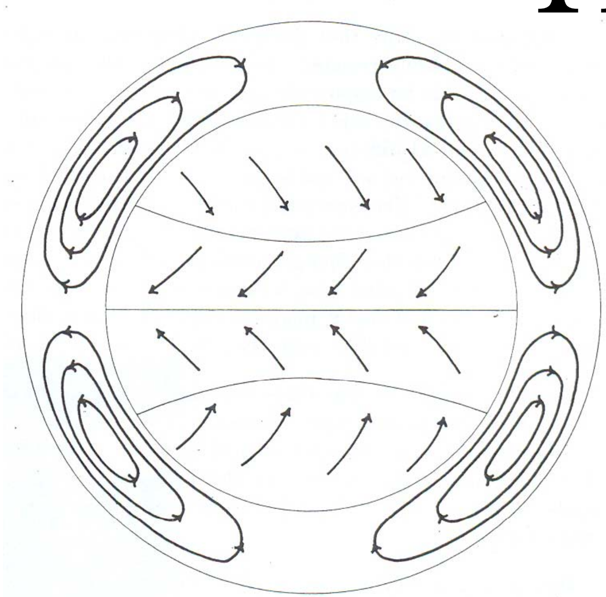
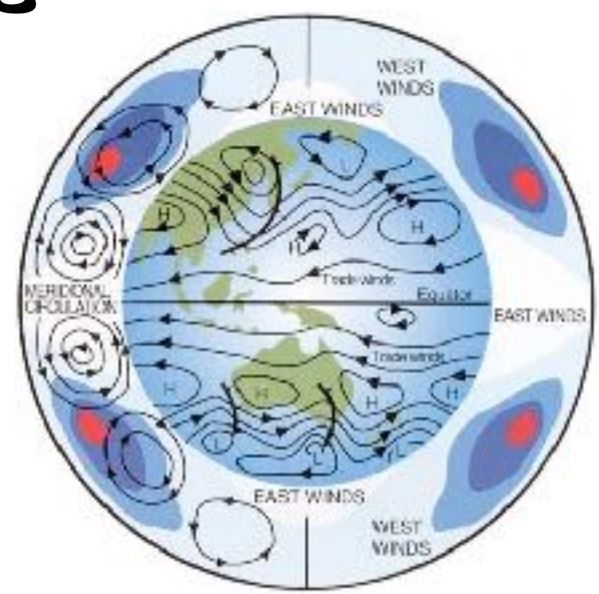
第三章:

Hadley 环流





授课教师：张洋

2022. 10. 13

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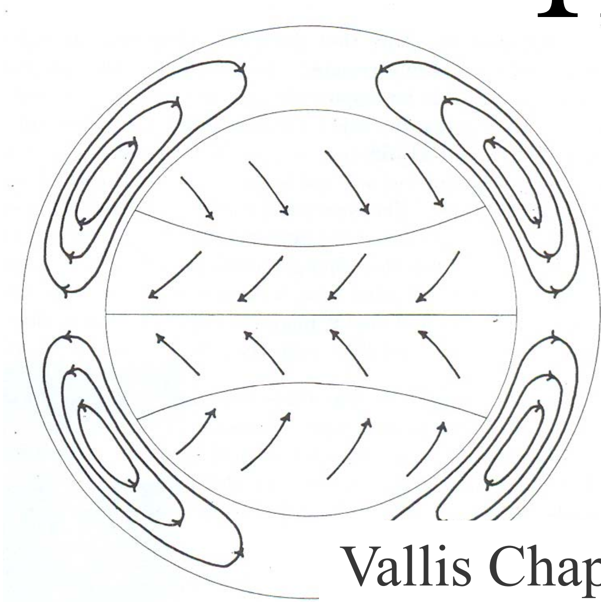

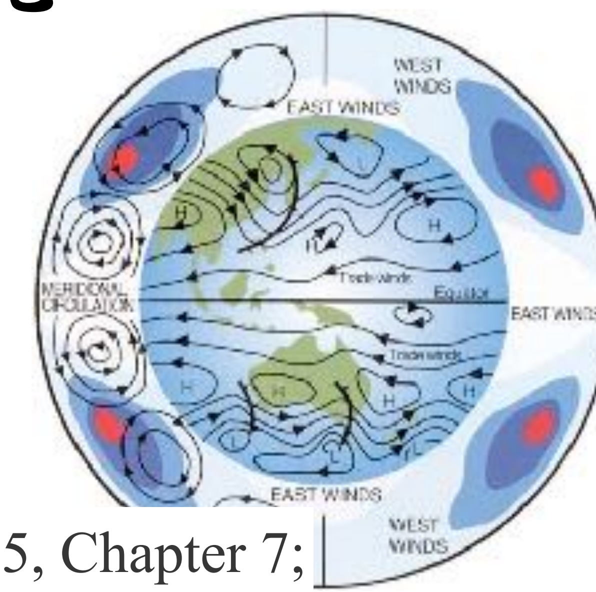

Reference reading:

Vallis Chapter 11.1-11.2; Lindzen 2005, Chapter 7;

Held and Hou 1980, JAS

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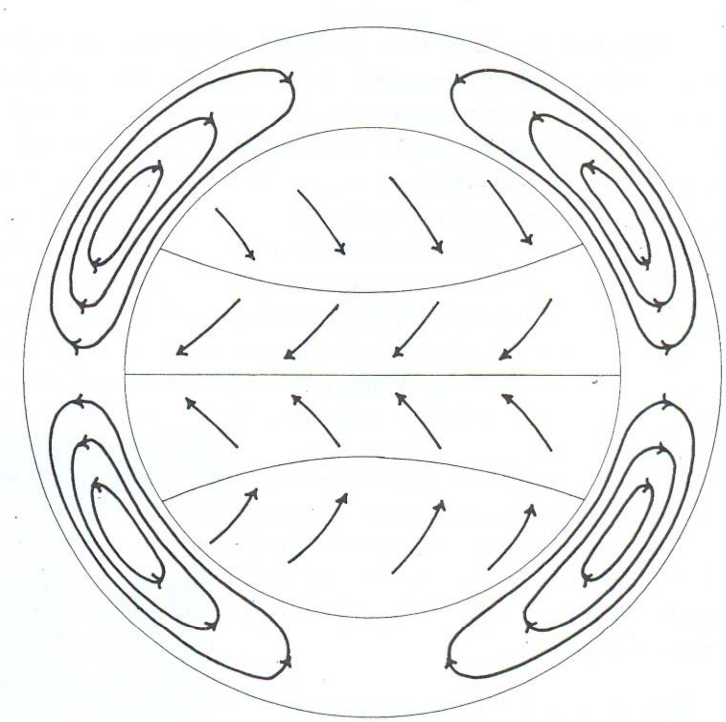
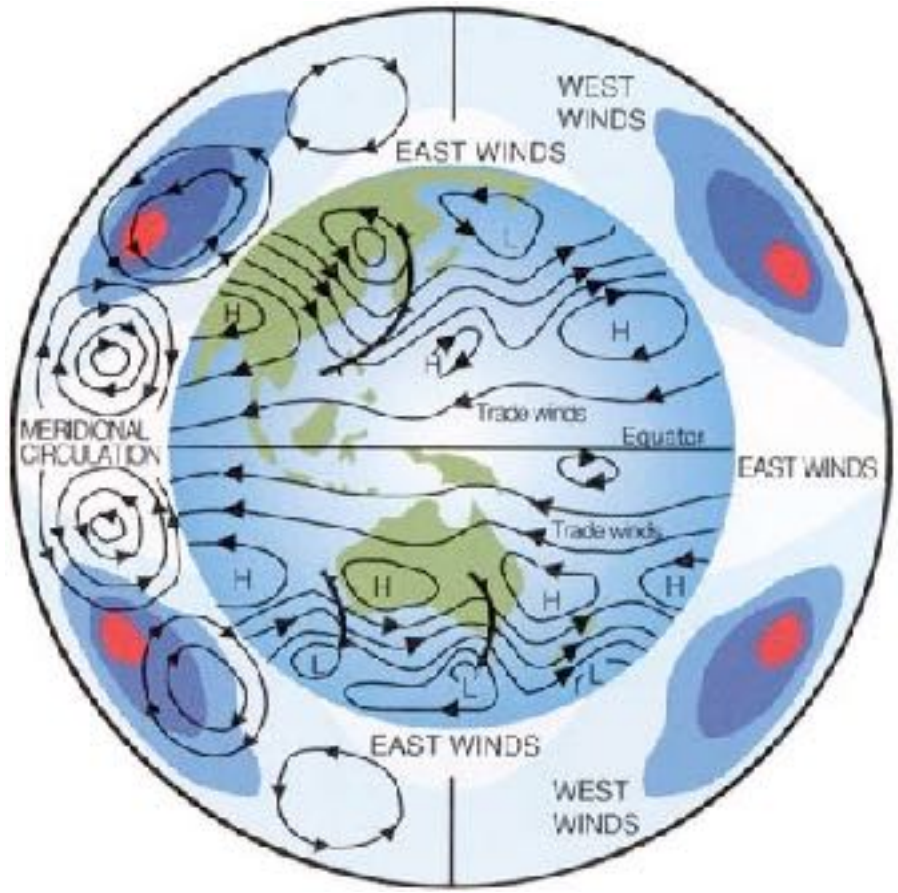
Zonally averaged meridional circulations
- Hadley Cell



