第四章：

中纬度的经向环流系统

- Ferrel cell, baroclinic eddies and the westerly jet

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第四章:

中纬度的经向环流系统

- Ferrel cell，baroclinic eddies and the westerly jet

Reference reading:
Vallis Chapter 11.7; PO Chapter 7.5

Outline

Review

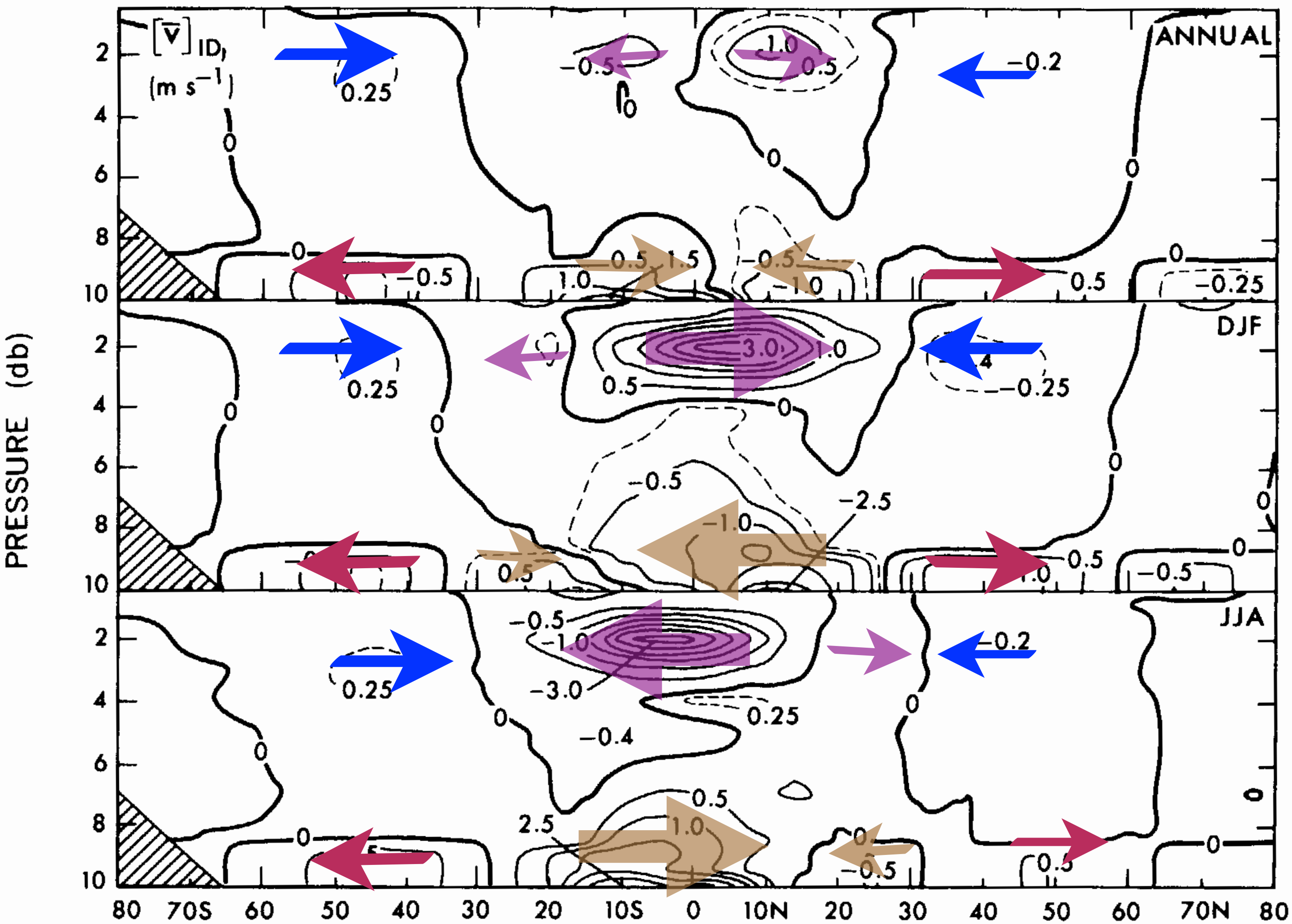
- Observations
- The Ferrel Cell
- Baroclinic eddies
  - Review: baroclinic instability and baroclinic eddy life cycle
  - Eddy-mean flow interaction
  - Transformed Eulerian Mean equation
- Eddy-driven jet
- The energy cycle

Observations

-Zonal mean fields

Review

- Meridional wind (v, 经向风)

