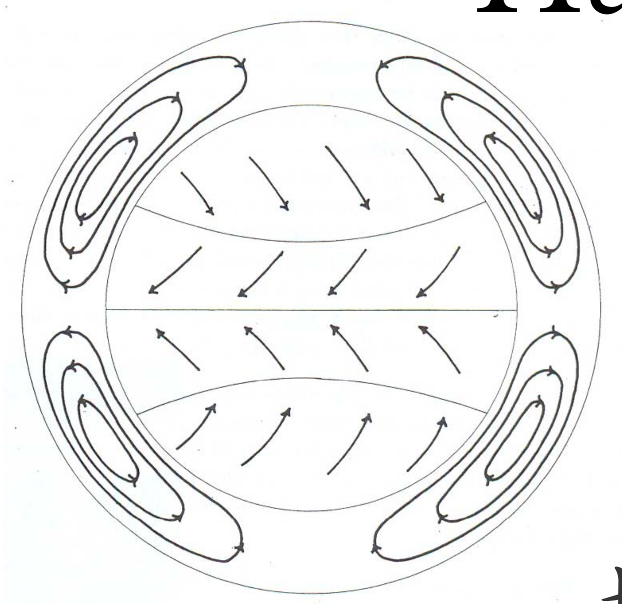
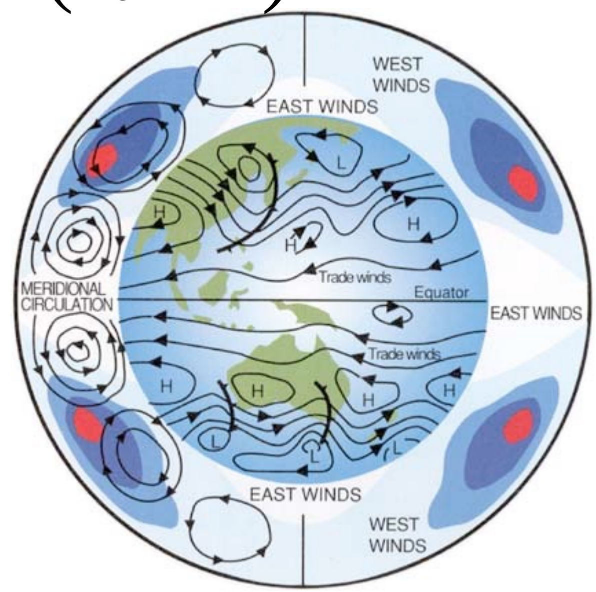
第三章:

Hadley 环流（三）





授课教师：张洋

2022.11.03

Held-Hou model (review)

-Summary

- Distribution of temperature constrained by the conservation of angular momentum and thermal wind balance.

$$\frac{\tilde{\Theta}(0)-\tilde{\Theta}(\phi)}{\Theta_o} = \frac{\Omega^2 a^2}{2gH}\frac{\sin^4\phi}{\cos^2\phi}$$

Smaller than the RE temp gradient

- Extent of Hadley Cell: $$\phi_H = \left(\frac{5}{3}\frac{gH\Delta_H}{\Omega^2 a^2}\right)^{1/2}$$

- Strength of Hadley Cell:

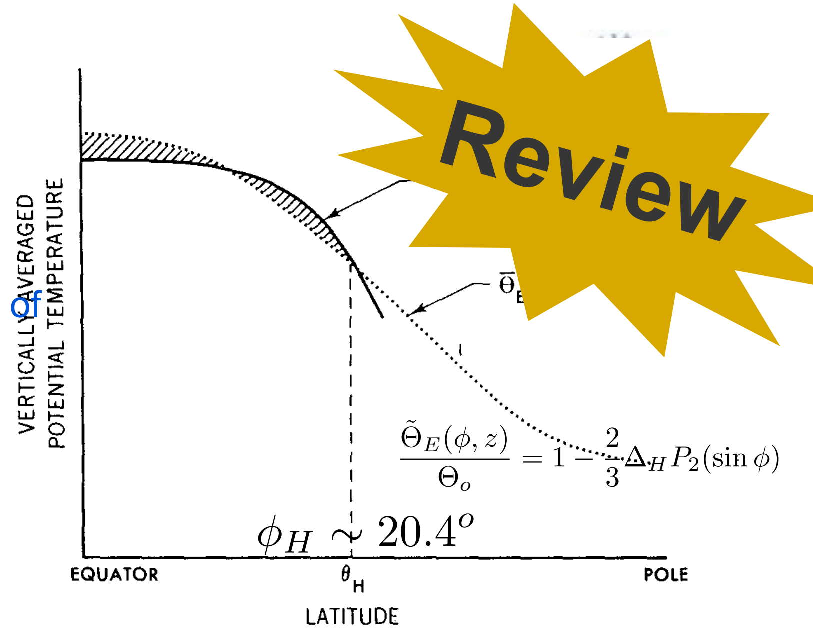
$$v \sim \frac{(gH)^{3/2}\,\Delta_H^{5/2}}{a^2\Omega^3\tau\Delta_V}$$

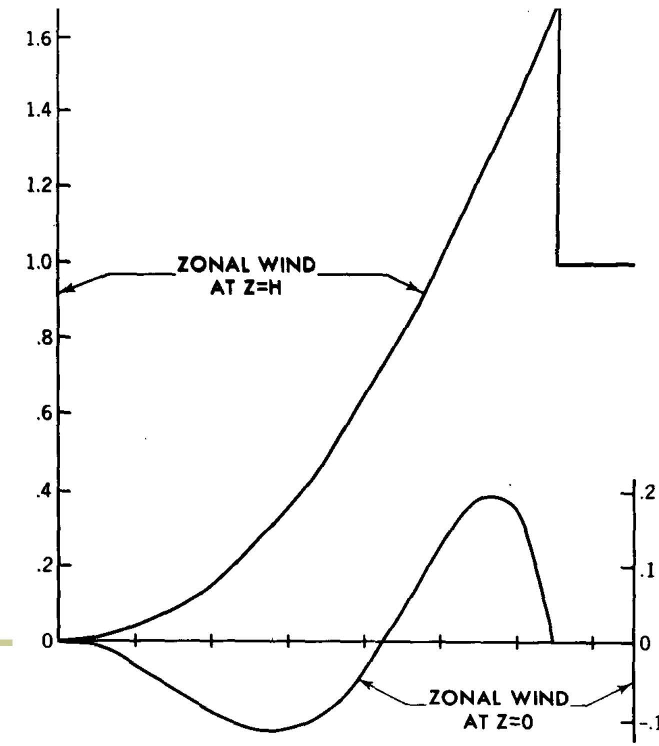
- Upper jet: $$[u] = \Omega a \frac{\sin^2\phi}{\cos\phi} \equiv U_M$$

- Surface winds:

$$Cu(0) \approx -\frac{25}{18}\frac{g^2 H^3 \Delta_H^3}{a^3\Omega^3\tau\Delta_V}\left[\left(\frac{\phi}{\phi_H}\right)^2 - \frac{10}{3}\left(\frac{\phi}{\phi_H}\right)^4 + \frac{7}{3}\left(\frac{\phi}{\phi_H}\right)^6\right]$$