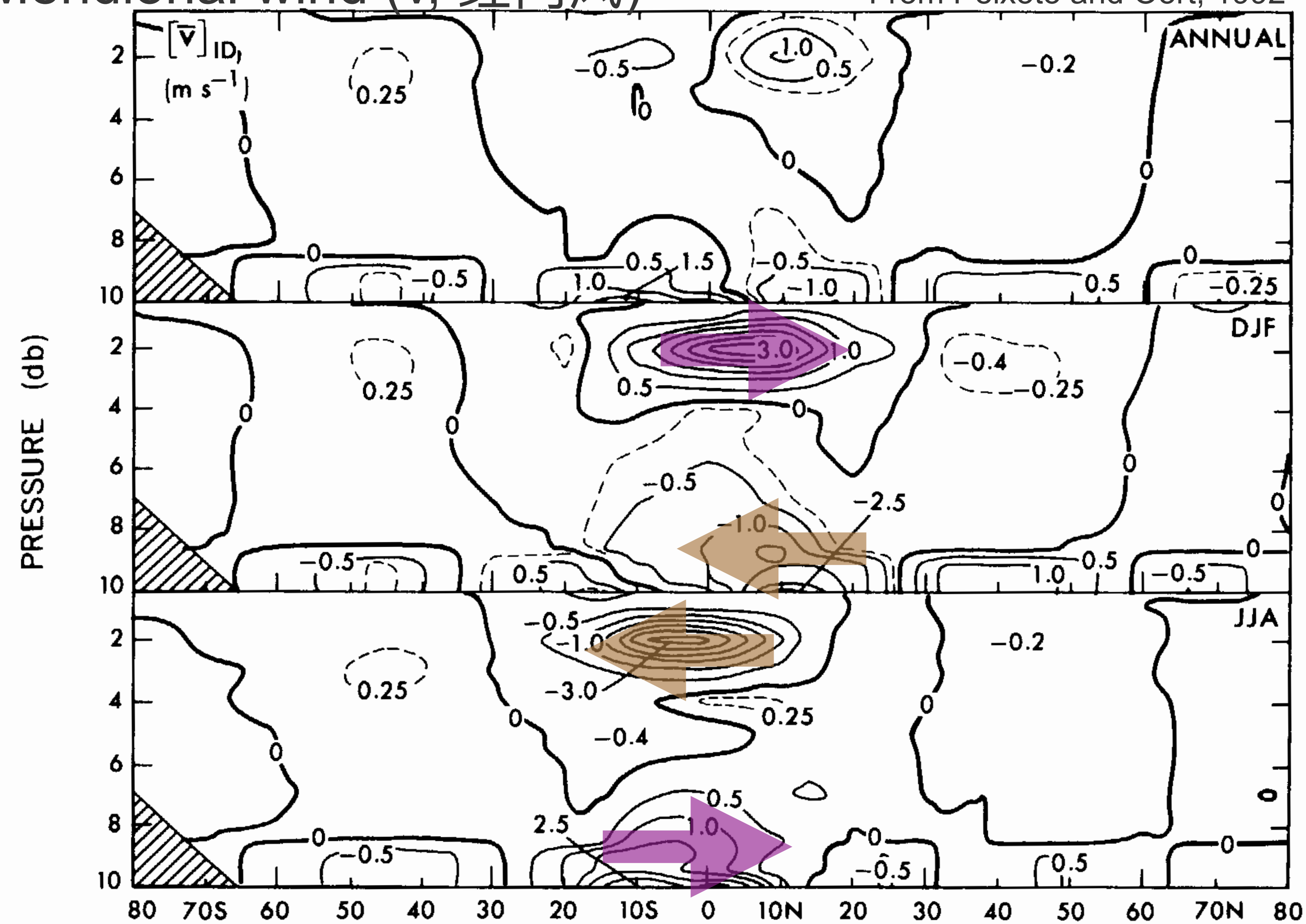


Hadley Cell

- Observations

- Meridional wind (v, 经向风)