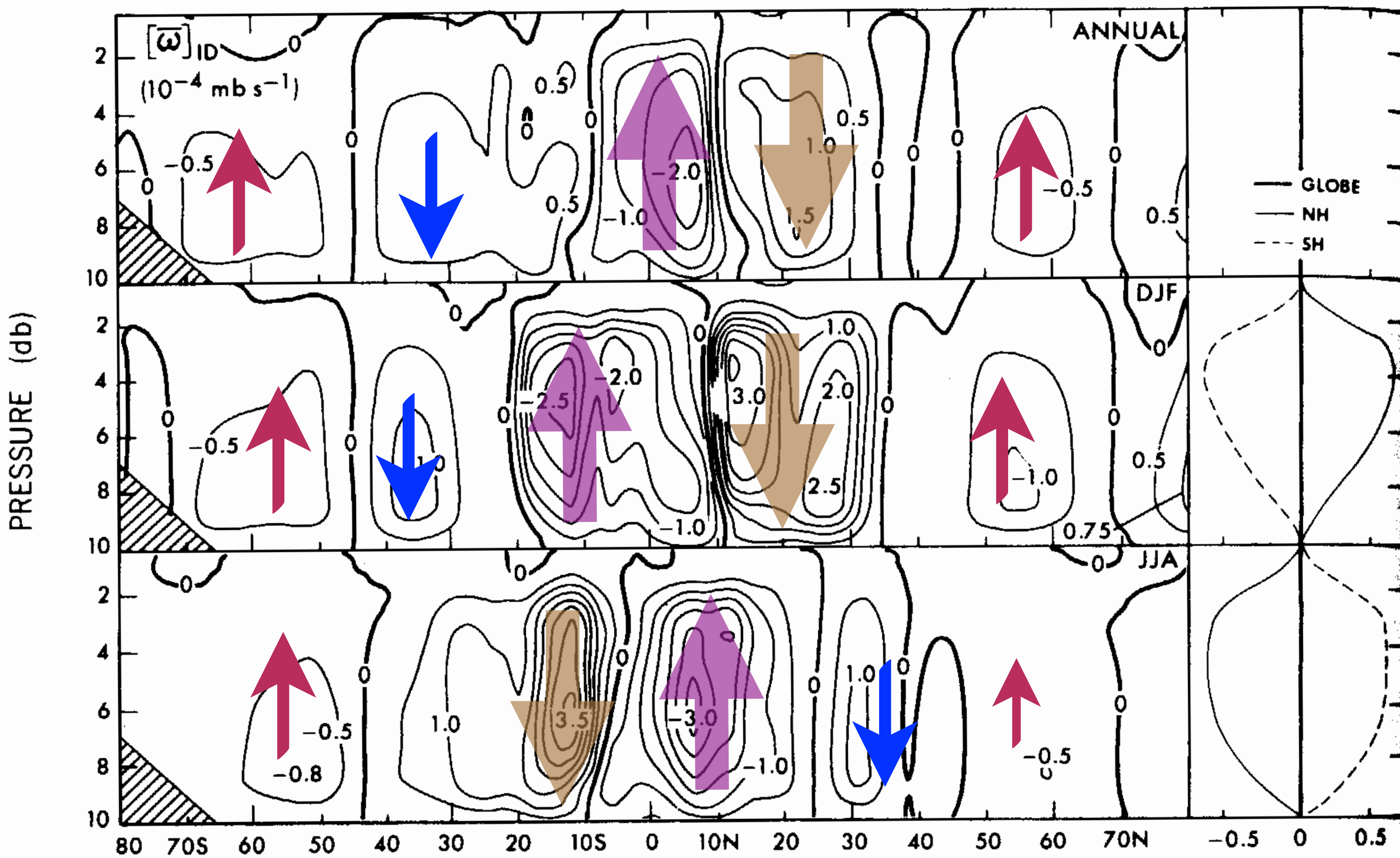


Observations

-Zonal mean fields

Review

- Vertical velocity (垂直速度)

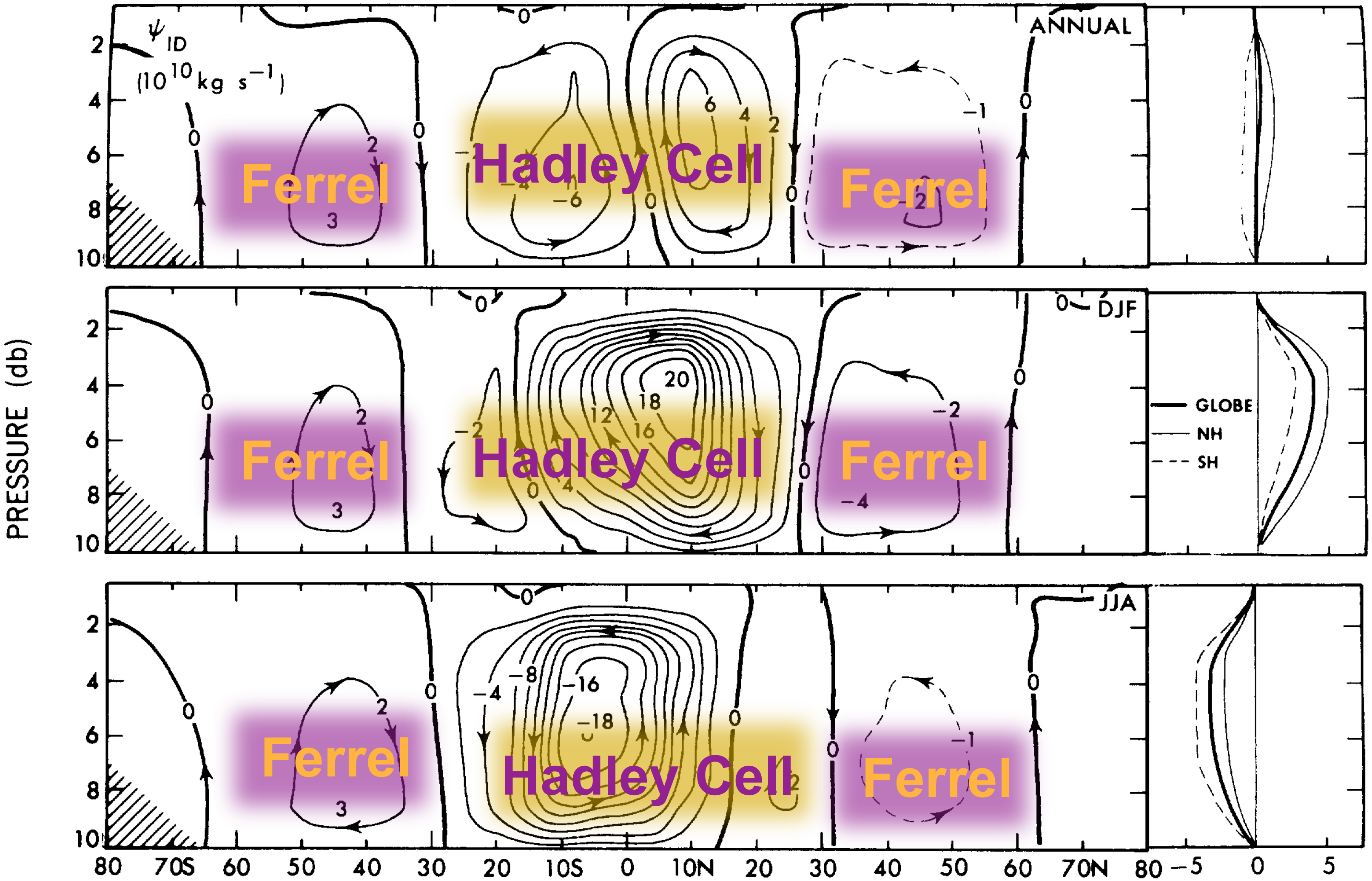


Observations

-Zonal mean fields

Review

- Stream function (流函数)