surface easterlies $$\phi < \left(\frac{3}{7}\right)^{1/2}\phi_H$$

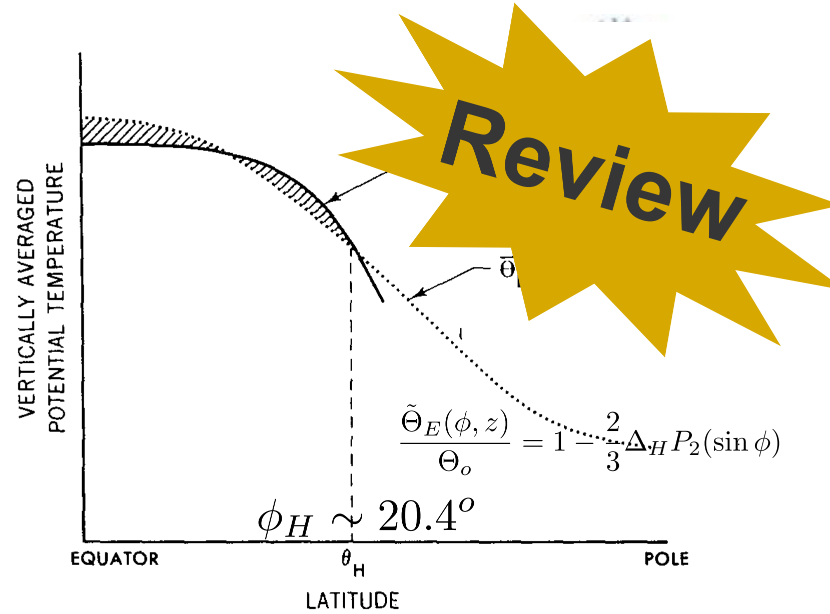


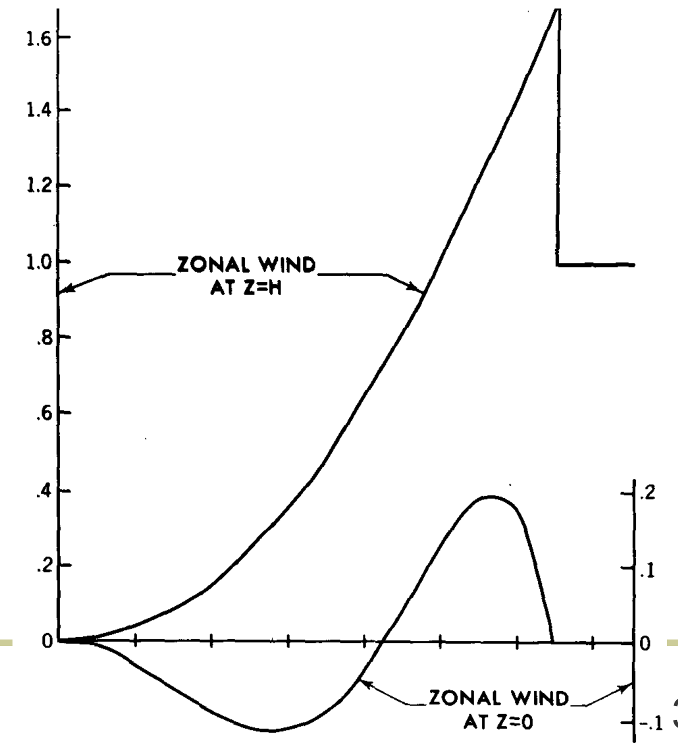


Held-Hou model (review)

-Discussion

- Upper jet: right place, but too large and discontinue.
- Extent of Hadley Cell: only a finite extent. Hadley cell cannot carry heat from equator to the pole, thus cannot be responsible to the observed equator-pole temperature difference. So does the wind, momentum and heat flux distribution.
- Axisymmetric flow: roles of eddies are neglected.
- Moisture effect is neglected.
- Seasonal variation and asymmetry on the equator?





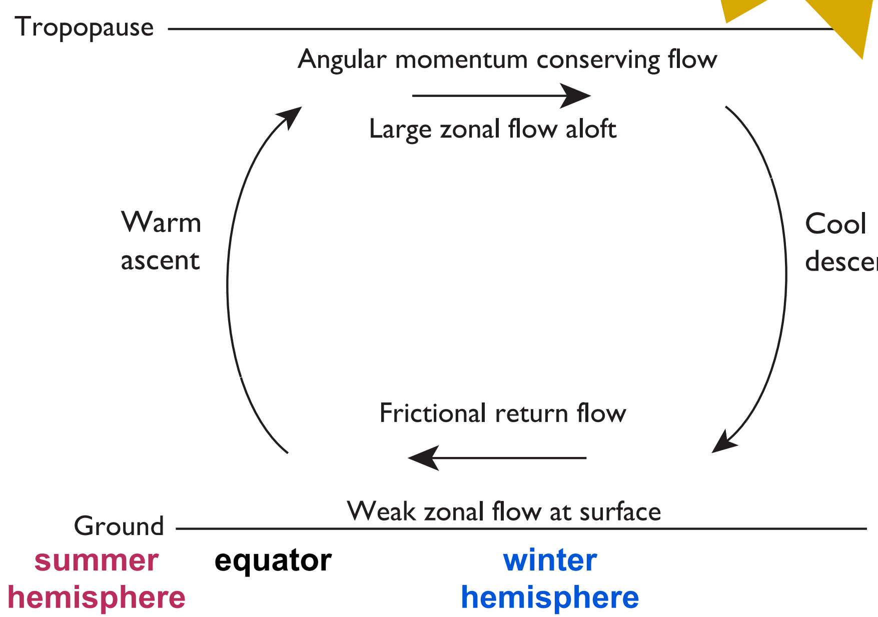
Hadley Cell - Theory: Asymmetry about the equator*

Review

- **Lindzen-Hou (1988)**

Still *make the assumptions:*

- the circulation is steady or quasi-steady (the flow adjusts to a steady circulation on a timescale faster than that on which the solar radiation varies);
- the upper branch conserves angular momentum; surface zonal winds are weak;
- the circulation is in thermal wind balance;



- the only **difference**: the **heating** is centered off the equator.

Asymmetry about the equator*
-Angular momentum

Review

- In an inviscid, **axisymmetric** flow, the angular momentum is conserved for the upper branch of Hadley Cell.

$$
\begin{aligned}
M &= (\Omega a \cos\phi + u) a \cos\phi \\
&= \Omega a^2 \cos^2 \phi_1
\end{aligned}
$$

$$
U_M = \frac{\Omega a (\cos^2 \phi_1 - \cos^2 \phi)}{\cos\phi}
$$

ϕ_1 - latitude of the rising motion

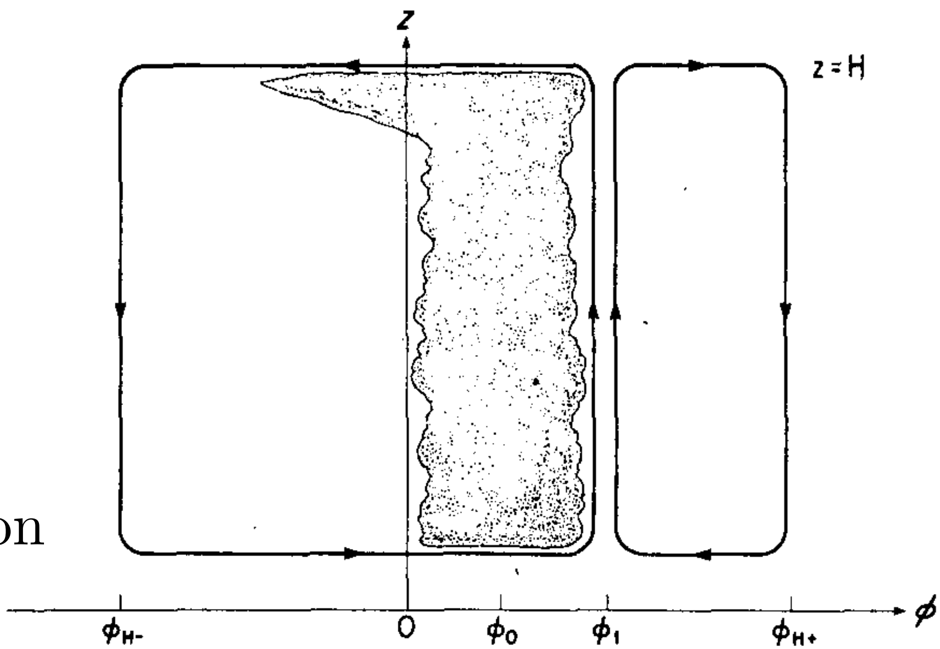


FIG. 3. Schematic illustration of the Hadley circulation.

Hadley Cell - Theory:
Asymmetry about the e

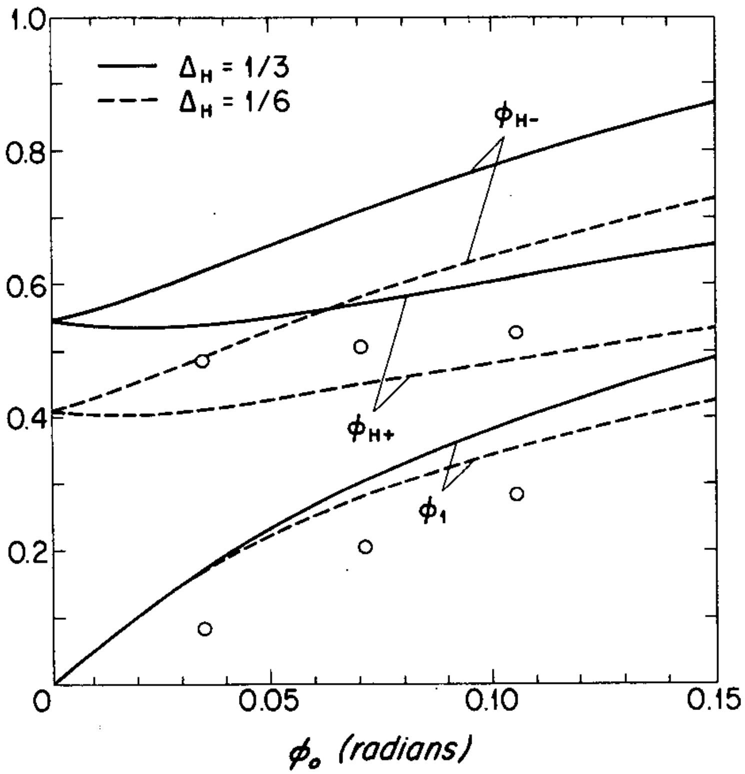


FIG. 4. ϕ_1, ϕ_{H^+} and ϕ_{H^-} as functions of ϕ_0 (see text for definitions). Open circles show results from numerical integration for ϕ_1 and ϕ_{H-1} when $\Delta_H = {}^1\!/_6$. (Note 1° of latitude $\approx$ 0.0175 radians.)

