第二章：

大气环流的外部强迫(III)

－Simple energy balance climate model

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大气环流的外部强迫（III）

Reference reading:
Lindzen 2005, Chapter 2

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The meridional energy transport by atmosphere and oceans can be described by the following equation:

\[
F_{rad}^{\text{top}} = \frac{1}{2\pi a^2 \cos \phi} \left( \frac{\partial}{\partial \phi} f(\phi) \right)
\]

Where \[f(\phi)\] represents the meridional energy transport by atmosphere and oceans.
\[ F_{\text{rad}}^{\text{top}} = \frac{1}{2\pi a^2 \cos \phi} \frac{\partial f(\phi)}{\partial \phi} \]

Wunsch (2005), J. Climate
\[ F_{rad}^{\text{top}} = \frac{1}{2\pi a^2 \cos \phi} \frac{\partial}{\partial \phi} f(\phi) \]

Atmosphere:
\[ F_{rad}^{\text{top}} - F_{rad}^{\text{sfc}} + F_{LH} + F_{SH} = \frac{1}{2\pi a^2 \cos \phi} \frac{\partial}{\partial \phi} f_A(\phi) \]

Ocean:
\[ F_{rad}^{\text{sfc}} - F_{LH} - F_{SH} = \frac{1}{2\pi a^2 \cos \phi} \frac{\partial}{\partial \phi} f_O(\phi) \]

Wunsch (2005), J. Climate
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\[ F_{\text{rad}}^{\text{top}} = \frac{1}{2\pi a^2 \cos \phi} \partial f(\phi) \]

Wunsch (2005), J. Climate
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- Simplest models in which the interactions between radiation and dynamic heat transport can be considered.

- Assumptions are made below:
  - One-dimensional, only latitude dependences are considered;
  - Global energy budgets are assumed to be expressed in $T_{sur}$;
  - Only annual mean conditions are considered;

$$c \frac{\partial T(x, t)}{\partial t} = \text{solar radiation} - \text{infrared cooling}$$

- divergence of heat flux

$$x = \sin \phi, \text{ where } \phi \text{ is latitude.}$$

$$c \frac{\partial T(x, t)}{\partial t} = F_{rad}^{top} - \frac{1}{2\pi a^2} \frac{\partial}{\partial x} f(x)$$
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\[ C \frac{\partial T(x,t)}{\partial t} = \text{solar radiation} - \text{infrared cooling} \]

- divergence of heat flux

\[ x = \sin \phi, \text{ where } \phi \text{ is latitude.} \]

solar radiation \( = Q_s(x)A(T) \)

\[ s(x) \rightarrow \text{latitudinal distribution of SW, whose integral from} \]
\[ \text{the equator to pole is unity} \]

\[ C \frac{\partial T(x,t)}{\partial t} = Q_s(x)A(T) - I(T) + F(T) \]

In equilibrium,

\[ Q_s(x)A(T) - I(T) + F(T) = 0 \]
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In equilibrium,

\[ Q_s(x)A(T) - I(T) + F(T) = 0 \]

The snow line case:
- Made assumptions below:
  - Planetary albedo is assumed to depend primarily on snow/ice cover;

\[ A(T) = \alpha, \quad \text{for } T < T_{snow} \]
\[ \quad \text{or} = \beta, \quad \text{for } T > T_{snow} \]
In equilibrium, 

$$Q_s(x)A(T) - I(T) + F(T) = 0$$

The snow line case:

- Made assumptions below:
  - Planetary albedo is assumed to depend primarily on snow / ice cover;
  - The infrared cooling $I = A + BT$

$$A(T) = \alpha, \quad \text{for } T < T_{snow}$$

$$\text{or } = \beta, \quad \text{for } T > T_{snow}$$
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In equilibrium,

\[ Q_s(x)A(T) - I(T) + F(T) = 0 \]

The snow line case:

- Made assumptions below:
  - Planetary albedo is assumed to depend primarily on snow / ice cover;
  - The infrared cooling \( I = A + BT \);
  - The primary feature of the heat transport is that it carries heat from warmer to colder regions. \( F(T) = C(\bar{T} - T) \)

\[ A(T) = \alpha, \quad \text{for } T < T_{\text{snow}} \]
\[ \text{or } = \beta, \quad \text{for } T > T_{\text{snow}} \]

Note: \( F(T) \) is the divergence of heat flux
In equilibrium,

\[ Q_s(x)A(T) - I(T) + F(T) = 0 \]

- One-dimensional, only latitude dependences are considered;
- Global energy budgets are assumed to be expressed in \( T_{sur} \);
- Planetary albedo is assumed to depend primarily on snow/ice cover;
- Only annual mean conditions are considered;
- The primary feature of the heat transport is that it carries heat from warmer to colder regions.

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In equilibrium,

\[ Qs(x)A(T) - I(T) + F(T) = 0 \]

The snow line case: **infrared cooling**

\[ I = A + BT \]

\[ F(T) = C(\bar{T} - T) \]

Assume:

\[ A(T) = \alpha = 0.4, \quad \text{for } T < T_{\text{snow}} \]

\[ = \beta = 0.7, \quad \text{for } T > T_{\text{snow}} \]

\[ = \frac{\alpha + \beta}{2}, \quad \text{for } T = T_{\text{snow}} \]

\[ T_{\text{snow}} = -10^\circ C \]

\[ s(x) = 1 - 0.241(3x^2 - 1) \]

\[ A = 211.1 \text{ Wm}^{-2}, \text{ and } B = 1.55 \text{ Wm}^{-2}(^\circ C)^{-1} \]
In equilibrium,

\[ Q_s(x)A(T) - I(T) + F(T) = 0 \]