Observations

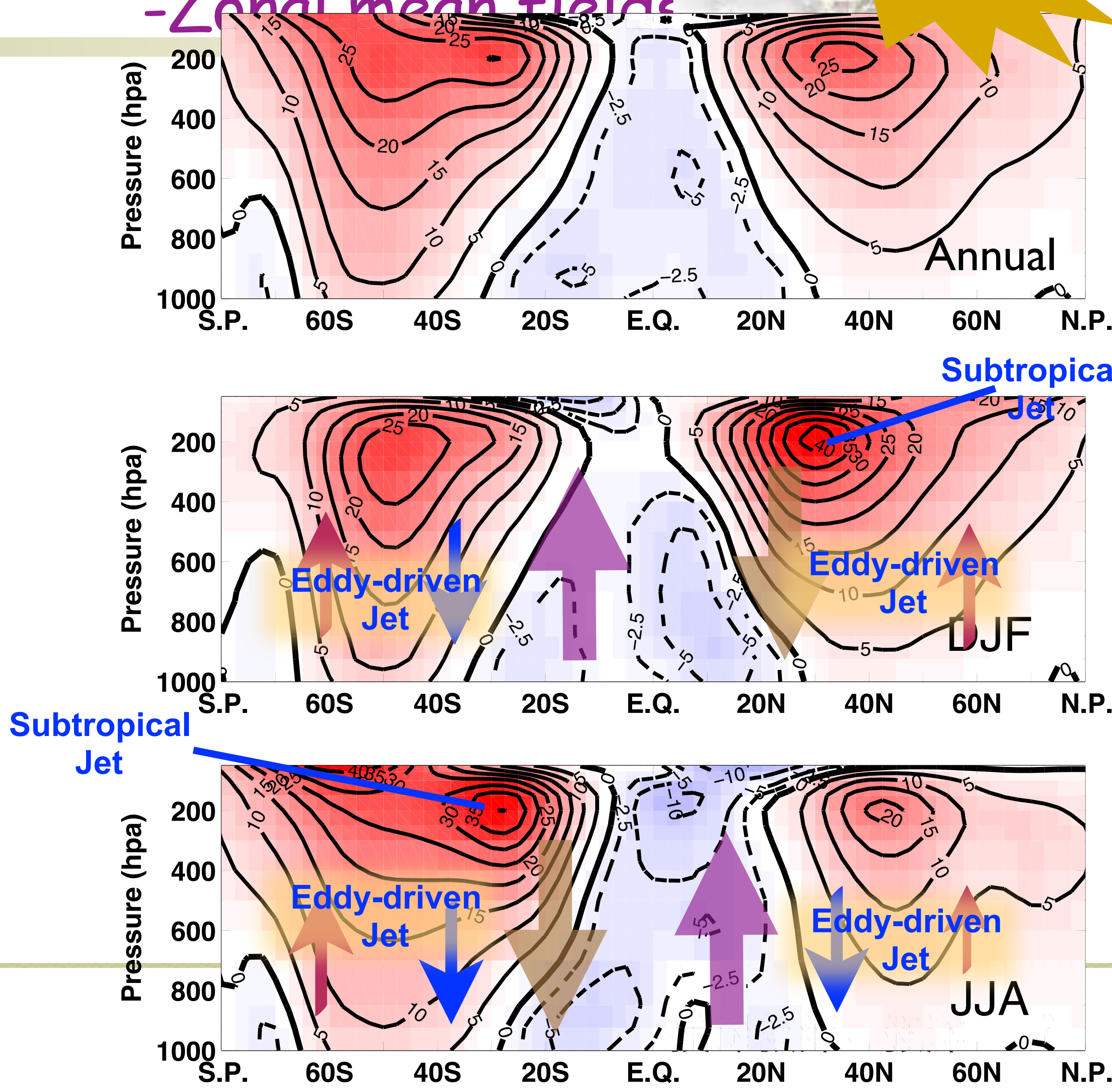
-Zonal mean fields

Review

- Zonal winds (U, 纬向风)