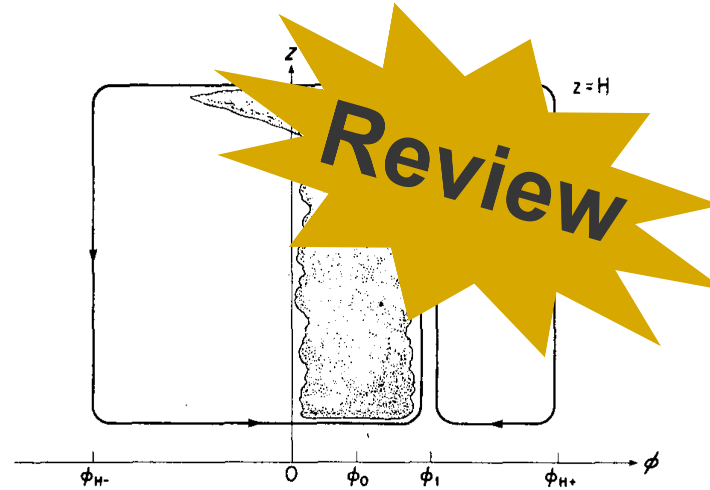


FIG. 3. Schematic illustration of the Hadley circulation.

- As the heating **shifts off the equator**, both cells and the center of the raising branch **shift poleward**, with a **wider** Hadley cell **cross the equator** and a **narrower** Hadley cell in **the summer hemisphere**;
- As the diabatic heating varies stronger in the meridional direction, **both cells shift poleward and become wider**.

Hadley Cell - Discussion: Asymmetry about the equator*

Review

- **Lindzen-Hou (1988)**
  - Quasi-steady assumption, however the seasonal cycle is temporally progressing;
  - The lack of angular momentum conservation in reality, especially when the angular momentum transport by **eddies** are significant.
  - The **moisture effect** is still neglected;

Hadley Cell :

Role of eddies*

- Qualitative considerations

$$\frac{D}{Dt}M = -\frac{1}{\rho}\frac{\partial p}{\partial \lambda} + a\cos\phi F_\lambda, \text{ where } \frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial}{\partial \lambda} + \frac{v}{a}\frac{\partial}{\partial \phi} + w\frac{\partial}{\partial z}$$

In an **axisymmetric flow** ([M]=M): $$\frac{D}{Dt}[M] = a\cos\phi[F_\lambda]$$

In a **3-D (with eddies) flow**, the zonally averaged angular momentum:

$$\frac{\partial [m]}{\partial t} + \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}([v][m]\cos\phi) + \frac{\partial}{\partial z}([w][m])$$
$$= -\frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}([m^*v^*]\cos\phi) - \frac{\partial}{\partial z}([m^*w^*]) + a\cos\phi[F_\lambda]$$
$$= -\frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}([u^*v^*]a\cos^2\phi) - \frac{\partial}{\partial z}([u^*w^*]a\cos\phi) + a\cos\phi[F_\lambda]$$

Neglecting vertical advection, vertical eddy fluxes and friction:

$$\frac{\partial [m]}{\partial t} + \frac{[v]}{a}\frac{\partial [m]}{\partial \phi} = -\frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}([u^*v^*]a\cos^2\phi)$$

Hadley Cell :

Role of eddies*

- Qualitative considerations

$$\frac{D}{Dt}M = -\frac{1}{\rho}\frac{\partial p}{\partial \lambda} + a\cos\phi F_\lambda, \text{ where } \frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial}{\partial \lambda} + \frac{v}{a}\frac{\partial}{\partial \phi} + w\frac{\partial}{\partial z}$$

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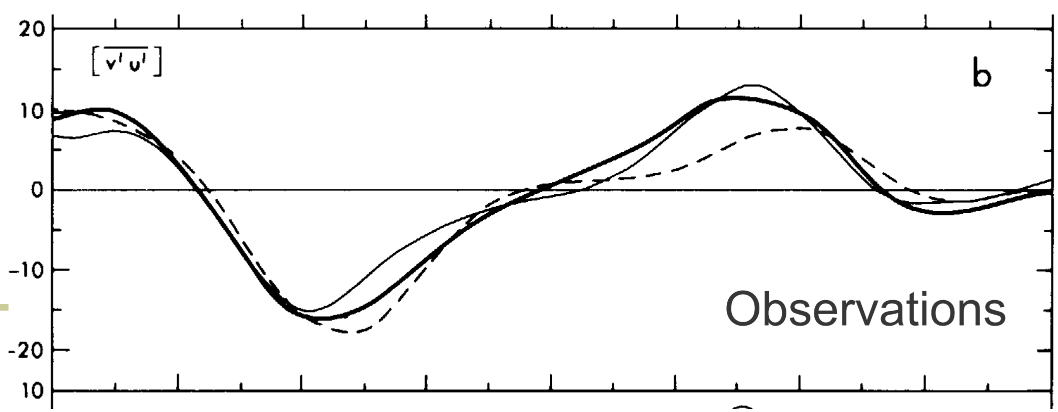
$$\frac{\partial[m]}{\partial t} + \frac{[v]}{a}\frac{\partial[m]}{\partial \phi} = -\frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}\left([u^*v^*]a\cos^2\phi\right)$$



Eddy momentum flux always acts to decrease the angular momentum of the zonal flow, and the zonal velocity is lower than U_M.

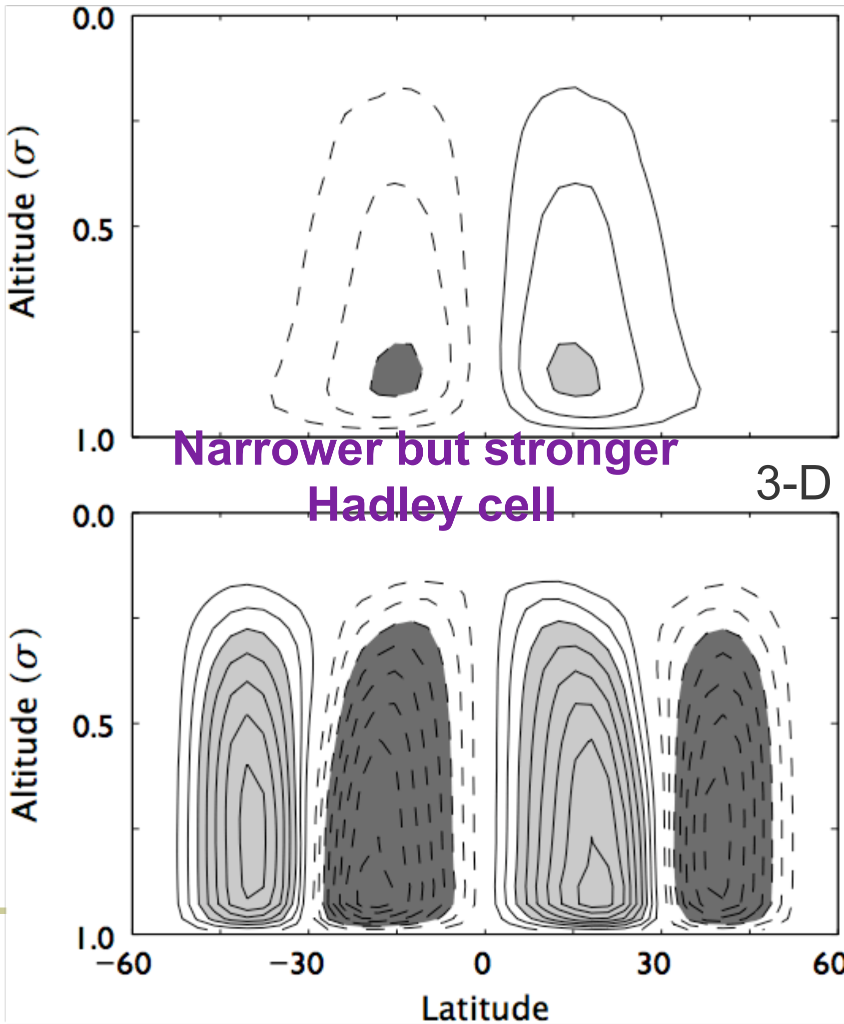
Hadley Cell :