The snow line case: infrared cooling \( I = A + BT \)

\[ F(T) = C(\bar{T} - T) \]

Hemisphere average:

\[ \bar{T} = \int_0^1 T \, dx \quad \bar{I} = \int_0^1 I \, dx \quad F(I) = (C/B)(\bar{I} - I) \]

Radiation balance

\[ \frac{\bar{I}}{Q} = \int_0^1 s(x)A(x) \, dx = (\beta - \alpha)(1.241x_s - 0.241x_s^3) + \alpha \]
In equilibrium,

\[ Q_s(x)A(T) - I(T) + F(T) = 0 \]

The snow line case:

\[ I/Q = \frac{C}{B} \frac{\bar{I}/Q + s(x)A(x, x_s)}{1 + C/B} \]

Determine C using current climate:

\[ I(x_s) = I(0.95) = I(T_{snow}), \text{ get the value of } \frac{C}{B} \]

Then

\[ Q(x_s) = \frac{(1 + \frac{C}{B})(A + BT_{snow})}{\frac{C}{B} \bar{I}/Q + s(x_s)\frac{C + \beta}{2}} \]
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The snow line case:

\[ Q(x_s) = \frac{(1 + \frac{C}{B})(A + BT_{\text{snow}})}{\frac{C}{B} \bar{I}/Q + s(x_s) \frac{\alpha + \beta}{2}} \]

The denominator:

\[ \text{den} = \frac{\alpha + \beta}{2} \times 1.241 + \alpha \frac{C}{B} + \frac{C}{B} (\beta - \alpha) \times 1.241 x_s \]

\[ - \frac{\alpha + \beta}{2} \times 0.723 x_s^2 \]

\[- \frac{C}{B} (\beta - \alpha) \times 0.241 x_s^3 \]
The snow line case:

\[
Q(x_s) = \frac{(1 + \frac{C}{B})(A + BT_{snow})}{C\bar{I}/Q + s(x_s)\frac{\alpha + \beta}{\alpha}} + \frac{C}{B} \frac{(\beta - \alpha)}{Q}\times 1.241x_s
\]

If \( C = 0 \), no heat flux, radiative equilibrium, then as \( x_s \) increases, den. decreases, \( Q \) increases.

太阳辐射越强，冰雪线越向两极移动

If \( C \) is nonzero,
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The snow line case:

\[ Q(x_s) = \frac{(1 + \frac{C}{B})(A + BT_{\text{snow}})}{\frac{C}{B} \bar{I}/Q + s(x_s)\frac{\alpha + \beta}{2}} \]

If C is nonzero,

The destabilizing effect of heat transport

There is a minimum value of Q, below which the climate will unstably proceed to a snow/ice covered earth.
The snow line case:

\[ Q(x_s) = \frac{(1 + \frac{C}{B})(A + BT_{\text{snow}})}{\frac{C}{B}I/Q + s(x_s)\frac{\alpha+\beta}{2}} \]

If \( C \) is nonzero,

**The destabilizing effect of heat transport**

There is a minimum value of \( Q \), below which the climate will unstably proceed to a snow/ice covered earth.

![Graph showing the relationship between \( Q/Q_0 \) and \( x_s \)](image)
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Assignment 2, Fall 2022

Question 1

Let's consider the energy balance at the top of the atmosphere (TOA), in the long-term average, under the assumption of a constant solar constant. The balance equation is:

\[ Q = Q_\odot \cdot s(x), \]

where \( s(x) = s_o \cdot P_0(x) + s_2 \cdot P_2(x), \)

with \( P_0(x) = 1, P_2(x) = \frac{1}{2} (3\omega^2 - 1), s_o = 1, s_2 = -0.473, \omega = \sin \phi, \phi \) is the latitude.

Given that the atmospheric energy flux is \( I \), which is a function of latitude, the long-term average is:

\[ I = Q_\odot \cdot i(x), \]

where \( i(x) = i_o \cdot P_0(x) + i_2 \cdot P_2(x), \)

with \( i_o = 0.687, i_2 = -0.165 \).

1. Write an equation for the energy balance at the top of the atmosphere, considering the atmospheric energy flux as \( I \), and the solar flux as \( Q_\odot \).

2. Write an equation for the long-term average atmospheric energy flux, considering the atmospheric energy flux as \( I \), and the solar flux as \( Q_\odot \).

Question 2

In the second chapter, we learned about Budyko's energy balance climate model (Simple Energy Balance Climate Model), and use this model to discuss the existence of climate feedbacks in the coupled oceanatmosphere system. We discover the existence of climate feedbacks in the coupled oceanatmosphere system. If we use the same parameters, such as \( A = 211.1 \text{ W/m}^2, B = 1.55 \text{ W/(m}^2\text{C}), Q_\odot = 340 \text{ W/m}^2, T_{\text{snow}} = -10 \text{ C}, s(z) = 1 - 0.241(3\omega^2 - 1) \), and the assumption:

\[ A(T) = \begin{cases} 
\alpha & T < T_{\text{snow}}, \\
\beta & T > T_{\text{snow}}, \\
\frac{\alpha + \beta}{2} & T = T_{\text{snow}}.
\end{cases} \]

If the solar constant is changed, let \( \alpha = 0.45 \), please discuss:

1. What is the difference in the global average temperature if the solar constant is changed?

2. What is the difference in the global average temperature if the solar constant is changed?

3. What is the difference in the global average temperature if the solar constant is changed?

4. What is the difference in the global average temperature if the solar constant is changed?