Midlatitude Jet
or
Polar-front Jet
or
Eddy-driven Jet

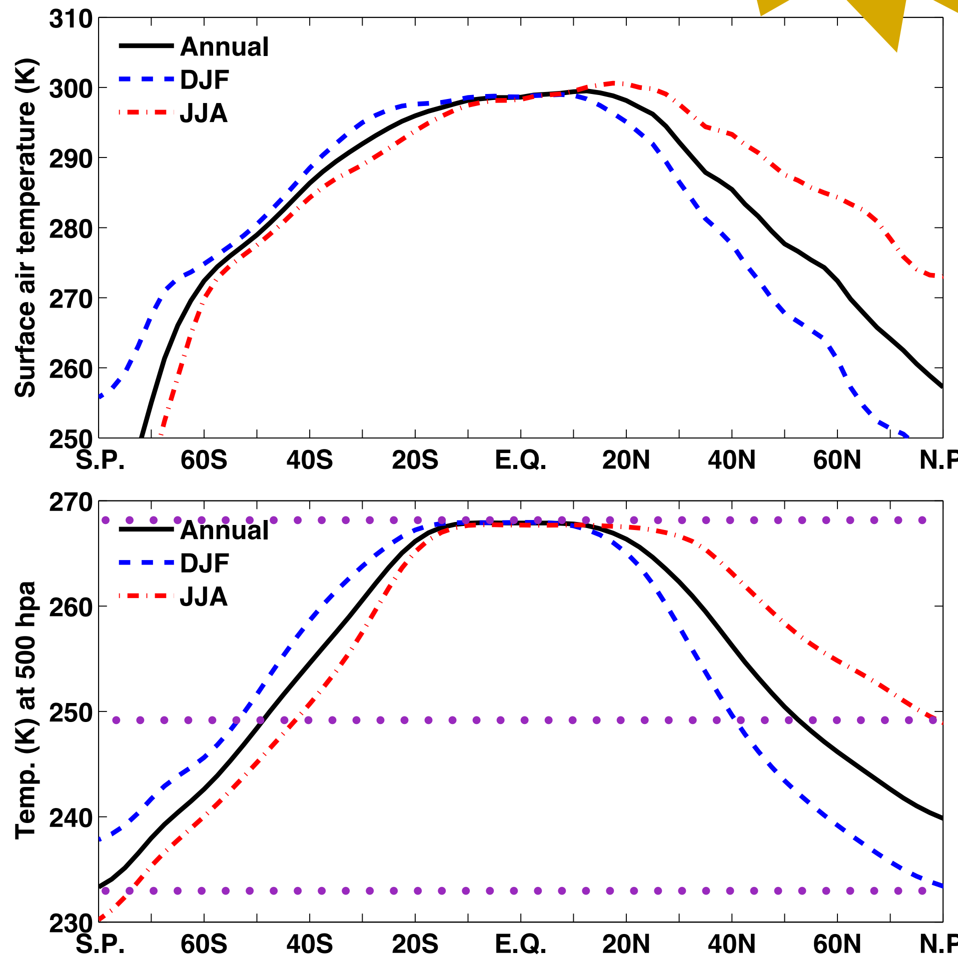
Surface westerly is always centered and strongest at 50 degree south and north, which is always considered as the **center of the eddy-driven jet**. It is also the **centric** latitude of Ferrel cell.



Observations
-Zonal mean fields

- Temperature (温度场)

Strong temperature gradient at midlatitudes, with obvious seasonal variation in the Northern Hemisphere compared to that in the Southern Hemisphere.

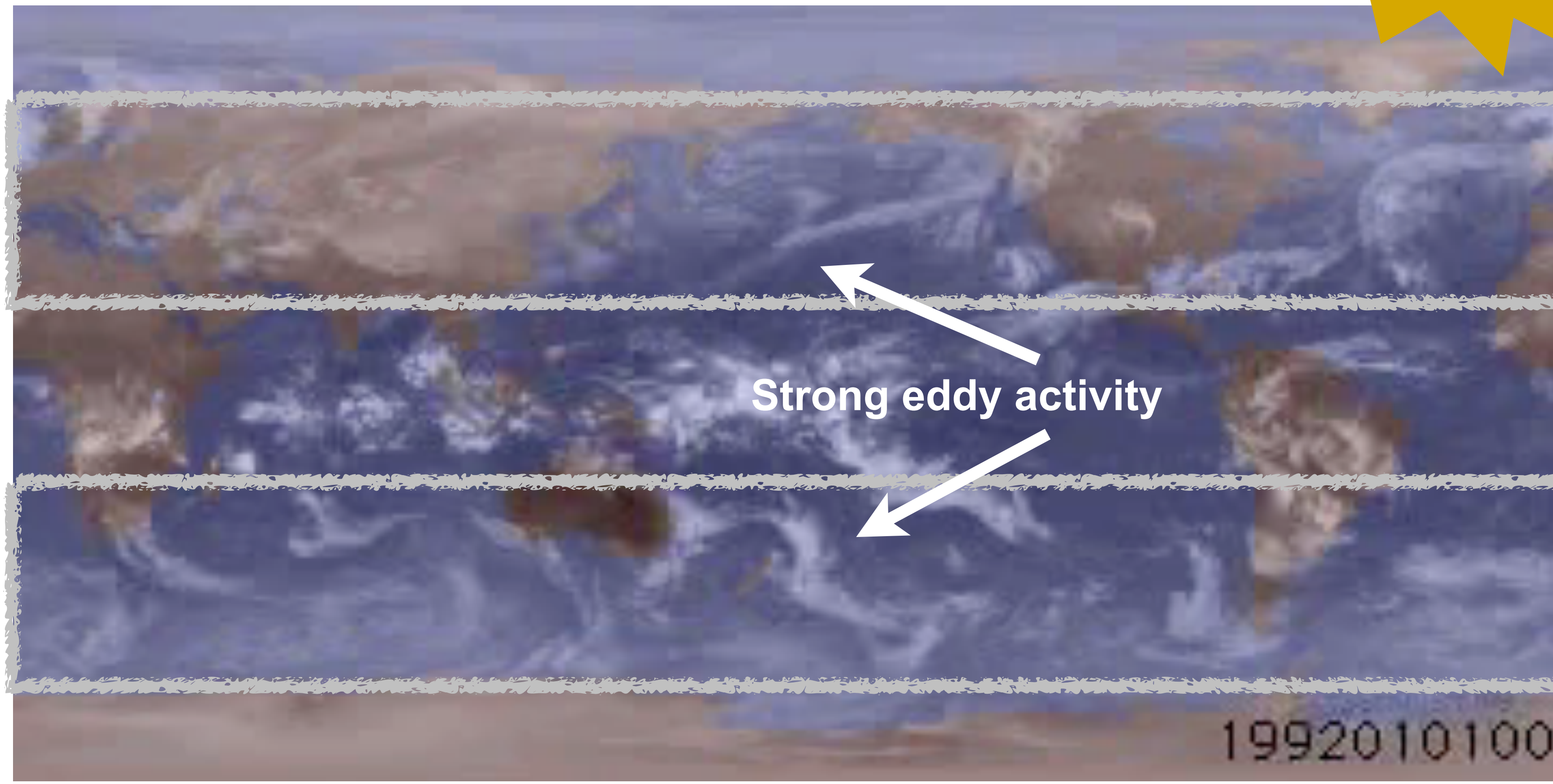


Review

Observations

- Eddy fields

Review



The British Atmospheric Data Centre (BADC)
www.badc.nerc.ac.uk/data/claus (infra-red)