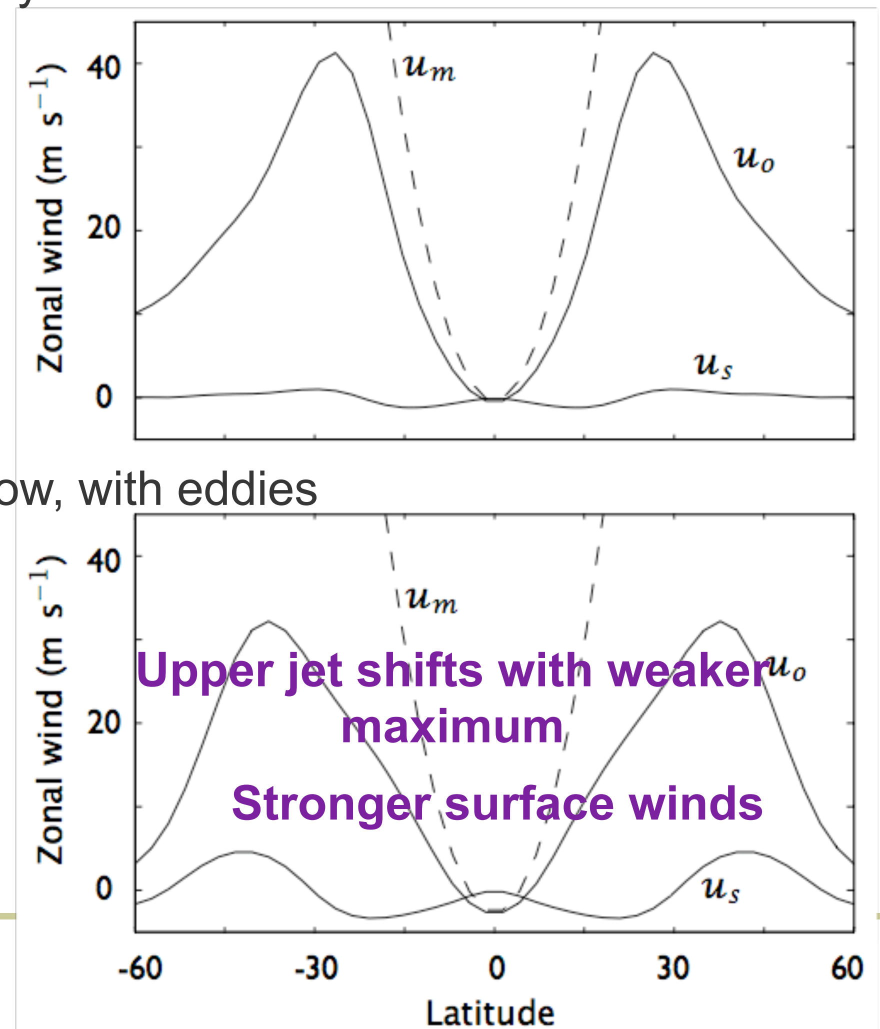
Role of eddies*

- Numerical solutions:

Vallis, 2006,
numerical results from idealized GCM

axisymmetric





Narrower but stronger Hadley cell

3-D flow, with eddies

Upper jet shifts with weaker maximum

Stronger surface winds

- Outline

- Observations
- Held-Hou theory (axisymmetric flow, a model that is symmetric about the equator)
- Lindzen-Hou theory (axisymmetric flow, a model that is asymmetry about the equator)
- The role of eddies
- Moisture effects
- Discussions

Hadley Cell :

Moisture effect*

- Momentum equation:

$$\left(\frac{du}{dt}\right)_p - fv = -\left(\frac{\partial \Phi}{\partial x}\right)_p + F_x$$

$$\left(\frac{dv}{dt}\right)_p + fu = -\left(\frac{\partial \Phi}{\partial y}\right)_p + F_y$$

- Continuity equation:

$$\nabla_p \cdot \boldsymbol{v} + \frac{\partial \omega}{\partial p} = 0$$

- Thermodynamic equation:

$$\left(\frac{d \ln \theta}{dt}\right)_p = \frac{Q}{c_p T}$$

- Water vapor (budget) equation:
  q - specific humidity

$$\left(\frac{dq}{dt}\right)_p = s(q) + D$$

$$\left(\frac{d}{dt}\right)_p = \left(\frac{\partial}{\partial t}\right)_p + u\left(\frac{\partial}{\partial x}\right)_p + v\left(\frac{\partial}{\partial y}\right)_p + \omega\frac{\partial}{\partial p}$$

Hadley Cell :

Moisture effect*

$$\left(\frac{dq}{dt}\right)_p = s(q) + D \qquad \left(\frac{d\ln\theta}{dt}\right)_p = \frac{Q}{c_p T}$$

D - molecular and turbulent eddy diffusion through the boundaries

$s(q)$ - souce-sink term $\quad s(q) = e - c$

e - the rate of evaporation per unit mass

c - the rate of condensation per unit mass

Diabatic heating $Q = Q_{\mathrm{RAD}} + Q_{\mathrm{LH}} + Q_{\mathrm{B}}$

$$Q_{\mathrm{LH}} = -L\left(\frac{dq}{dt}\right)$$

Q_{B} - diabatic heating due to boundary layer processes, i.e. sensible heat

Hadley Cell :

Moisture effect*

- Momentum equation:

$$\left(\frac{du}{dt}\right)_p - fv = -\left(\frac{\partial \Phi}{\partial x}\right)_p + F_x$$

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- Water vapor (budget) equation:
  q - specific humidity

$$\left(\frac{dq}{dt}\right)_p = -\frac{Q_{\mathrm{LH}}}{L}$$

$$\left(\frac{d}{dt}\right)_p = \left(\frac{\partial}{\partial t}\right)_p + u\left(\frac{\partial}{\partial x}\right)_p + v\left(\frac{\partial}{\partial y}\right)_p + \omega\frac{\partial}{\partial p}$$

Hadley Cell :

Moisture effect*

$$\left(\frac{d\ln\theta}{dt}\right)_p = \frac{Q_{\mathrm{RAD}} + Q_{\mathrm{LH}} + Q_{\mathrm{B}}}{c_p T}$$

$$\left(\frac{dq}{dt}\right)_p = -\frac{Q_{\mathrm{LH}}}{L}$$

Dry adiabatic: $\left(\frac{d\ln\theta}{dt}\right)_p = 0$

e.g. following a dry adiabatically-upraising parcel:

$$\frac{d\theta}{dz} = 0, \quad \frac{\partial T}{\partial z} + \frac{g}{c_p} = 0$$

Moist adiabatic: $\left(\frac{d\ln\theta}{dt}\right)_p = \frac{Q_{\mathrm{LH}}}{c_p T} = -\frac{L}{c_p T}\left(\frac{dq}{dt}\right)_p$

$$\left(\frac{d}{dt}\right)_p = \left(\frac{\partial}{\partial t}\right)_p + u\left(\frac{\partial}{\partial x}\right)_p + v\left(\frac{\partial}{\partial y}\right)_p + \omega\frac{\partial}{\partial p}$$

Hadley Cell :

Moisture effect*

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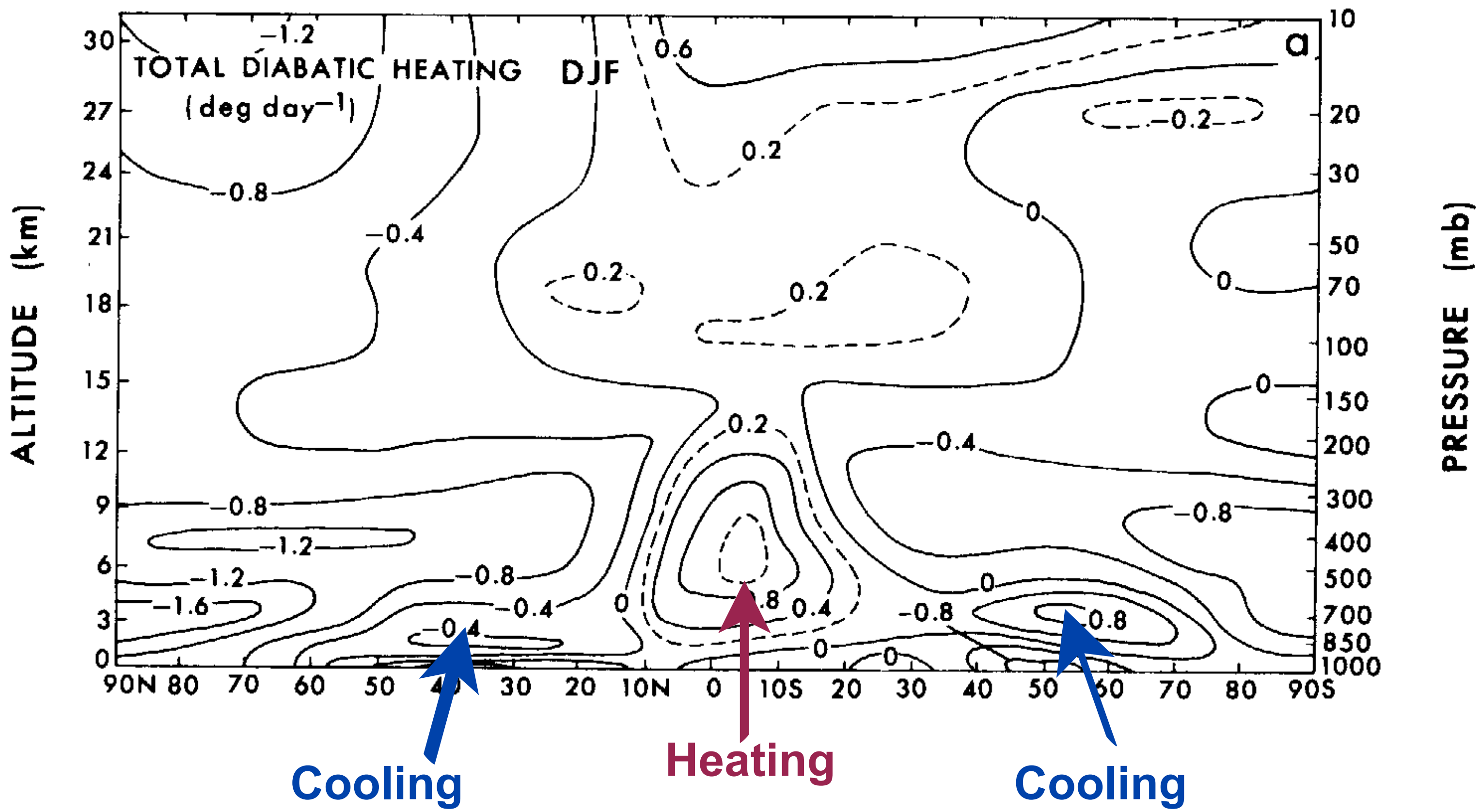
Moisture effect*

