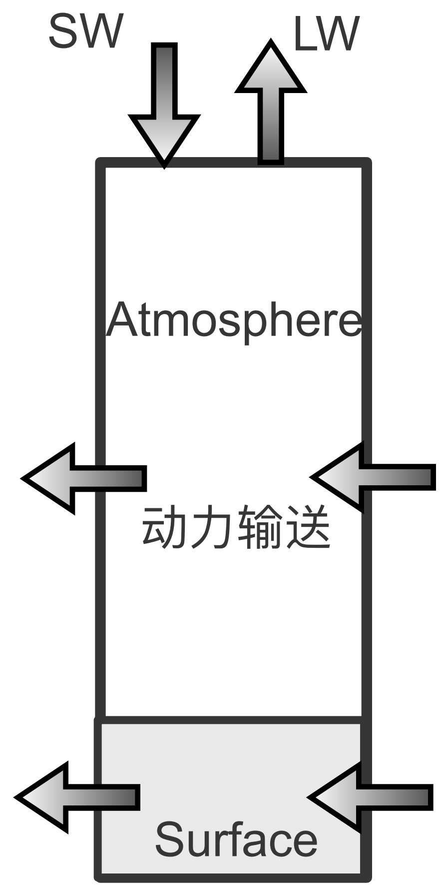
Assume:

$$\begin{aligned} \mathcal{A}(T) = & \quad \alpha = 0.4, & \text{for } T < T_{snow} \\ = & \quad \beta = 0.7, & \text{for } T > T_{snow} \\ = & \quad \tfrac{\alpha+\beta}{2}, & \text{for } T = T_{snow} \end{aligned}$$

$$T_{snow} = -10^oC$$

$$s(x) = 1 - 0.241(3x^2 - 1)$$

$$A = 211.1\, Wm^{-2}, \text{ and } B = 1.55\, Wm^{-2}(^oC)^{-1}$$



Simple energy balance climate models

In equilibrium,

$$\boxed{Qs(x)\mathcal{A}(T) - I(T) + F(T) = 0}$$

The snow line case: infrared cooling $I = A + BT$

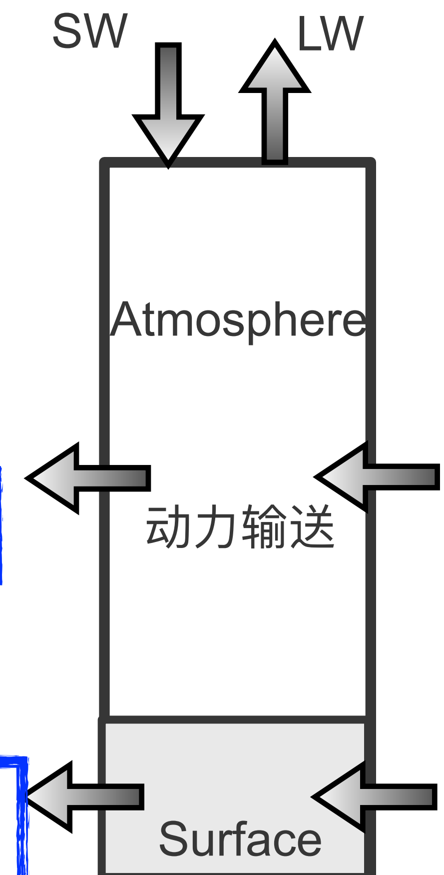
$$F(T) = C(\bar{T} - T)$$

Hemisphere average:

$$\bar{T} = \int_0^1 T dx \qquad \bar{I} = \int_0^1 I dx \qquad \boxed{F(I) = (C/B)(\bar{I} - I)}$$

Radiation balance

$$\begin{aligned} \bar{I}/Q &= \int_0^1 s(x)\mathcal{A}(x)dx \\ &= (\beta - \alpha)(1.241x_s - 0.241x_s^3) + \alpha \end{aligned}$$



Simple energy balance climate models

In equilibrium,

$$\boxed{Qs(x)\mathcal{A}(T) - I(T) + F(T) = 0}$$

The snow line case:

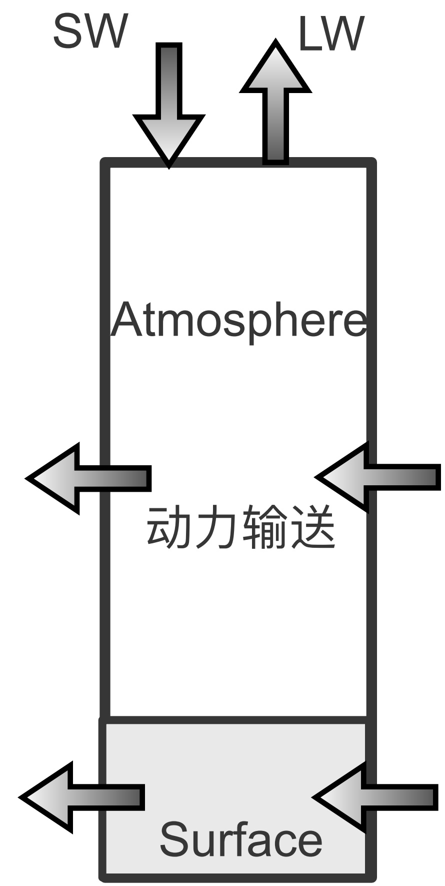
$$I/Q = \frac{\frac{C}{B}\bar{I}/Q + s(x)\mathcal{A}(x, x_s)}{1 + \frac{C}{B}}$$

Determine C using current climate:

$$I(x_s) = I(0.95) = I(T_{snow}), \text{ get the value of } \frac{C}{B}$$

Then

$$Q(x_s) = \frac{(1 + \frac{C}{B})(A + BT_{snow})}{\frac{C}{B}\bar{I}/Q + s(x_s)\frac{\alpha+\beta}{2}}$$



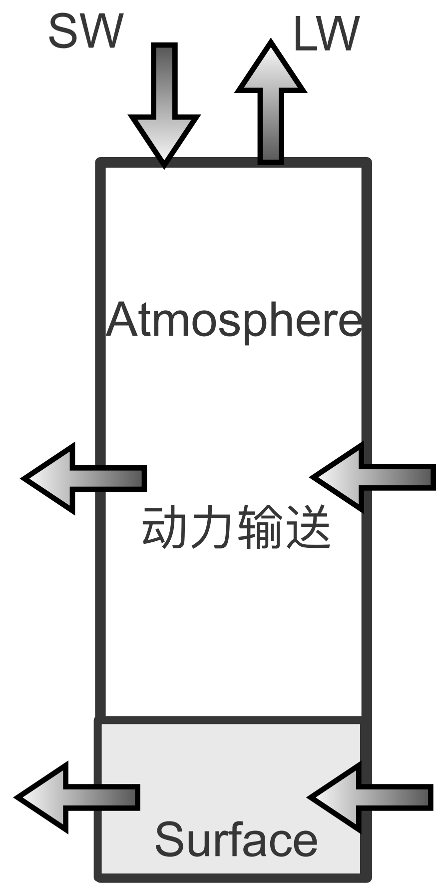
Simple energy balance climate models

The snow line case:

$$Q(x_s) = \frac{(1+\frac{C}{B})(A+BT_{snow})}{\frac{C}{B}\bar{I}/Q + s(x_s)\frac{\alpha+\beta}{2}}$$

The denominator:

$$\begin{aligned} den &= \frac{\alpha+\beta}{2}\times 1.241 + \alpha\frac{C}{B} \\ &+\frac{C}{B}(\beta-\alpha)\times 1.241x_s \\ &-\frac{\alpha+\beta}{2}\times 0.723x_s^2 \\ &-\frac{C}{B}(\beta-\alpha)\times 0.241x_s^3 \end{aligned}$$



Simple energy balance climate models

The snow line case:

$$Q(x_s) = \frac{(1+\frac{C}{B})(A+BT_{snow})}{\frac{C}{B}\bar{I}/Q + s(x_s)\frac{\alpha+\beta}{2}}$$

$$\begin{aligned} den &= \boxed{\frac{\alpha+\beta}{2}\times 1.241} + \alpha\frac{C}{B} \\ &+ \frac{C}{B}(\beta-\alpha)\times 1.241 x_s \\ &\boxed{-\frac{\alpha+\beta}{2}\times 0.723 x_s^2} \\ &- \frac{C}{B}(\beta-\alpha)\times 0.241 x_s^3 \end{aligned}$$

If C=0, no heat flux, radiative equilibrium, then as x_s increases, den. decreases, Q increases.