Observations

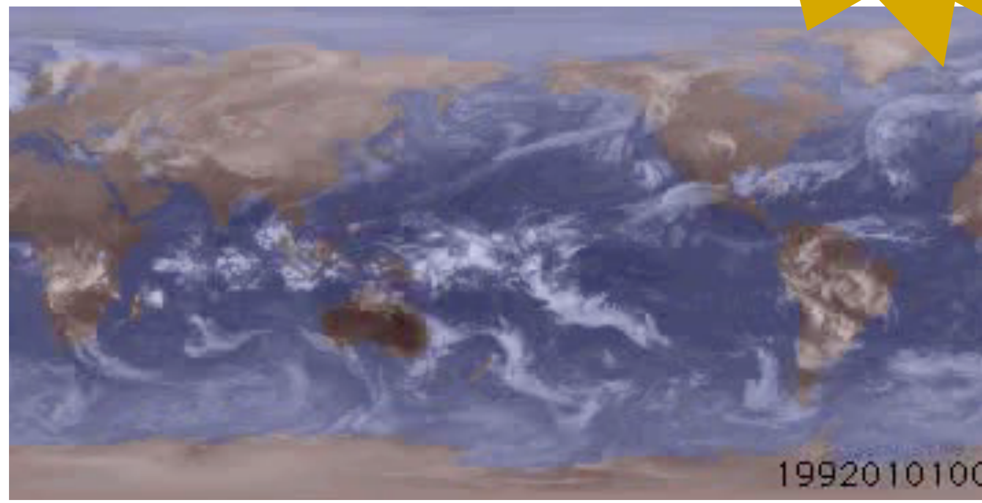
- Eddy fields

Review

Strong **baroclinic eddy** activity

$L_R \sim O(1000km)$

Synoptic time scale (2-8 days)



$$
\begin{aligned}
[\overline{AB}] &= [\overline{(\bar{A}+A')(\bar{B}+B')}] = [\bar{A}\bar{B}] + [\overline{A'B'}] \\
&= [([\bar{A}]+\bar{A}^*)([\bar{B}]+\bar{B}^*)] + [\overline{A'B'}] \\
&= [\bar{A}][\bar{B}] + [\bar{A}^*\bar{B}^*] + [\overline{A'B'}]
\end{aligned}
$$

$$A = [\bar{A}] + \bar{A}^* + A^{'}$$

Observations

- Eddy fields

- Kinetic energy:

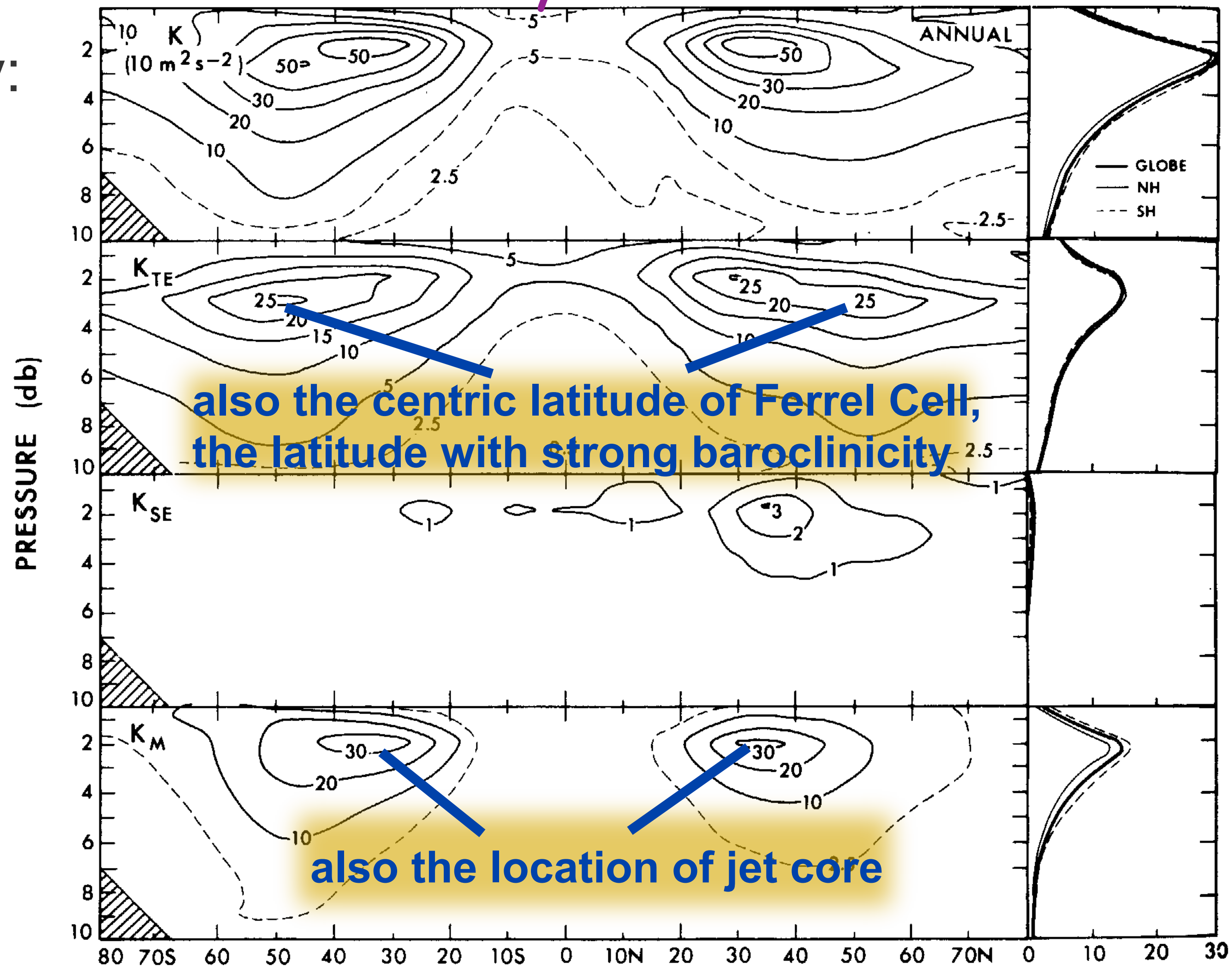
$$A = [\bar{A}] + \bar{A}^* + A'$$

$$K = K_M + K_{SE} + K_{TE}$$

$$K_M = \frac{1}{2}([\bar{u}]^2 + [\bar{v}]^2)$$

$$K_{\mathrm{SE}} = \frac{1}{2}[\bar{u}^{*2} + \bar{v}^{*2}]$$

$$K_{\mathrm{TE}} = \frac{1}{2}[\overline{u'^2} + \overline{v'^2}]$$