From Peixoto and Oort, 1992

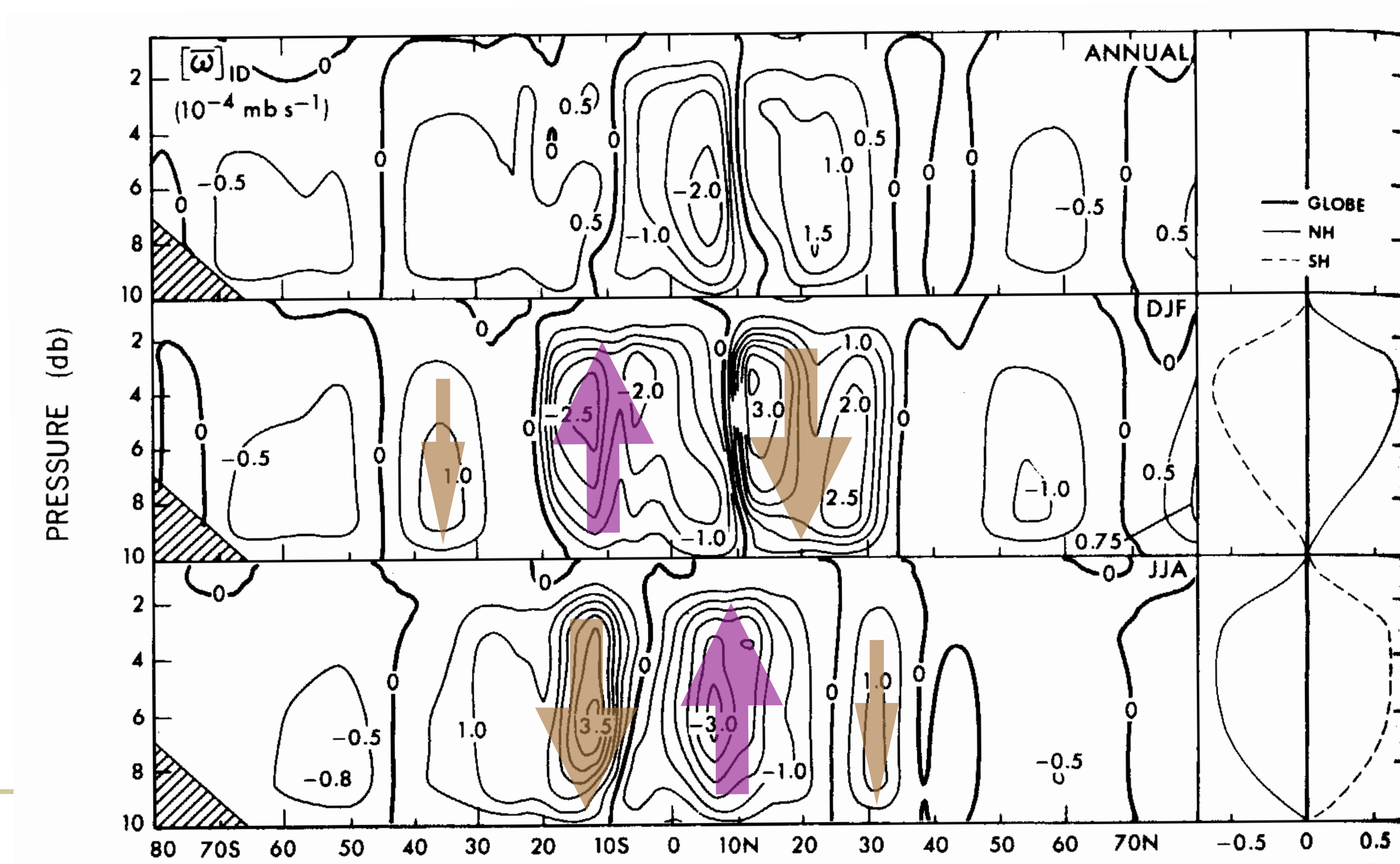


Hadley Cell

- Observations

- Vertical velocity (垂直速度)

From Peixoto and Oort, 1992



- Stream function (流函数)

纬向平均的连续方程：

$$\frac{\partial[\bar{v}]\cos\phi}{R\cos\phi\partial\phi}+\frac{\partial[\bar{\omega}]}{\partial p}=0$$

引入流函数：

$$[\bar{v}]=g\frac{\partial\psi}{2\pi R\cos\phi\partial p}$$

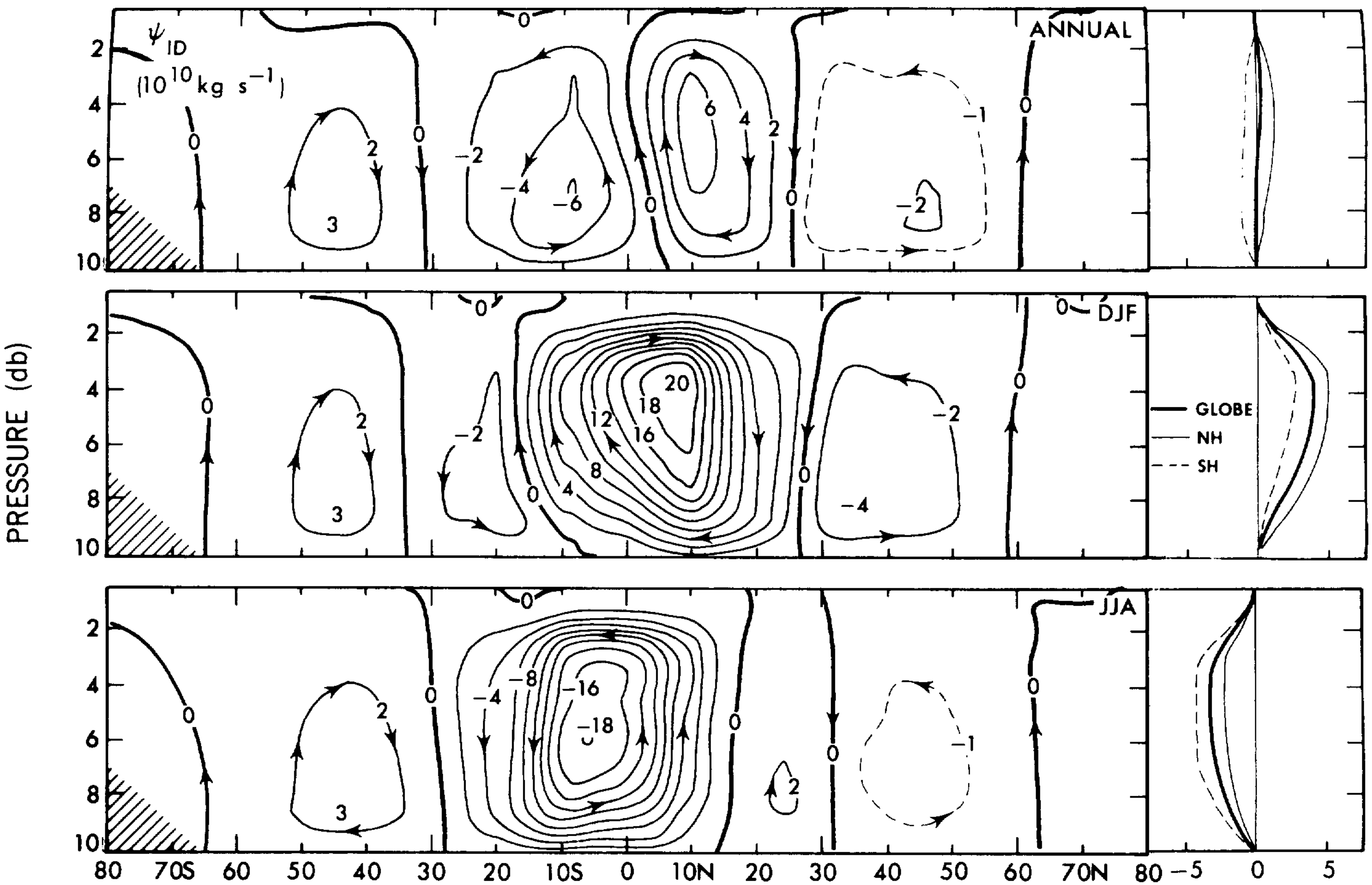
$$[\bar{\omega}]=-g\frac{\partial\psi}{2\pi R^2\cos\phi\partial\phi}$$

Hadley Cell

- Observations

- Stream function (流函数)