Diabatic heating $Q = Q_{\mathrm{RAD}} + Q_{\mathrm{LH}} + Q_{\mathrm{B}}$



Hadley Cell :

Moisture effect*

$$Q = Q_{\mathrm{RAD}} + Q_{\mathrm{LH}} + Q_{\mathrm{B}}$$

$$Q_{\mathrm{RAD}}$$

Radiative cooling,
weaker meridional variation

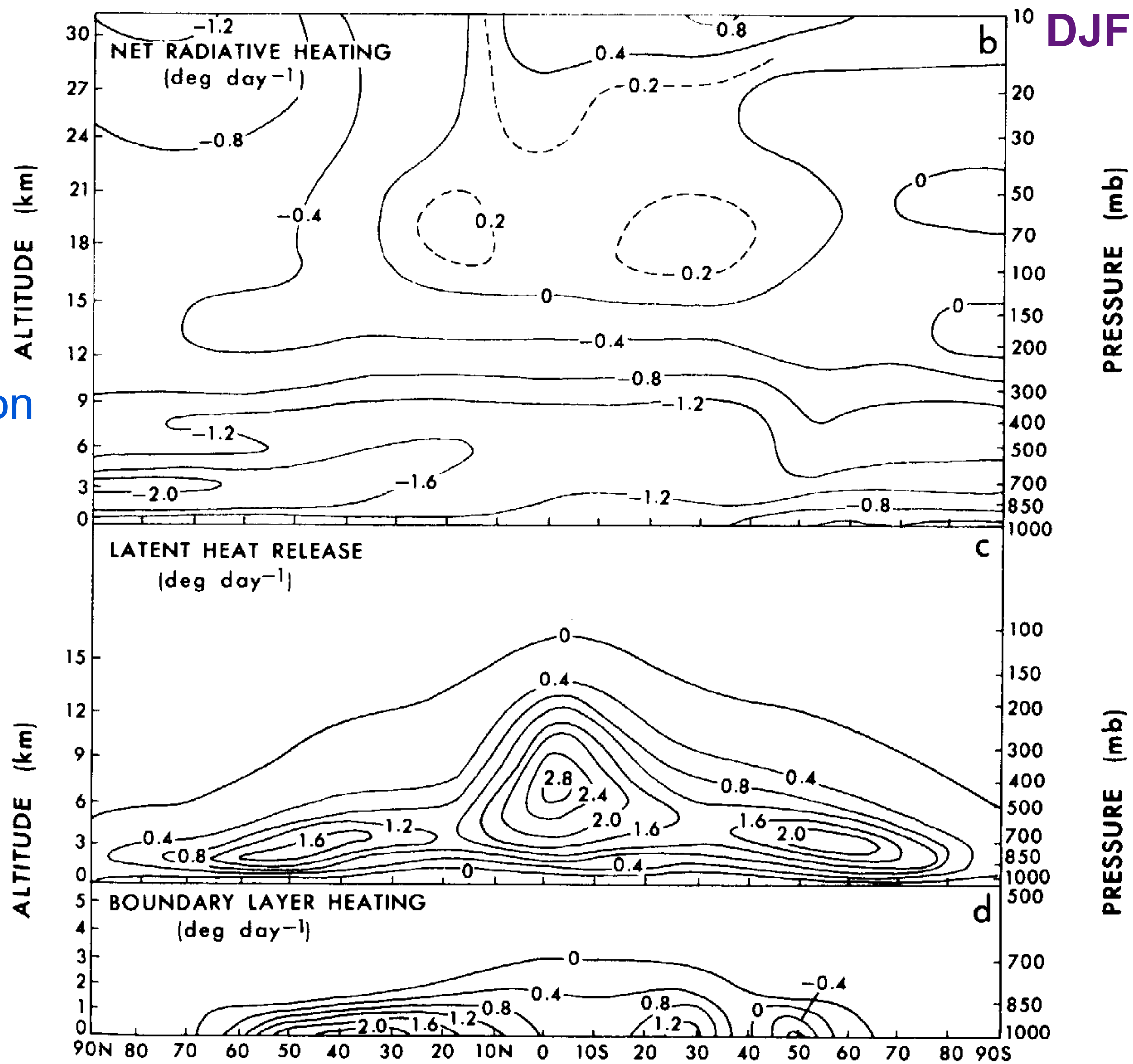
$$Q_{\mathrm{LH}}$$

Heating, strong meridional
and vertical variation

$$Q_{\mathrm{B}}$$

Heating, strongest at
midlatitudes

(Newell et al, 1970)



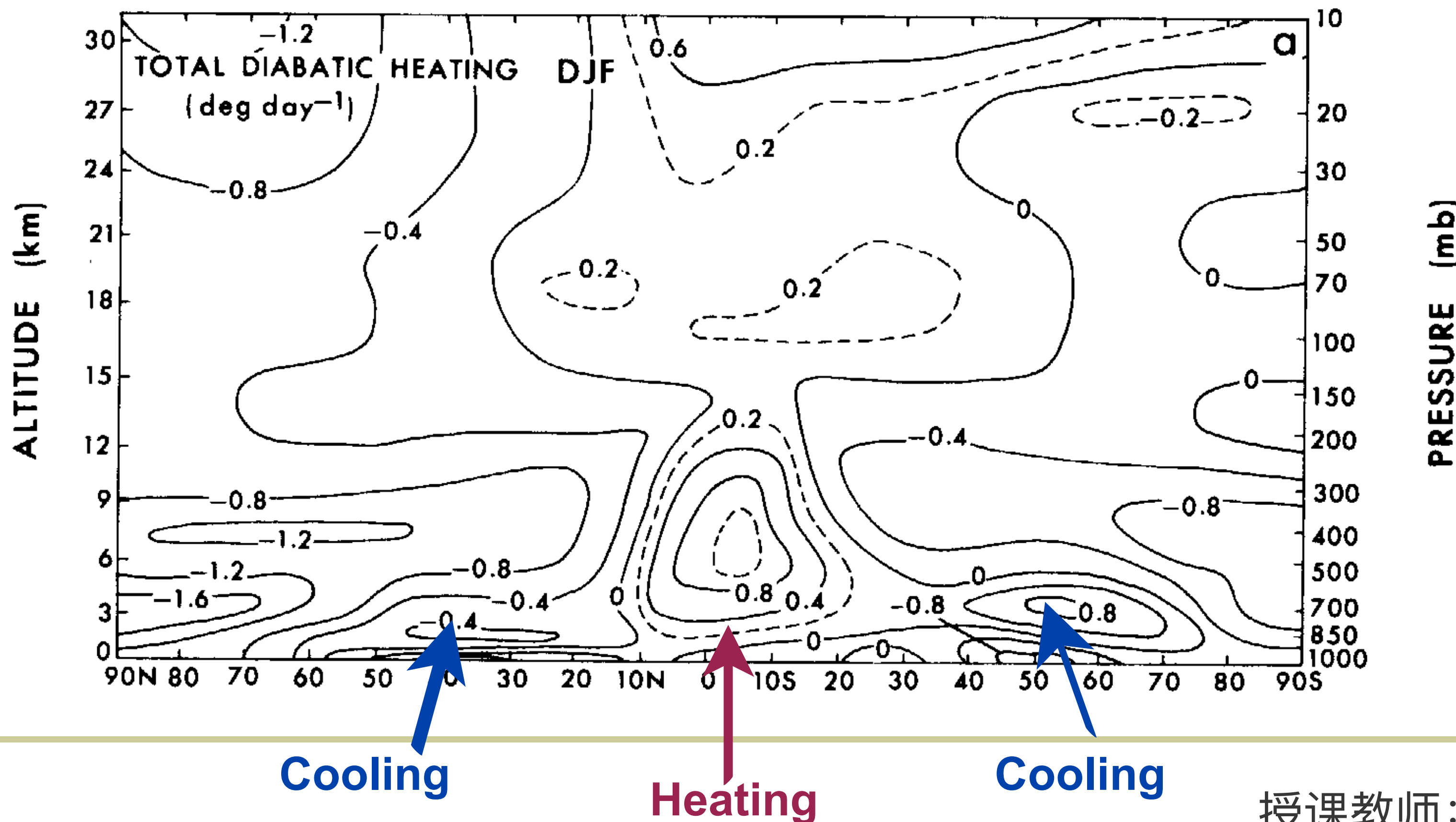
Hadley Cell :