Observat

- Kinetic energy:

$$A = [\bar{A}] + \bar{A}^* + A'$$

$$K = K_M + K_{SE} + K_{TE}$$

$$K_M = \frac{1}{2}([\bar{u}]^2 + [\bar{v}]^2)$$

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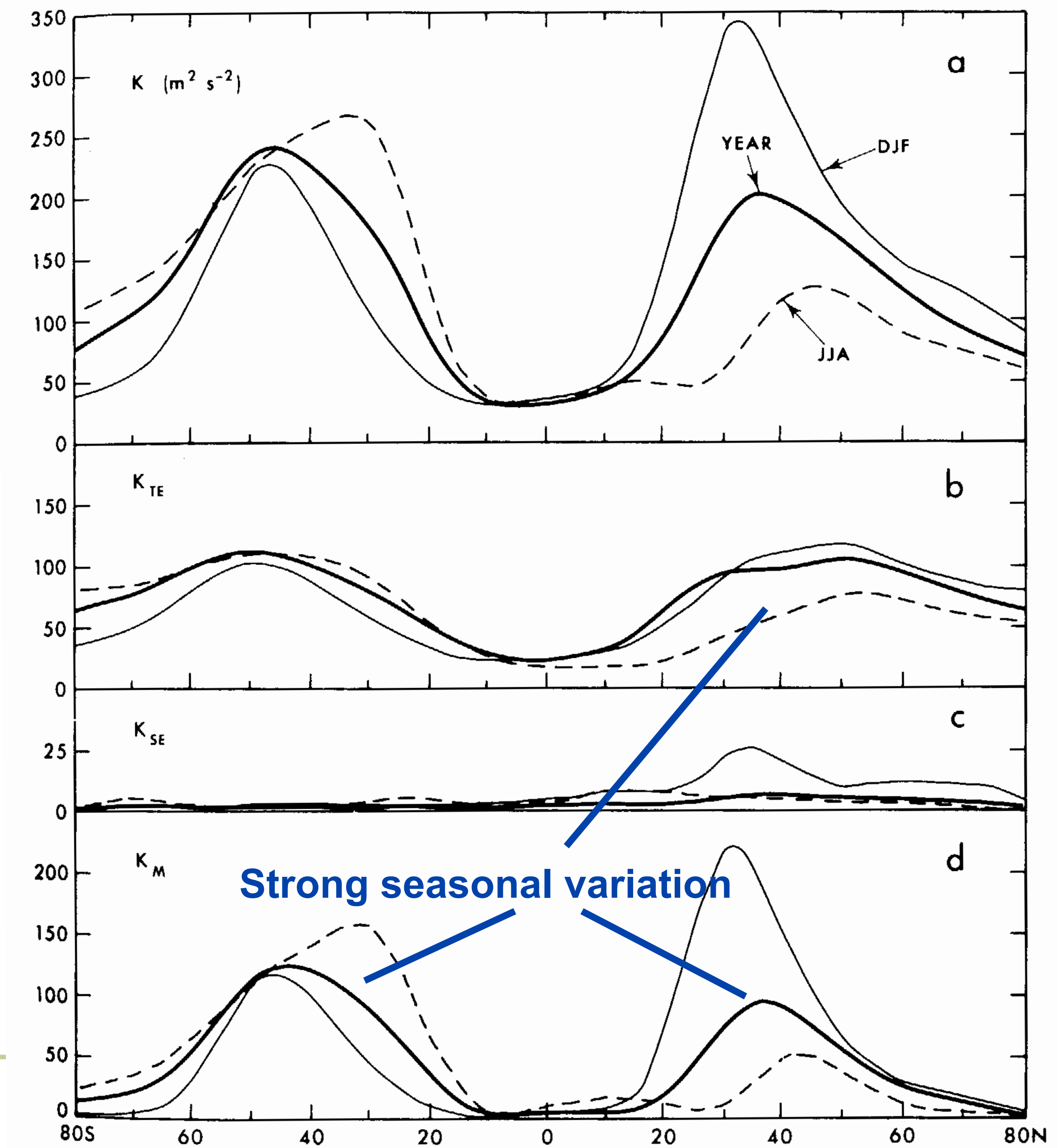
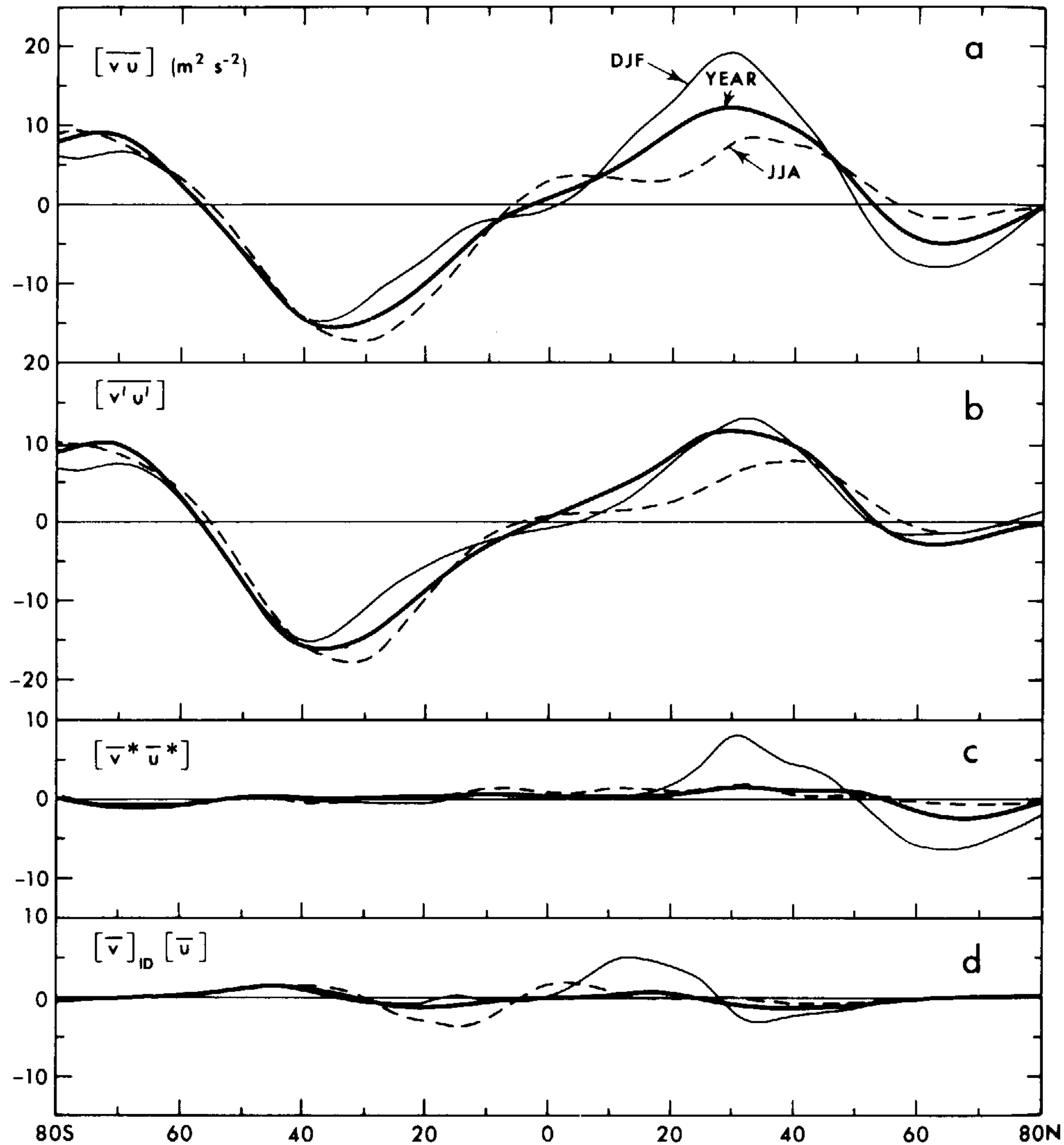