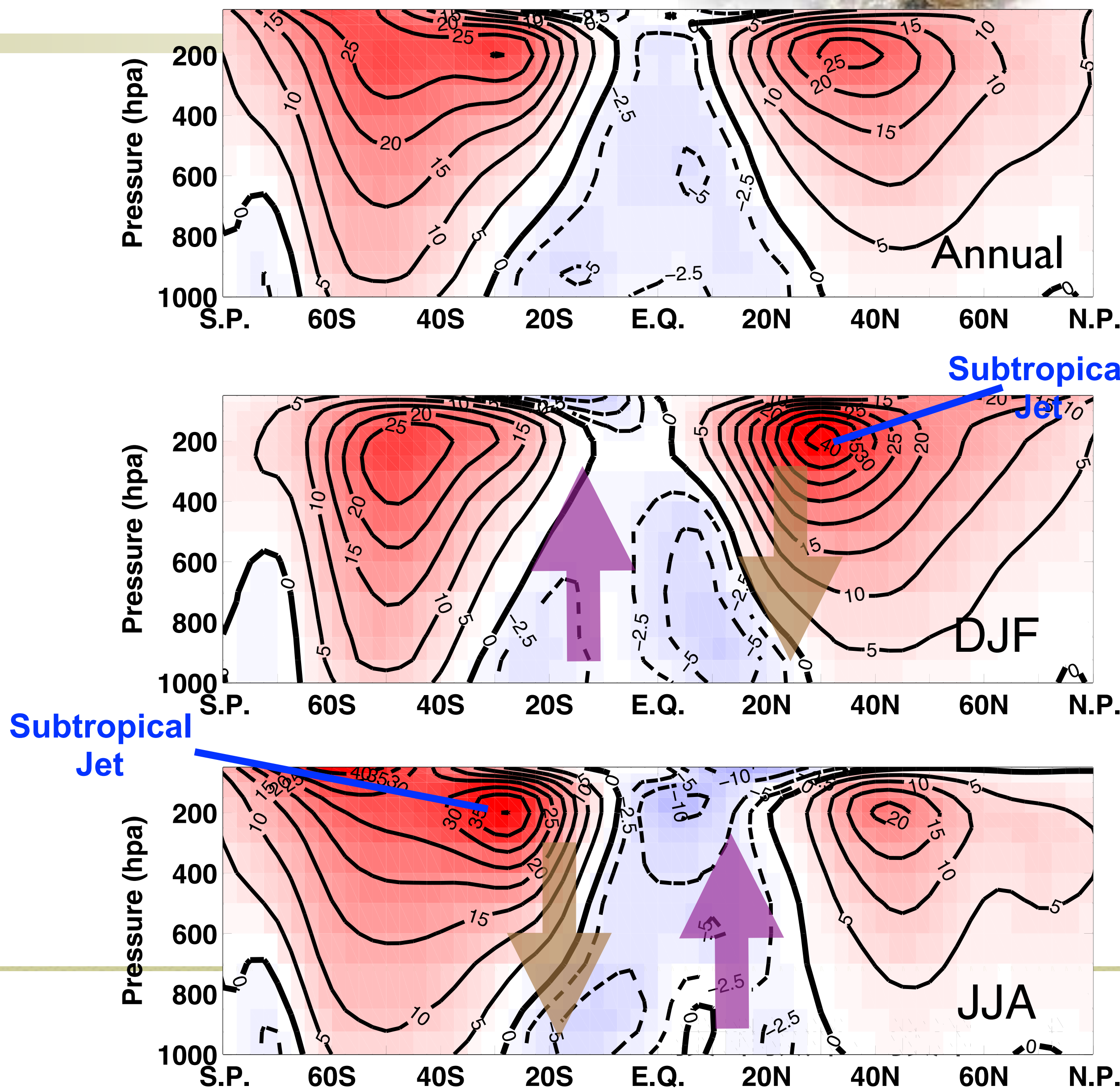
From Peixoto and Oort, 1992



Hadley Cell

- Observations

- Zonal winds (U, 纬向风)



- Observations

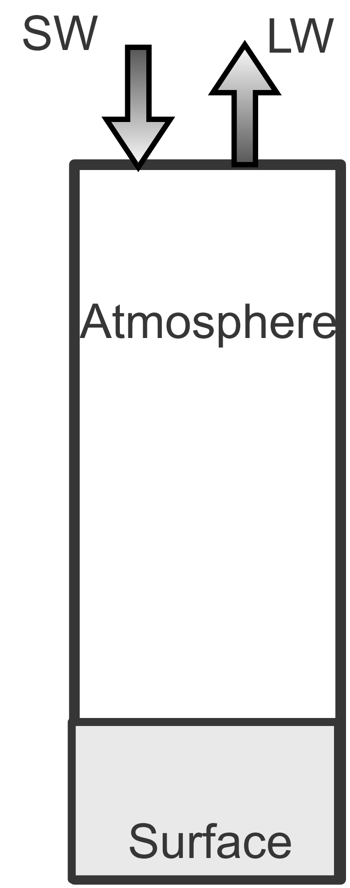
- RE temperature gradient and observed temperature distribution

$$\begin{aligned} \text{solar radiation} &= \text{infrared cooling} \\ Qs(x)\mathcal{A}(T) &= I(T) \end{aligned}$$

$$\text{infrared cooling } I = A + BT$$

or

$$\boxed{I = \sigma T_{rad}^4}$$



- Observations

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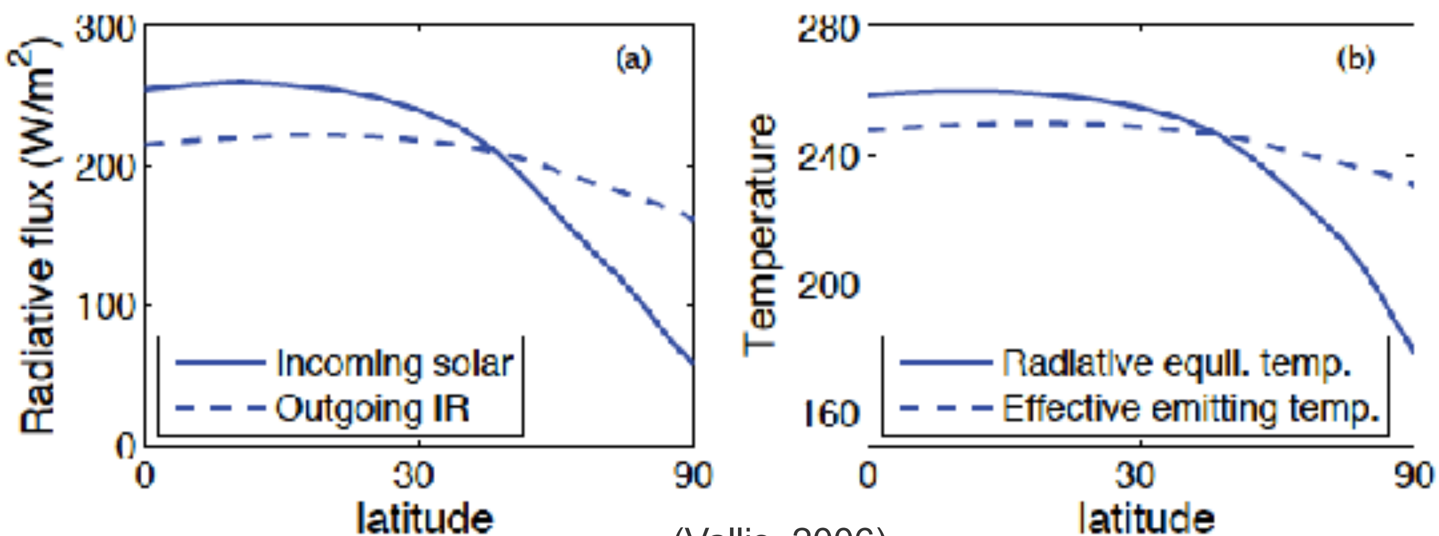
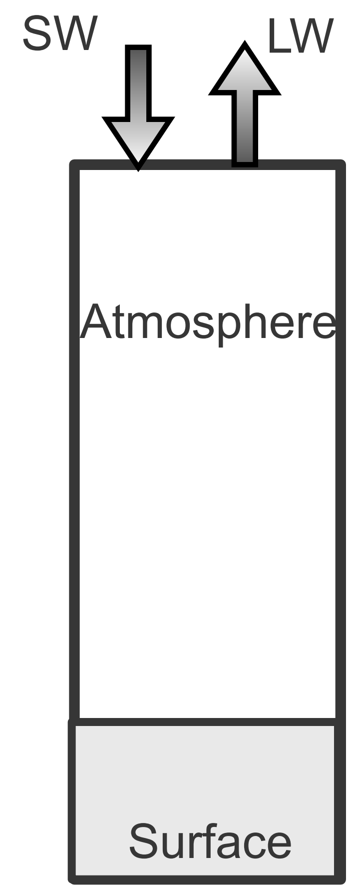


(Vallis, 2006)



- Summary （小结）
  - **Temperature field:** the equator-pole temperature gradient is much smaller than the RE temperature gradient.
  - **Wind fields:** **meridional winds** strongest at tropopause and surface; **vertical velocity** strongest at mid-level of the troposphere.
  - **Jets (zonal winds):** strong subtropical jet at **upper level** with its maximum in the latitudes at the edge or just poleward of the descending branch of the Hadley cell; **surface winds**-easterlies near the equator and westerlies in the extratropics.
  - **Strong seasonal variations:** in summer or winter, Hadley cell always appears as a strong single cell across the equator with the ascending branch in the tropics of the summer hemisphere.