太阳辐射越强，冰雪线越向两极移动

If C is nonzero,

Simple energy balance climate models

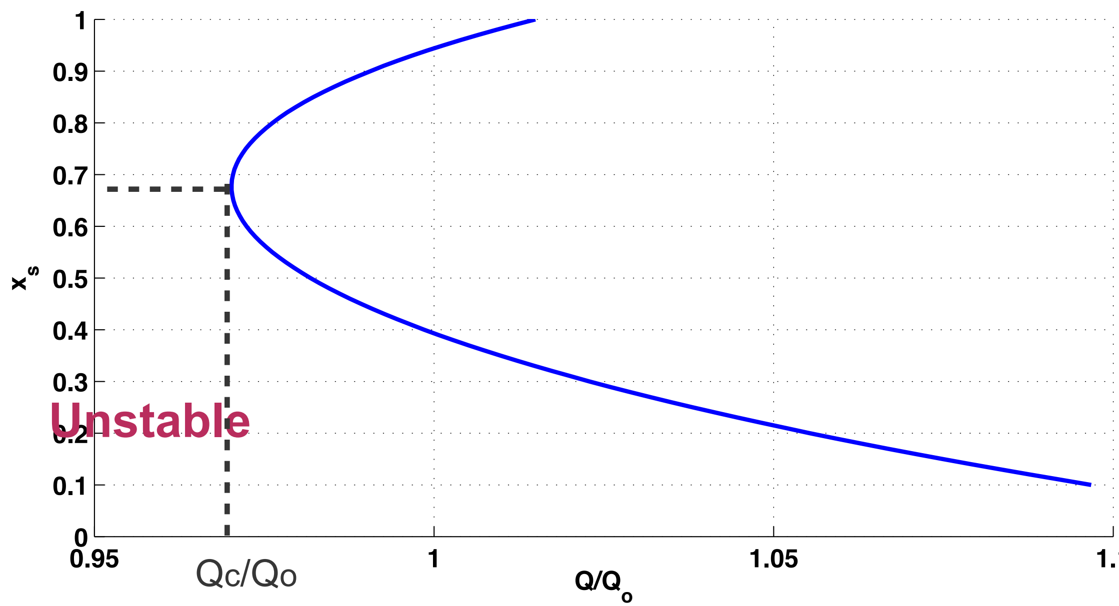
The snow line case:

$$Q(x_s) = \frac{(1+\frac{C}{B})(A+BT_{snow})}{\frac{C}{B}\bar{I}/Q + s(x_s)\frac{\alpha+\beta}{2}}$$

If C is nonzero,

The destabilizing effect of heat transport

There is a minimum value of Q, below which the climate will unstably proceed to a snow/ice covered earth.



Simple energy balance climate models

The snow line case:

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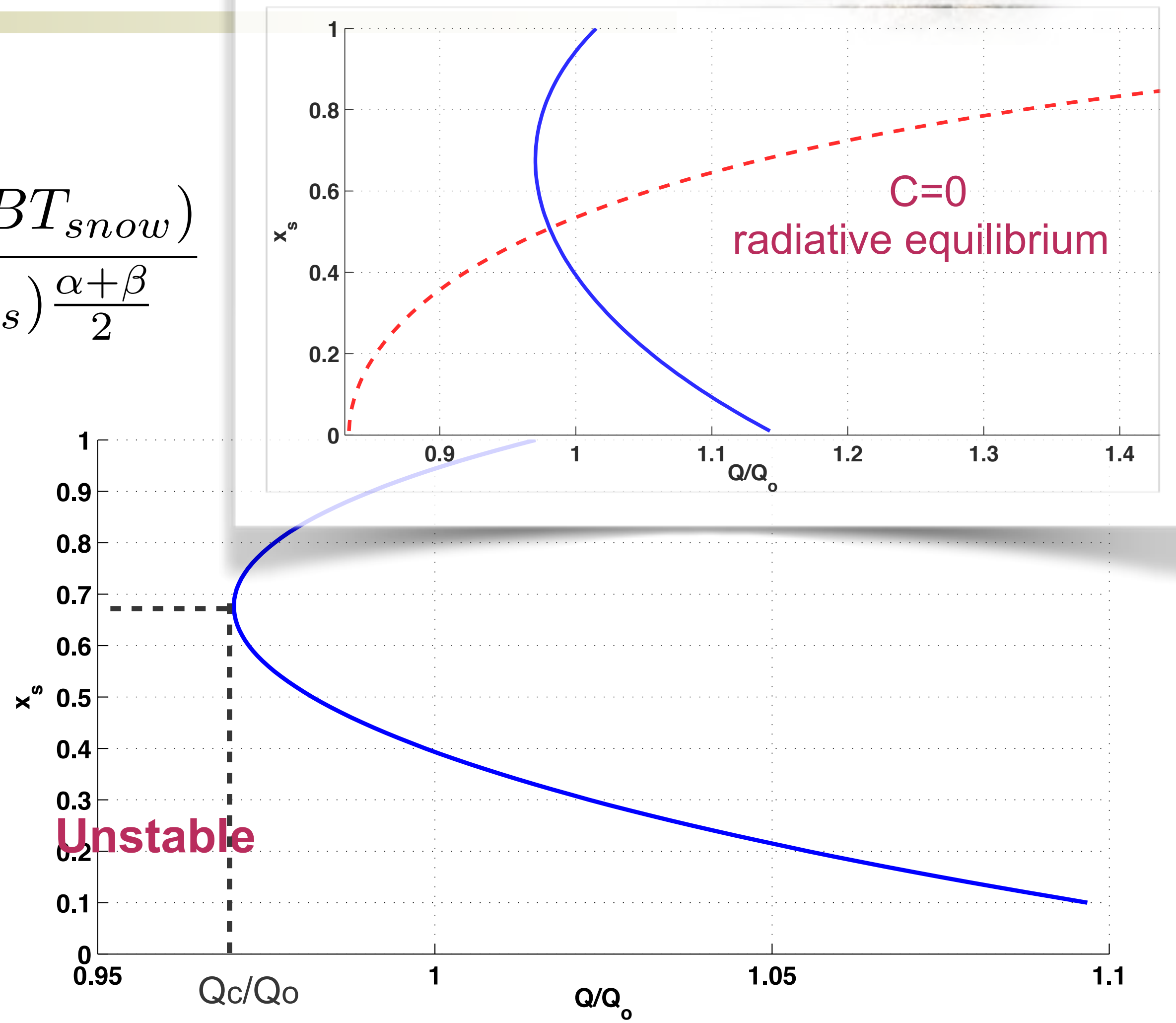