Moisture effect*

Diabatic heating $Q = Q_{\mathrm{RAD}} + Q_{\mathrm{LH}} + Q_{\mathrm{B}}$

Recall previous calculation:

$$\frac{D\Theta}{Dt} = \frac{\Theta_E - \Theta}{\tau}$$



Hadley Cell：

Moisture effect*

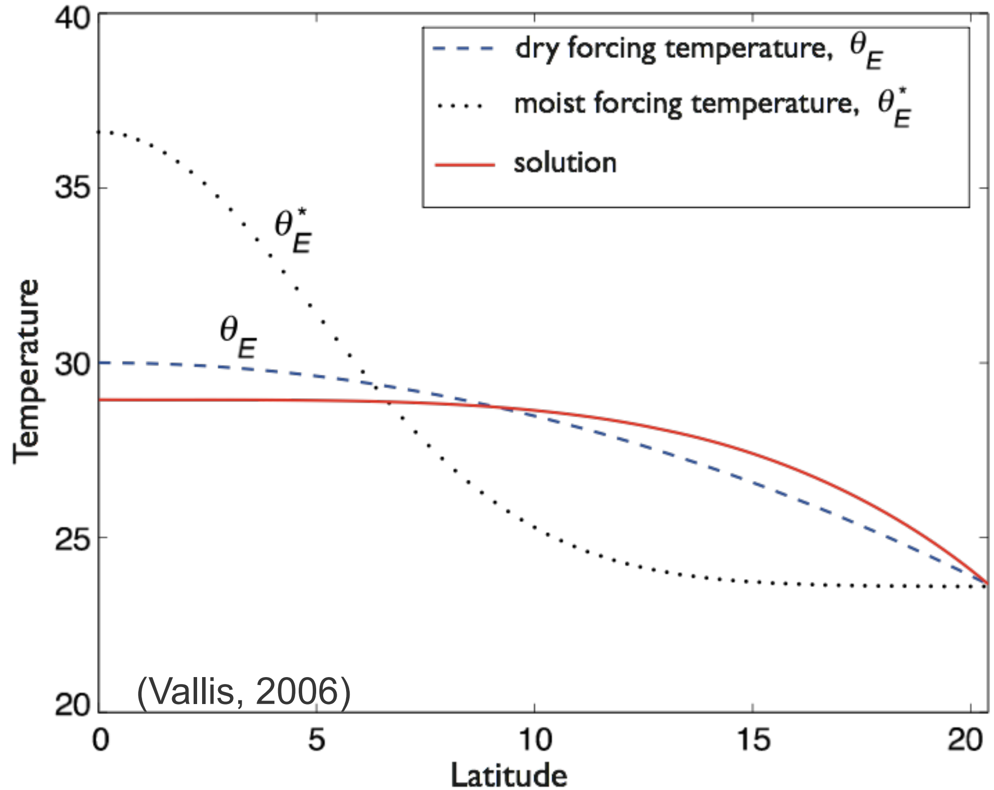


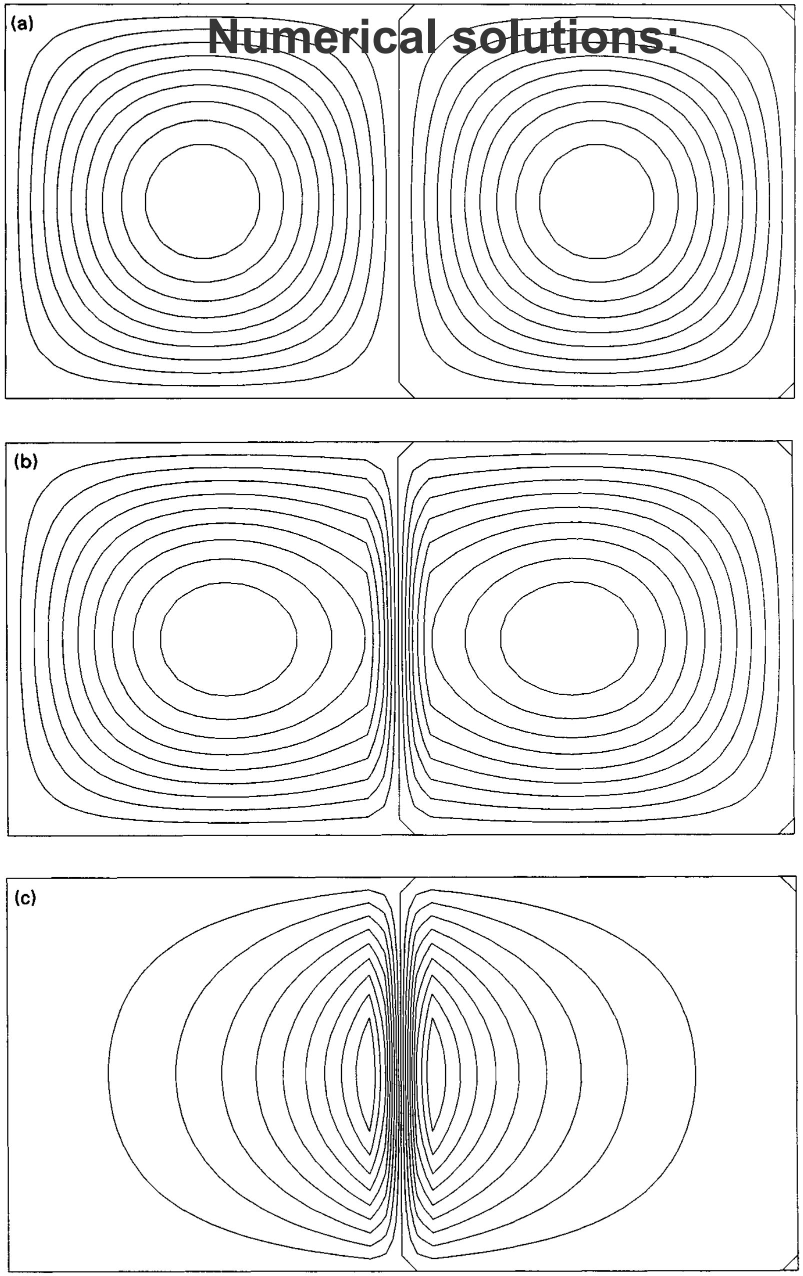
Fig. 11.8 Schema of the effects of moisture on a model of the Hadley Cell. The temperature of the solution (solid line) is the same as that of a dry model, because this is determined from the angular-momentum-conserving wind. The heating distribution (as parametrized by a forcing temperature) is peaked near the equator in the moist case, leading to a more vigorous overturning circulation.

$$w \approx \frac{H}{\Theta_o \Delta_V} \frac{\Theta_E - \Theta}{\tau} = \frac{5g\Delta_H^2 H^2}{18a^2\tau\Omega^2\Delta_V}$$

Hadley cell becomes stronger

Numerical solutions:

Moisture effect*



Dry circulation

Upward branch of the MOIST Hadley cell becomes much narrower and more intense than the downward branch due to the enhanced efficiency of moist convection.

Mild condensation

Strong condensation

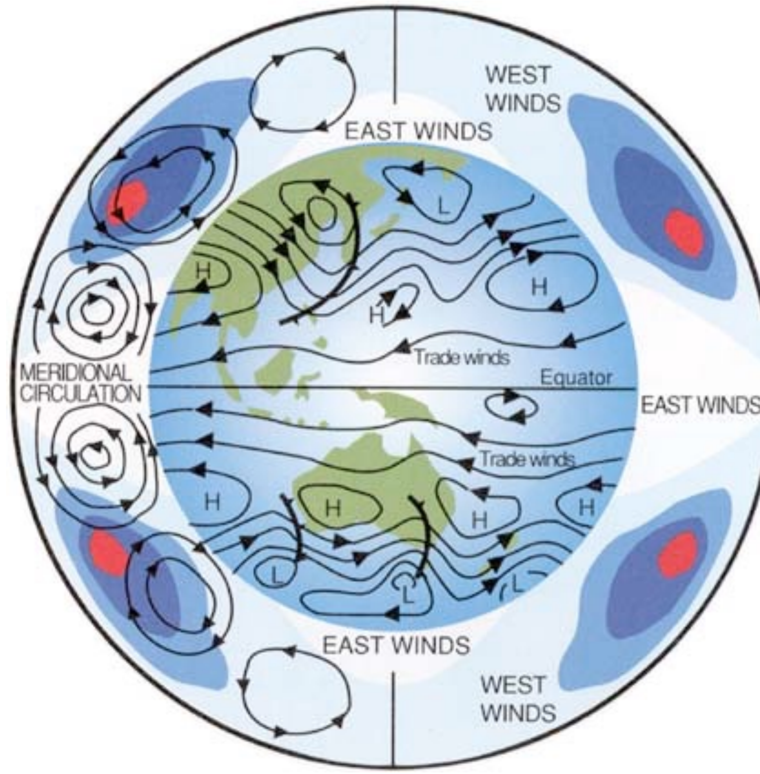
(Vallis, 2006)