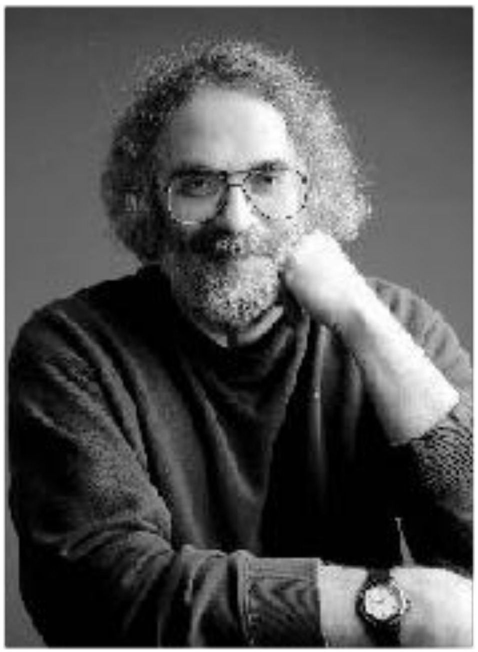
- Outline

- Observations
- Held-Hou theory (axisymmetric flow, a model that is symmetric about the equator)
- Lindzen-Hou theory (axisymmetric flow, a model that is asymmetry about the equator)
- Moisture effects
- The role of eddies
- Discussions

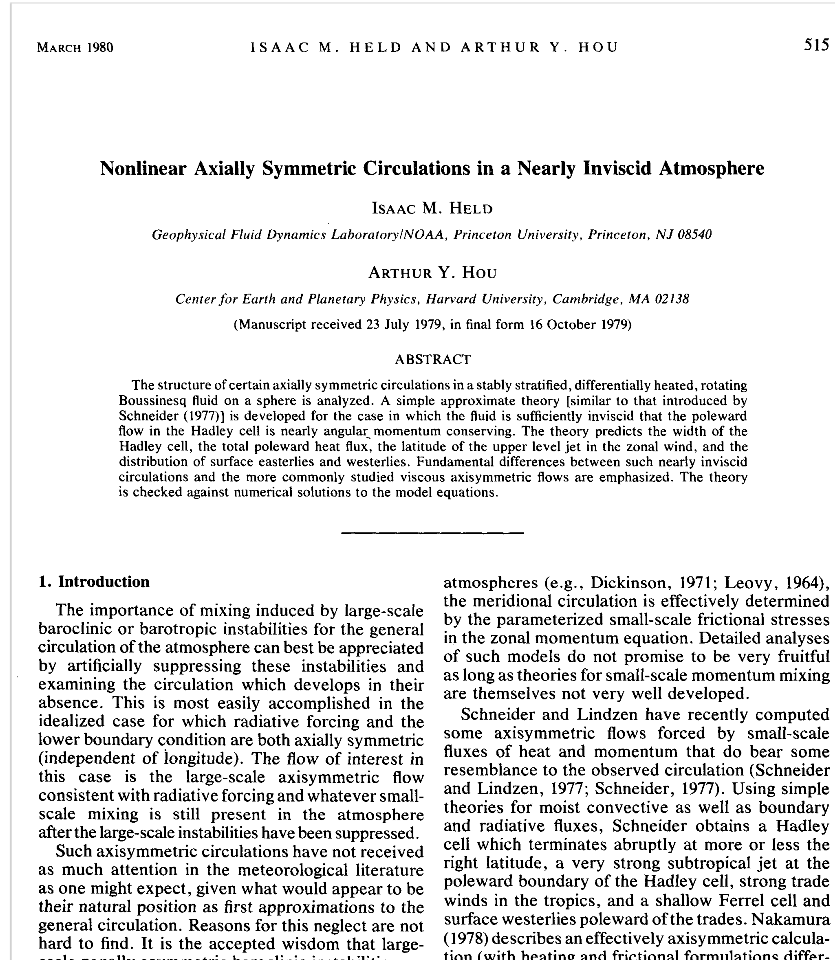
Hadley Cell

- Theories

- **Held-Hou model (1980)**



Isaac M. Held



Nonlinear Axially Symmetric Circulations in a Nearly Inviscid Atmosphere

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(Manuscript received 23 July 1979, in final form 16 October 1979)

ABSTRACT

The structure of certain axially symmetric circulations in a stably stratified, differentially heated, rotating Boussinesq fluid on a sphere is analyzed. A simple approximate theory [similar to that introduced by Schneider (1977)] is developed for the case in which the fluid is sufficiently inviscid that the poleward flow in the Hadley cell is nearly angular momentum conserving. The theory predicts the width of the Hadley cell, the total poleward heat flux, the latitude of the upper level jet in the zonal wind, and the distribution of surface easterlies and westerlies. Fundamental differences between such nearly inviscid circulations and the more commonly studied viscous axisymmetric flows are emphasized. The theory is checked against numerical solutions to the model equations.

1. Introduction

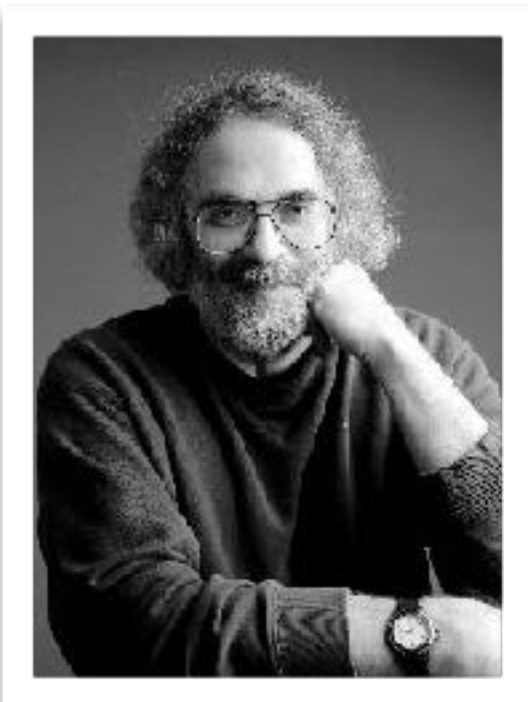
The importance of mixing induced by large-scale baroclinic or barotropic instabilities for the general circulation of the atmosphere can best be appreciated by artificially suppressing these instabilities and examining the circulation which develops in their absence. This is most easily accomplished in the idealized case for which radiative forcing and the lower boundary condition are both axially symmetric (independent of longitude). The flow of interest in this case is the large-scale axisymmetric flow consistent with radiative forcing and whatever small-scale mixing is still present in the atmosphere after the large-scale instabilities have been suppressed.

Such axisymmetric circulations have not received as much attention in the meteorological literature as one might expect, given what would appear to be their natural position as first approximations to the general circulation. Reasons for this neglect are not hard to find. It is the accepted wisdom that large-scale zonally asymmetric baroclinic instabilities are atmospheres (e.g., Dickinson, 1971; Leovy, 1964), the meridional circulation is effectively determined by the parameterized small-scale frictional stresses in the zonal momentum equation. Detailed analyses of such models do not promise to be very fruitful as long as theories for small-scale momentum mixing are themselves not very well developed.

Schneider and Lindzen have recently computed some axisymmetric flows forced by small-scale fluxes of heat and momentum that do bear some resemblance to the observed circulation (Schneider and Lindzen, 1977; Schneider, 1977). Using simple theories for moist convective as well as boundary and radiative fluxes, Schneider obtains a Hadley cell which terminates abruptly at more or less the right latitude, a very strong subtropical jet at the poleward boundary of the Hadley cell, strong trade winds in the tropics, and a shallow Ferrel cell and surface westerlies poleward of the trades. Nakamura (1978) describes an effectively axisymmetric calculation (with heating and frictional formulations differ-

Hadley Cell - Theories

- **Held-Hou model (1980)**



Isaac M. Held

Meet the model (diagram)
↓
Conservation of angular momentum
↓
Thermal wind balance
↓
Distribution of temperature
↓
Latitudinal extent of Hadley Cell
↓
Strength of Hadley Cell
↓
Distribution of upper westerly
↓
Distribution of surface winds

Hadley Cell

- Theories

- **Held-Hou model (1980)**

Make assumptions:

- the circulation is steady;
- the upper branch conserves angular momentum; surface zonal winds are weak;
- the circulation is in thermal wind balance.