Simple energy balance climate models

Assignment 2, Fall 2022

请在题目一和题目二中任选一题作为本章的作业题目。另外的一题可作为选做题。

Question 1 [edit | edit source]

假设在大气层顶（TOA），在多年全年平均的情况下，入射的太阳辐射随纬度的分布满足

$$Q = Q_o \cdot s(x),$$
$$s(x) = s_o \cdot P_o(x) + s_2 \cdot P_2(x),$$

其中，$P_o(x) = 1, P_2(x) = \frac{1}{2}(3x^2 - 1), s_o = 1, s_2 = -0.473, x = sin\phi, \phi$ 为纬度。

如果假设大气层顶的向外净长波辐射为I，其随纬度的分布满足

$$I = Q_o \cdot i(x),$$
$$i(x) = i_o \cdot P_o(x) + i_2 \cdot P_2(x),$$

其中，$i_o = 0.687, i_2 = -0.165$。假设行星反照率为$\alpha$，且忽略它随纬度和经度的变化:

1. 请写出在能量平衡的情况下，大气和海洋的总经向能量输送应满足什么条件?
2. 按此条件，请利用合适的边界条件，请推导出大气和海洋的总经向能量输送将如何随纬度ϕ和太阳辐射强度Q_o变化。

Question 2 [edit | edit source]

在第二章中，我们学习了Budyko的能量平衡气候模型（Simple Energy Balance Climate Model），并用此模型来讨论了存在冰雪线的情况下，气候系统的稳定性问题。冰雪线的南北移动会引起行星反照率及热量的动力输送的变化。我们发现这两种过程的相互作用能够引起气候系统的不稳定。如果使用同样的参数，即$A = 211.1W/m^2$, $B = 1.55W/(m^{2\circ}\mathrm{C}), Q_o = 340W/m^2$，$T_{snow} = -10^\circ\mathrm{C}, s(x) = 1 - 0.241(3x^2 - 1)$，并同样假设:

$$A(T) = \begin{cases} \alpha & T < T_{\text{snow}}, \\ \beta & T > T_{\text{snow}}, \\ \dfrac{\alpha+\beta}{2} & T = T_{\text{snow}}. \end{cases}$$

如果行星反照率发生了改变，使得$\alpha = 0.45$，请讨论:

1. 为了保持全球平均温度$\bar{T}$不变，对于当前的Q和冰雪线，β应该取多少?
2. 对于新的 α和β值，C的取值应该为多少?
3. 请计算出新的$Q(x_s)$。
4. 请与课堂上所讲的$\alpha = 0.4, \beta = 0.7$的情况相比较（请将两种情况下$Q(x_s)$的曲线画在同一张图上），讨论两种情况的异同，尤其是气候稳定性在α和β值改变后发生了何种变化，请讨论为什么会发生这些变化?