- Summary

- Observations
- Held-Hou theory (axisymmetric flow, a model that is symmetric about the equator)
- Lindzen-Hou theory (axisymmetric flow, a model that is asymmetry about the equator)
- The role of eddies
- Moisture effects
- Discussions

Hadley Cell, Subtropical Jet and the Subtropical High** (open discussion)

- Hadley cell and subtropical jet:
  - 副热带急流常由Hadley环流所产生，出现在其末端所在的纬度；

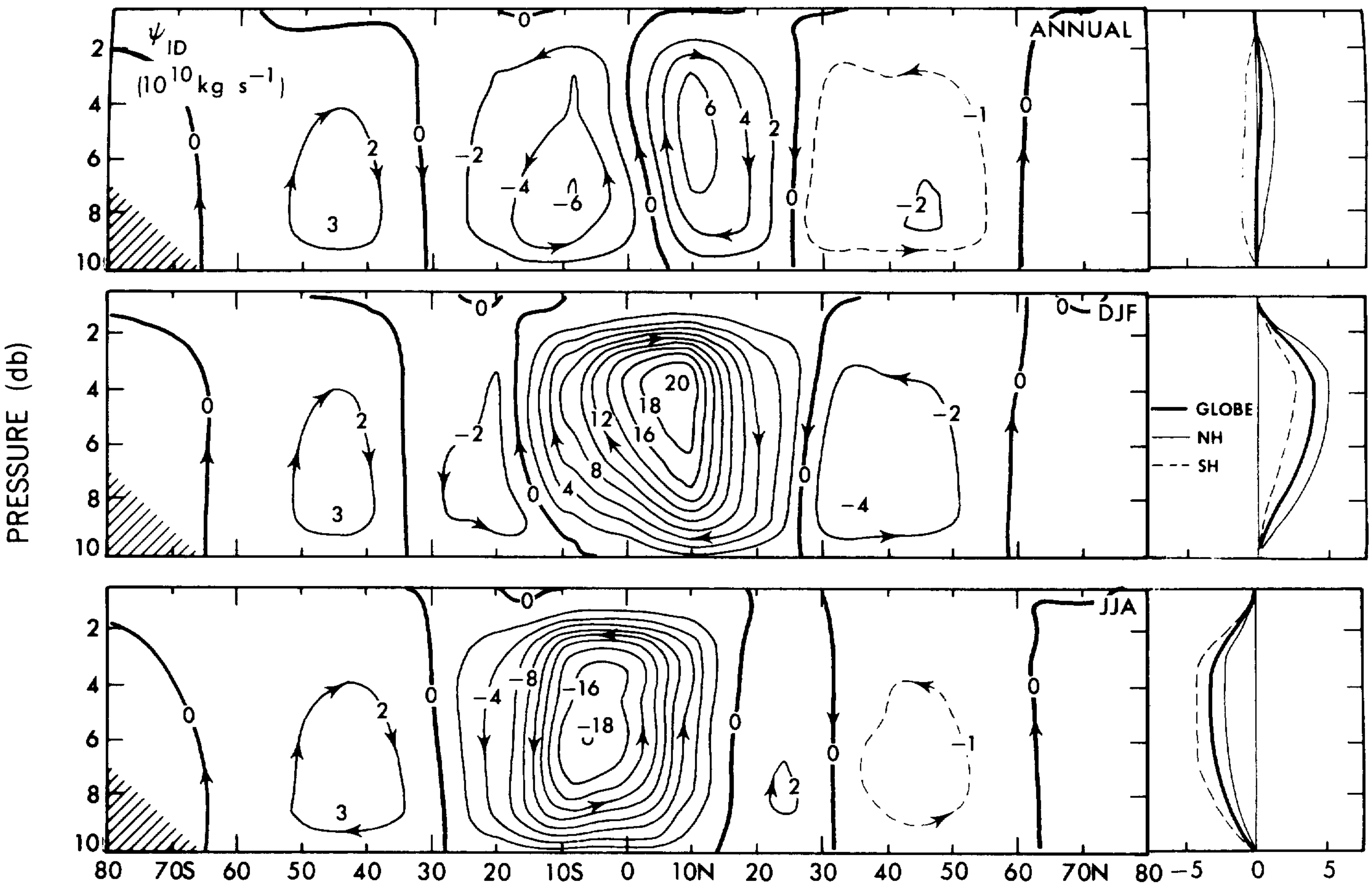


Hadley Cell

- Observations

- Stream function (流函数)

From Peixoto and Oort, 1992



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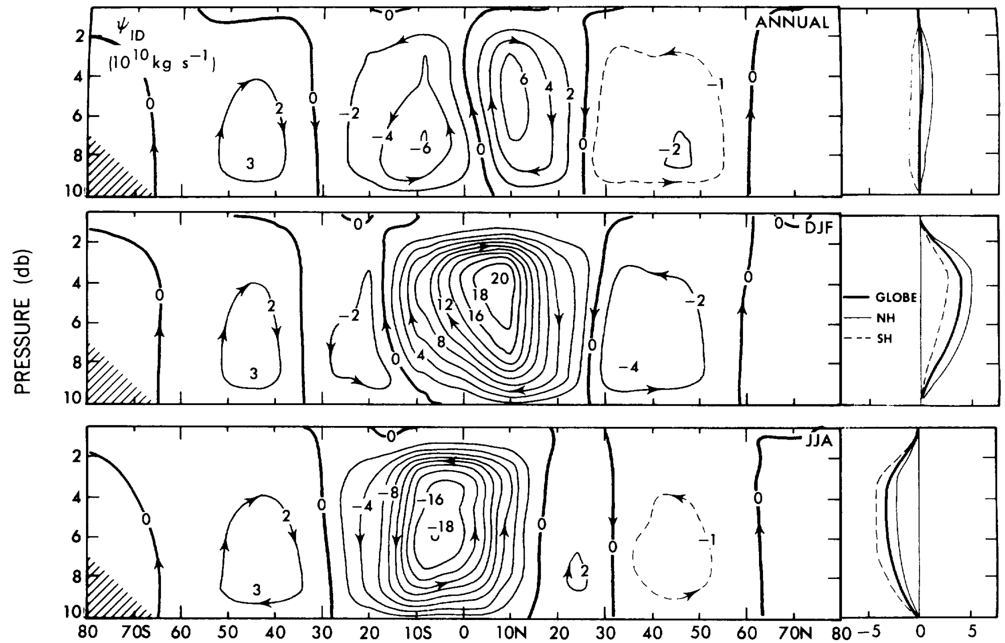
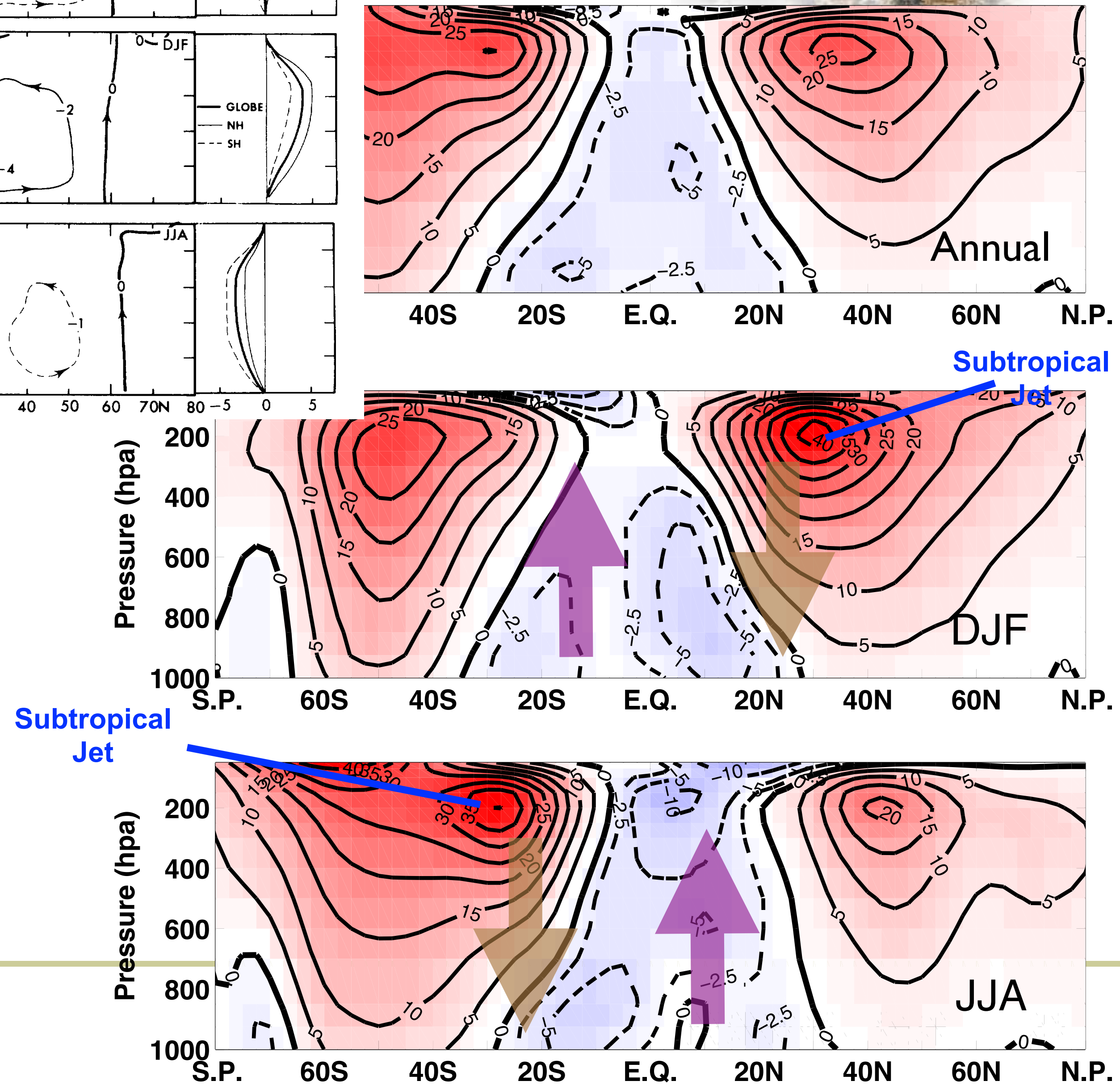