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- Theories

- **Held-Hou model (1980)**

Make assumptions:

(Vallis, 2006)

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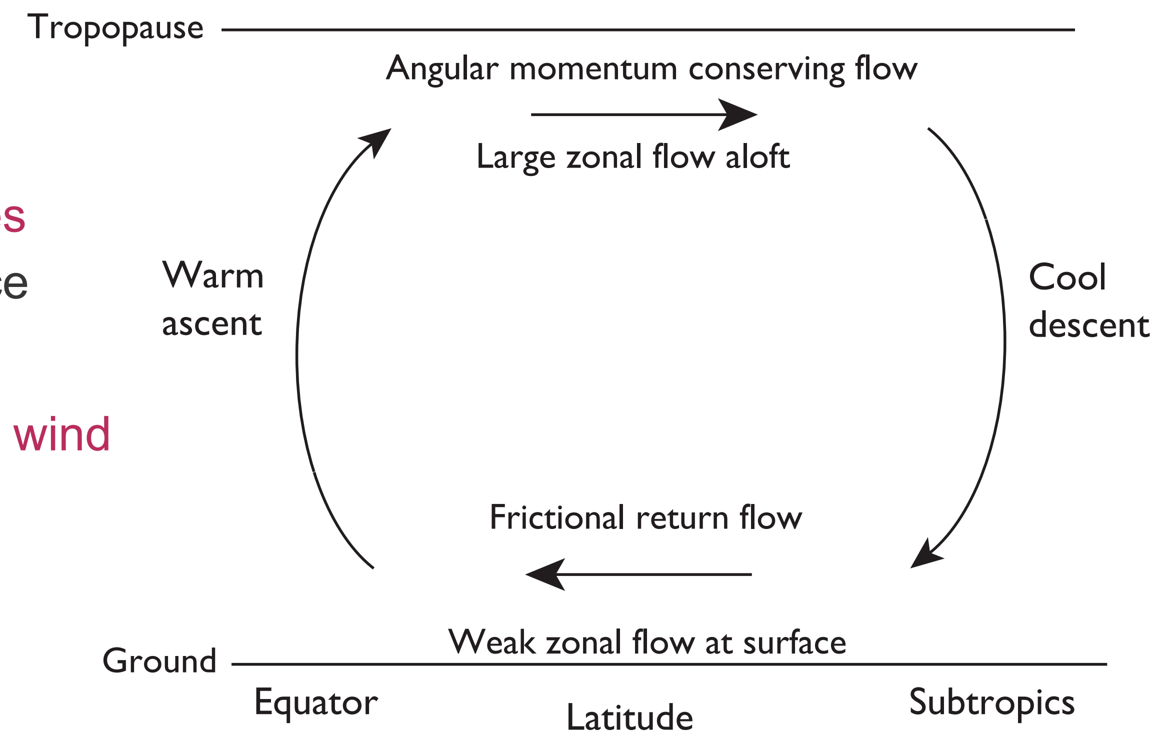
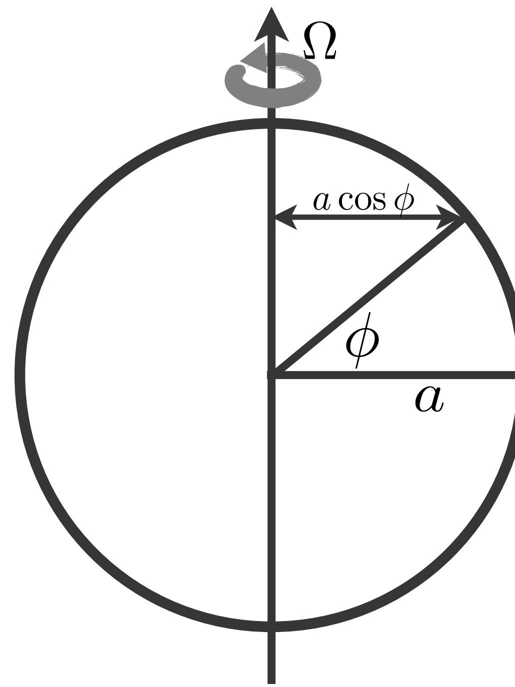


Angular momentum

- Definition (per unit mass): $\vec{M} = \vec{r} \times \vec{v}_a$

- Since the earth moves with its axis with an angular velocity $\vec{\Omega}$, the component of angular momentum about its axis is $\vec{M} \cdot \vec{n}$



- Absolute angular momentum about its axis is:

$$\begin{aligned} M = \vec{M} \cdot \vec{n} &= (\vec{r} \times \vec{v}_a) \cdot \vec{n} \\ &= [\vec{r} \times (\vec{\Omega} \times \vec{r} + \vec{v})] \cdot \vec{n} \end{aligned}$$

- After vector calculation, we have:

$$\begin{aligned} M &= M_\Omega + M_E \\ &= \Omega a^2 \cos^2\phi + ua\cos\phi \end{aligned}$$

Held-Hou model

-Angular momentum

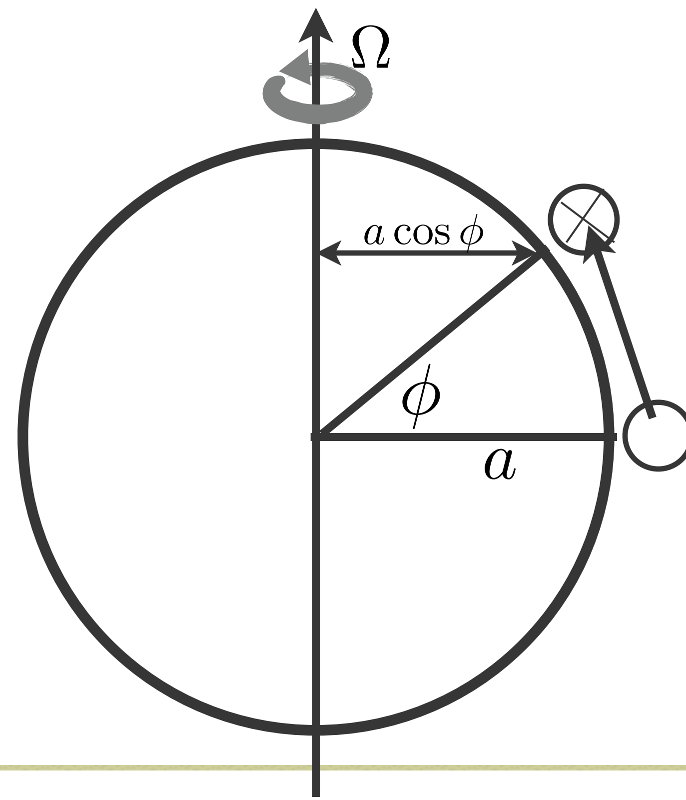
- The absolute angular momentum per unit mass is

$$M = (\Omega a \cos\phi + u) a \cos\phi$$

Due to earth's solid rotation

Deviation from the solid rotation

a is the radius of the earth



- Zonal momentum equation:

$$\frac{Du}{Dt} - fv - \frac{uv}{a}\tan\phi = -\frac{1}{a\cos\phi\rho}\frac{\partial p}{\partial\lambda} + F_\lambda$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial}{\partial\lambda} + \frac{v}{a}\frac{\partial}{\partial\phi} + w\frac{\partial}{\partial z}$$

$$\frac{D}{Dt}M = -\frac{1}{\rho}\frac{\partial p}{\partial\lambda} + a\cos\phi F_\lambda$$

Held-Hou model
-Angular momentum

- The absolute angular momentum per unit mass is

$$M = (\Omega a\cos\phi + u)a\cos\phi$$

Due to earth's solid rotation

Deviation from the solid rotation

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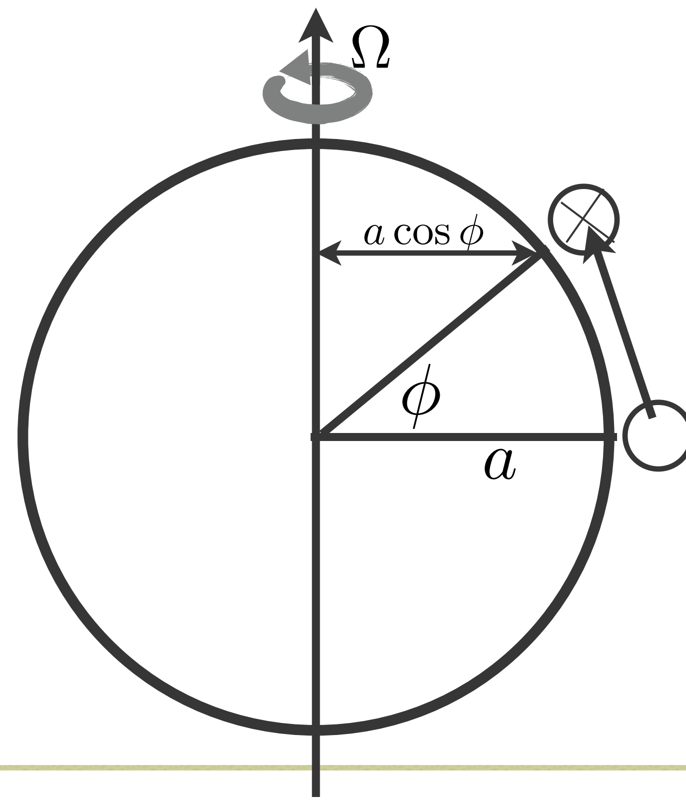


$$\frac{D}{Dt}M = -\frac{1}{\rho}\frac{\partial p}{\partial\lambda} + a\cos\phi F_\lambda$$

In an **axisymmetric flow** ([M]=M)

$$\frac{D}{Dt}[M] = a\cos\phi[F_\lambda]$$

In an inviscid (frictionless), **axisymmetric** flow, the angular momentum is conserved.

Held-Hou model

-Angular momentum

$$[M] = (\Omega a \cos\phi + [u]) a \cos\phi$$

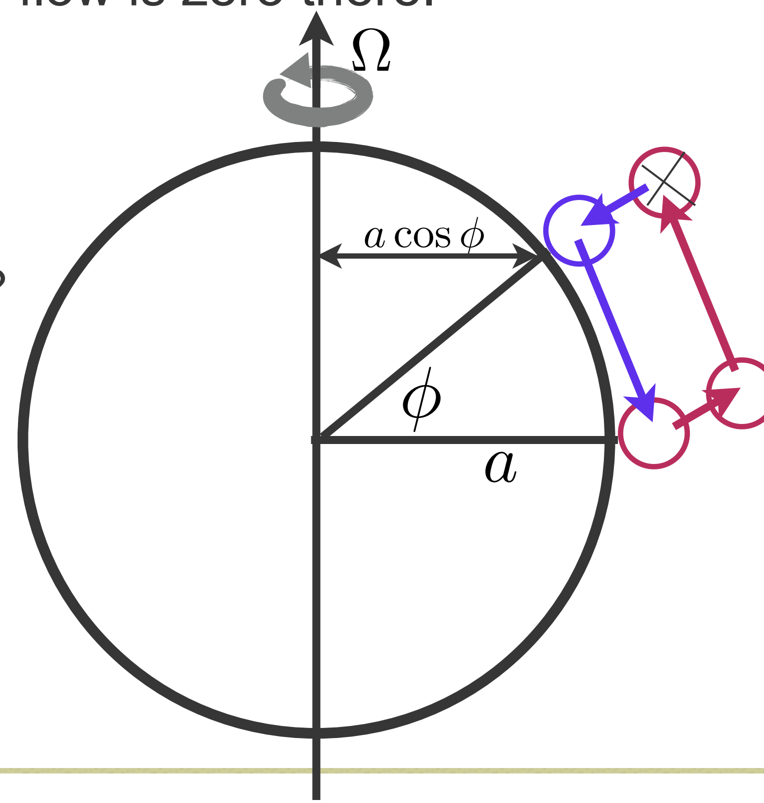
- At the equator, as the parcels rise from the surface, where the flow is weak, we assume that the zonal flow is zero there.

$$[u] = \Omega a \frac{\sin^2\phi}{\cos\phi} \equiv U_M$$

Then, what is the U_M at 10, 20, 30 degree?

Answers: 14, 57, 134 m/s, respectively

Combined with the weak surface flow, this indicates strong vertical shear of the zonal wind.



Held-Hou model
-Thermal wind relation

- Angular momentum:

$$[u] = \Omega a \frac{\sin^2\phi}{\cos\phi} \equiv U_M$$

- Thermal wind relation:

From steady state momentum equation

$$fu + \frac{u^2\tan\phi}{a} = -\frac{1}{a}\frac{\partial\Phi}{\partial\phi} \qquad \Phi = \frac{p}{\rho_s}$$

At z=H and Z=0

$$f[u(H) - u(0)] + \frac{\tan\phi}{a}[u^2(H) - u^2(0)] = -\frac{1}{a}\frac{\partial}{\partial\phi}[\Phi(H) - \Phi(0)]$$

Hydrostatic balance: $\frac{\partial\Phi}{\partial z} = g\frac{\Theta}{\Theta_o}$ Vertical integral from 0 to H $\frac{\Phi(H) - \Phi(0)}{H} = g\frac{\tilde{\Theta}}{\Theta_o}$

$\tilde{\Theta}$ - vertically averaged potential temperature

Θ_o - reference potential temperature

$$f[u(H) - u(0)] + \frac{\tan\phi}{a}[u^2(H) - u^2(0)] = -\frac{gH}{a\Theta_o}\frac{\partial\tilde{\Theta}}{\partial\phi}$$

Held-Hou model
-Thermal wind relation

- Angular momentum:

$$\boxed{[u] = \Omega a \frac{\sin^2\phi}{\cos\phi} \equiv U_M}$$

- Thermal wind relation:

$$\boxed{f[u(H) - u(0)] + \frac{\tan\phi}{a}[u^2(H) - u^2(0)] = -\frac{gH}{a\Theta_o}\frac{\partial\tilde{\Theta}}{\partial\phi}}$$

Set $u(0) = 0$

$$2\Omega\sin\phi\frac{\Omega a\sin^2\phi}{\cos\phi} + \frac{\tan\phi}{a}\frac{\Omega^2 a^2\sin^4\phi}{\cos^2\phi} = -\frac{gH}{a\Theta_o}\frac{\partial\tilde{\Theta}}{\partial\phi}$$

Integrate with respect to ϕ

$$\boxed{\frac{\tilde{\Theta}(0) - \tilde{\Theta}(\phi)}{\Theta_o} = \frac{\Omega^2 a^2}{2gH}\frac{\sin^4\phi}{\cos^2\phi}}$$

Held-Hou model
-Temperature distribution

- Angular momentum:

$$[u] = \Omega a \frac{\sin^2\phi}{\cos\phi} \equiv U_M$$

- Thermal wind relation:

$$f[u(H) - u(0)] + \frac{\tan\phi}{a}[u^2(H) - u^2(0)] = -\frac{gH}{a\Theta_o}\frac{\partial\tilde{\Theta}}{\partial\phi}$$

$$\frac{\tilde{\Theta}(0) - \tilde{\Theta}(\phi)}{\Theta_o} = \frac{\Omega^2 a^2}{2gH}\frac{\sin^4\phi}{\cos^2\phi}$$

Conservation of angular momentum and the maintenance of thermal wind completely determine the variation of temperature within the Hadley Cell !