- Zonal mean zonal winds (U, 纬向风)

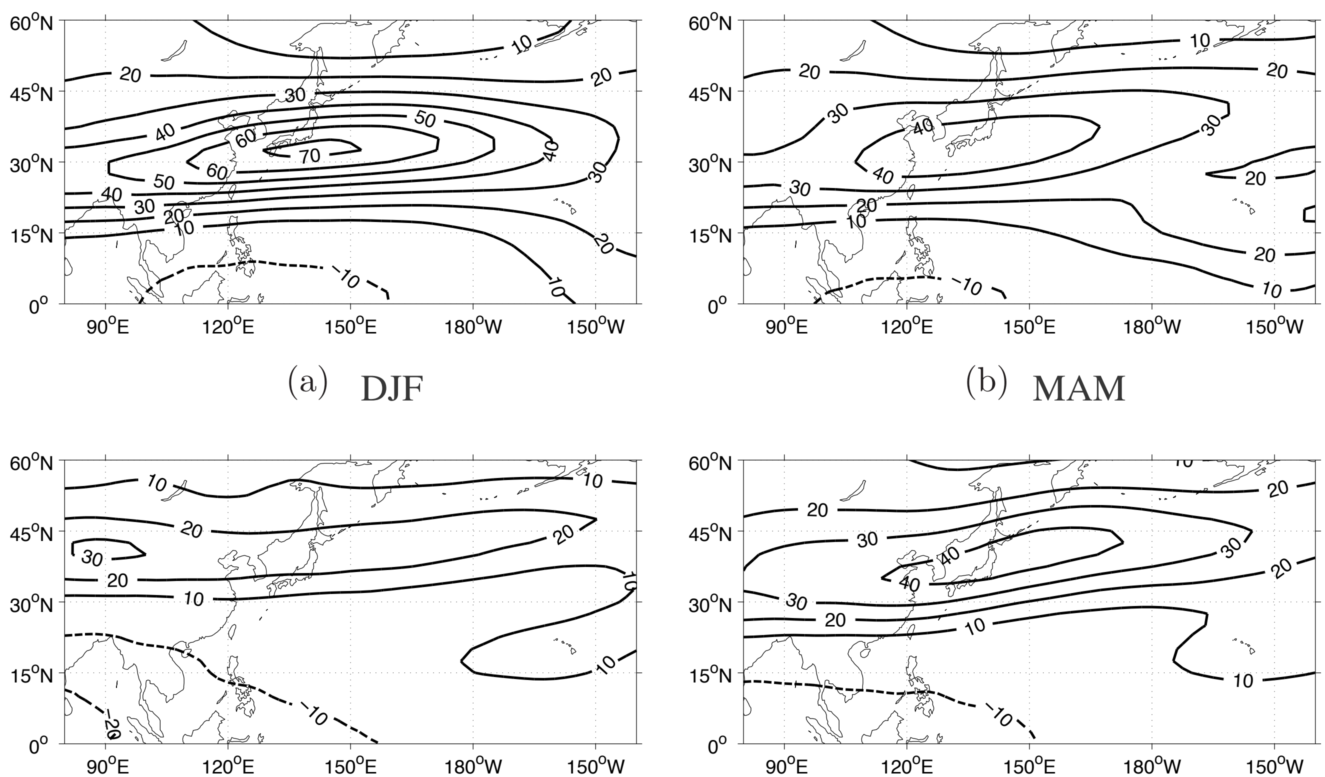


Hadley Cell, Subtropical Jet and the Subtropical High** (open discussion)

- Hadley cell and subtropical jet:
  - 副热带急流常由Hadley环流所产生，出现在其末端所在的纬度；

Climatology of zonal wind distribution

Seasonal migration of the EAJS is associated with the seasonal variation of Hadley Cell

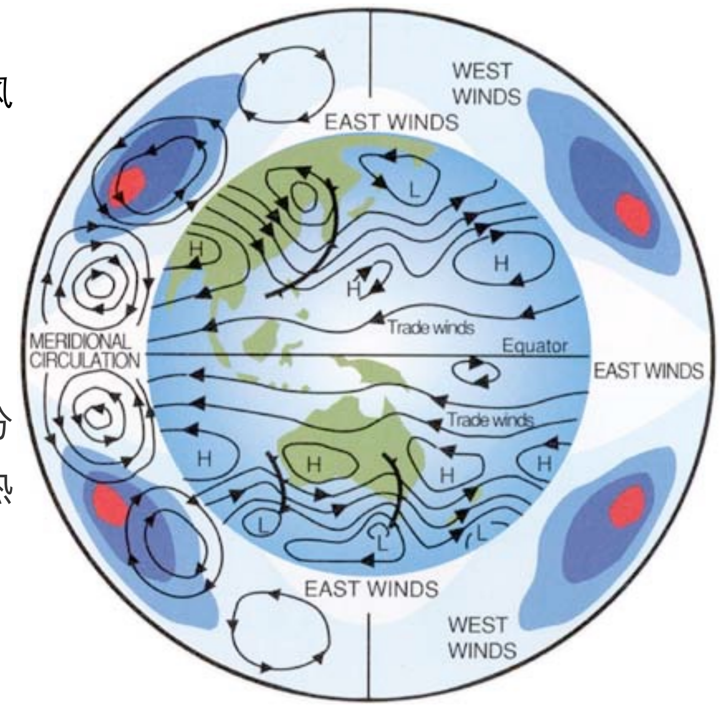


(a) DJF (b) MAM

(c) JJA (d) SON

Hadley Cell, Subtropical Jet and the Subtropical High** (open discussion)

- Hadley cell and subtropical jet:
  - 副热带急流常由Hadley环流所产生，出现在其末端所在的纬度；
  - 东亚的副热带急流对东亚大气环流的季节转换、亚洲夏季风的爆发、中国东部雨带由南向北推进都有重要影响
- Hadley cell and subtropical high:
  - 在冬季，副高所在位置为Hadley环流的下沉区；
  - 在夏季，经典的Hadley环流已经很弱。有研究认为，夏季非绝热加热（凝结潜热释放、地表感热通量等）的不均匀分布，使得副热带形成闭合的高压中心。也有研究认为，副热带高压的维持来源于槽后的下沉气流。
- Zonal winds and subtropical high：
  - 有研究发现，西风减弱，东风增强时，副高北上西进；
  - 西风增强，东风减弱，副高南撤东退。



Assignment 3, Fall 2022

Held-Hou(1980) 讨论了当外部强迫的经向分布呈二次勒让德多项式，即

$\frac{\Theta_E(\phi, z)}{\Theta_o} = 1 - \frac{2}{3}\Delta_H P_2(\sin\phi) + \Delta_v(\frac{z}{H} - \frac{1}{2})$的情况下，哈德莱环流内的风场、温度场、环流的空间范围等将怎样随纬度和外力强迫的强度而变化。

如果将外力强迫的空间分布改为 $\frac{\Theta_E(\phi, z)}{\Theta_o} = 1 - \Delta_H(\sin^3\phi - \frac{1}{4}) + \Delta_v(\frac{z}{H} - \frac{1}{2})$,

1. 请推导出哈德莱环流内的高空风场和垂直平均位温场 $\frac{\tilde{\Theta}}{\Theta_o}$ 将如何随纬度分布;
2. 同样利用小角度假设，请推导出环流的空间范围 ϕ_H 的表达式。如果设 $r \equiv \frac{gH}{\Omega^2 a^2}$, 请分别画出当 $\Delta_H = 1/3$ 和 $\Delta_H = 1/6$ 时, 与Held-Hou的情况相比，ϕ_H 怎样随 r 而变化。
3. 选做题目：在此情况下，近地面风场的分布有怎样变化。