Held-Hou model
-Extent of Hadley Cell

- Thermodynamics

$$\frac{D\Theta}{Dt} = \frac{\Theta_E - \Theta}{\tau}$$

Newtonian cooling: the cooling rate linearly depends on the local temperature perturbation

- Radiative equilibrium temperature

$$\frac{\Theta_E(\phi, z)}{\Theta_o} = 1 - \frac{2}{3}\Delta_H P_2(\sin\phi) + \Delta_v(\frac{z}{H} - \frac{1}{2})$$

Δ_H, Δ_V - fractional temperature difference between equator and pole, ground and top of the flow

P_2 - second Legendre polynomial, $P_2(x) = \frac{1}{2}(3x^2 - 1)$

Θ_o - reference potential temperature, equivalent to global mean RE temp

Vertical average: $\frac{\tilde{\Theta}_E(\phi, z)}{\Theta_o} = 1 - \frac{2}{3}\Delta_H P_2(\sin\phi)$

Held-Hou model

-Extent of Hadley Cell

- Extent of Hadley Cell ϕ_H, with following considerations:

  - Temperature should be continuous at the edge:

$$\tilde{\Theta}(\phi_H) = \tilde{\Theta}_E(\phi_H)$$

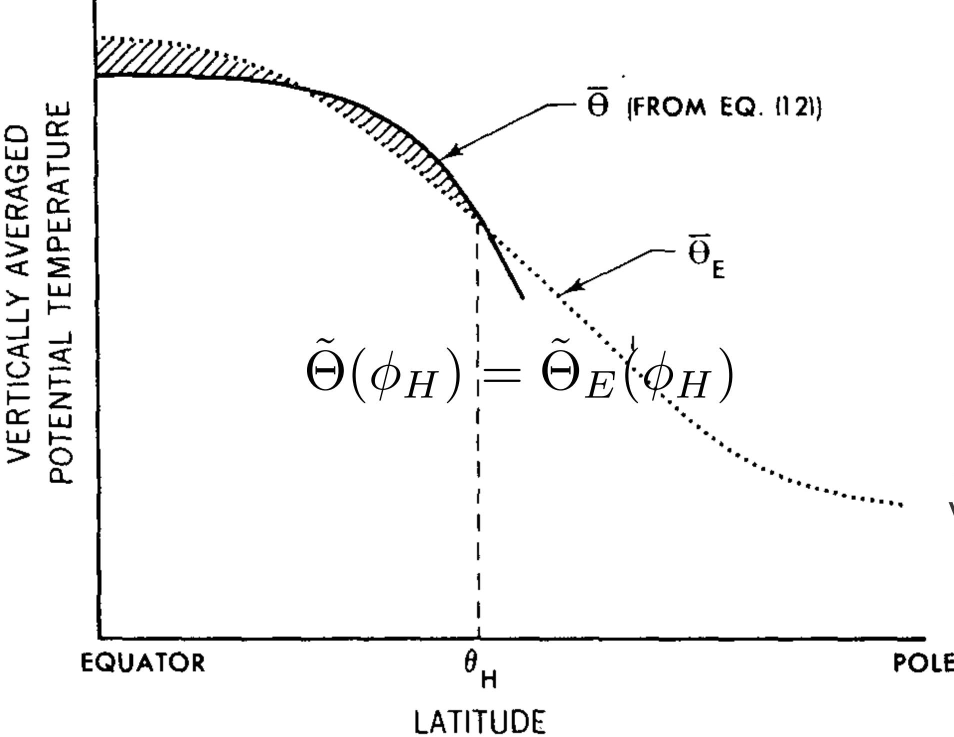
  - Hadley cell does not produce net heating but just carry heat poleward over the extent of Hadley Cell:

$$\int_0^{\phi_H} \tilde{\Theta} \cos\phi d\phi = \int_0^{\phi_H} \tilde{\Theta}_E \cos\phi d\phi$$

Held-Hou model

-Extent of Hadley Cell

$$\int_0^{\phi_H} \tilde{\Theta} \cos\phi d\phi = \int_0^{\phi_H} \tilde{\Theta}_E \cos\phi d\phi$$



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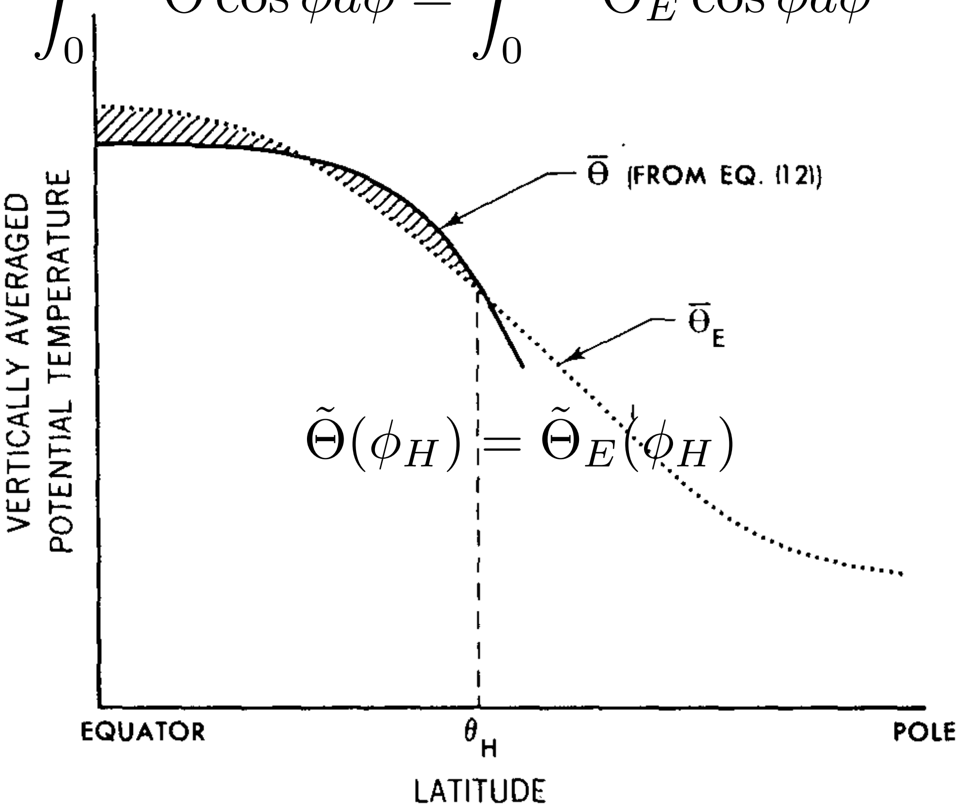
Held-Hou model

-Extent of Hadley Cell

$$\int_0^{\phi_H} \tilde{\Theta} \cos\phi d\phi = \int_0^{\phi_H} \tilde{\Theta}_E \cos\phi d\phi$$



$$\tilde{\Theta}(\phi_H) = \tilde{\Theta}_E(\phi_H)$$

Assume small ϕ, $\sin\phi \sim \phi$

$$\frac{\tilde{\Theta}(0)}{\Theta_o} \approx \frac{\tilde{\Theta}_E(0)}{\Theta_o} - \frac{5}{18}\frac{gH\Delta_H^2}{\Omega^2 a^2}$$

$$\phi_H = \left(\frac{5}{3}\frac{gH\Delta_H}{\Omega^2 a^2}\right)^{1/2}$$

set $R = \frac{gH\Delta_H}{\Omega^2 a^2}$, then, $\phi_H = \left(\frac{5}{3}R\right)^{1/2}$