Observati...

- Momentum flux:

The total momentum flux is strongest around 30-40 degree north and south, which is mainly due to the contribution of transient eddies.

In N.H., the contributions from the zonal mean flow and the stationary eddies are comparable, but centered in the tropic and subtropic, respectively.



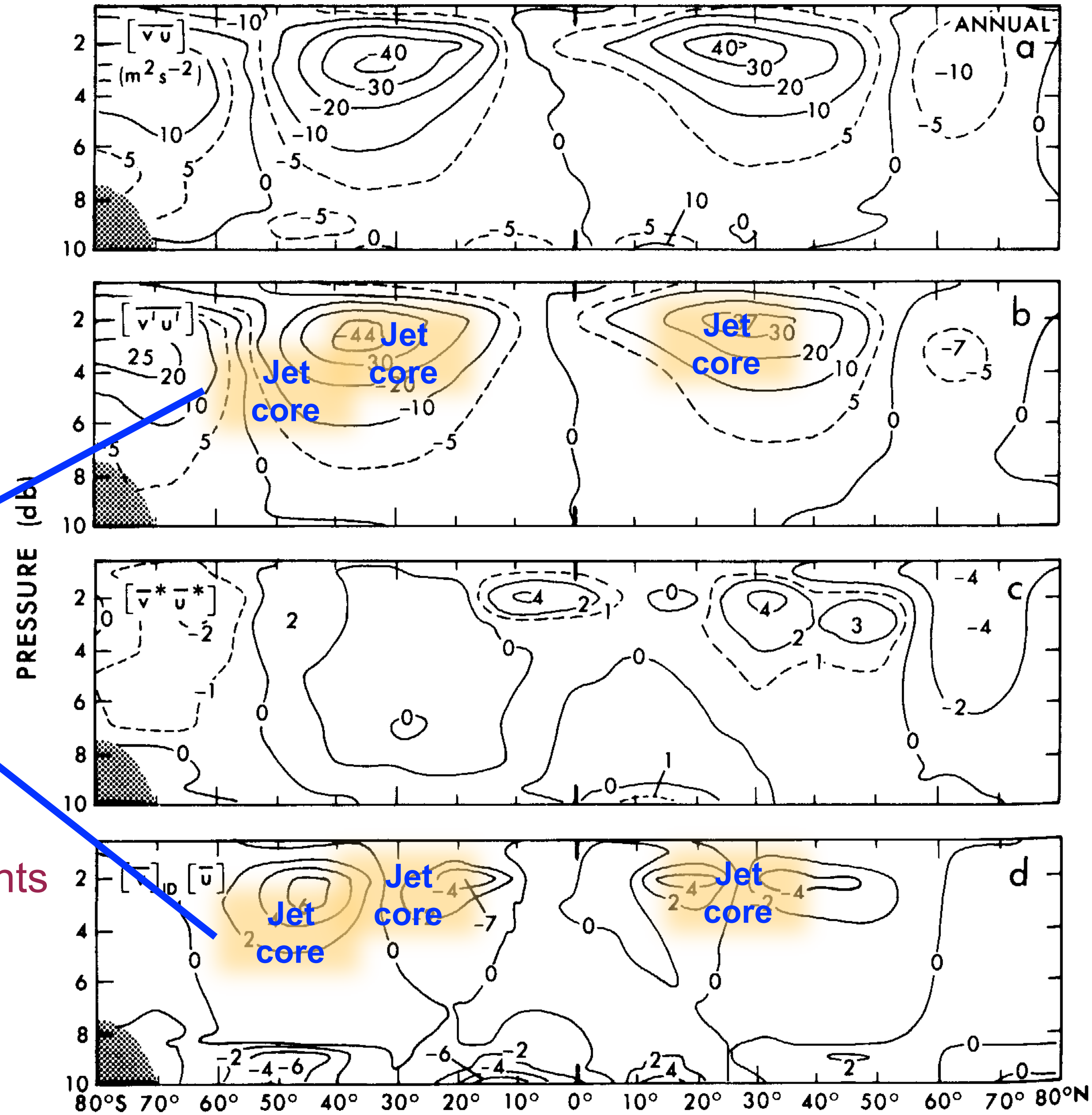
Observations

- Momentum flux:

The eddy components are centered at upper level, near tropopause.

The relation with jets

The zonal-mean components are centered near tropopause and surface.



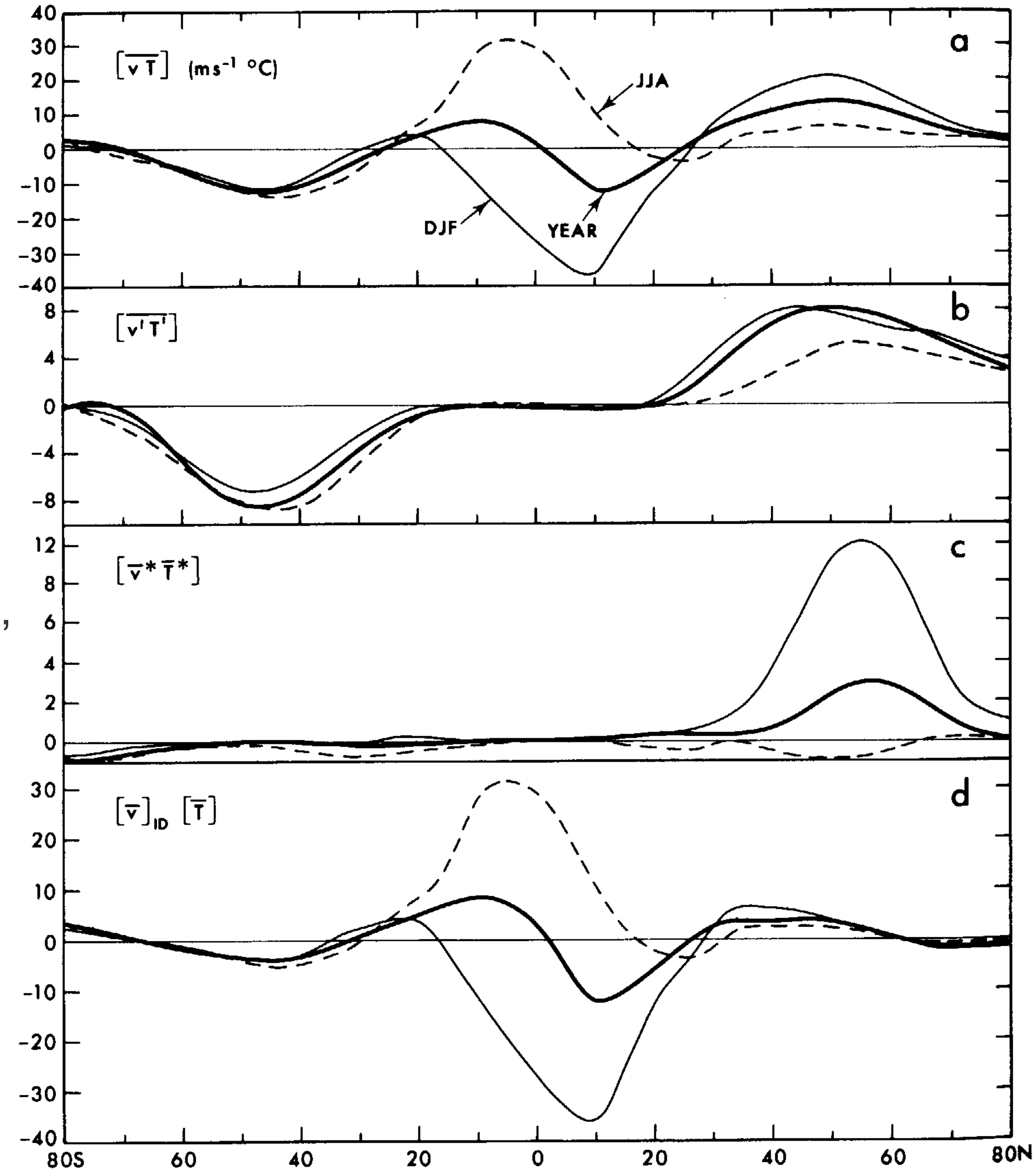
Observatio

- Heat flux:

Transient components: strongest at 40-50 degree, with obvious seasonal variation in N.H..

Stationary components: strongest at mid-latitude in N.H., whose directions are reversed from winter to summer.

Zonal mean flow: centered in the tropics, whose directions are reversed from winter to summer.

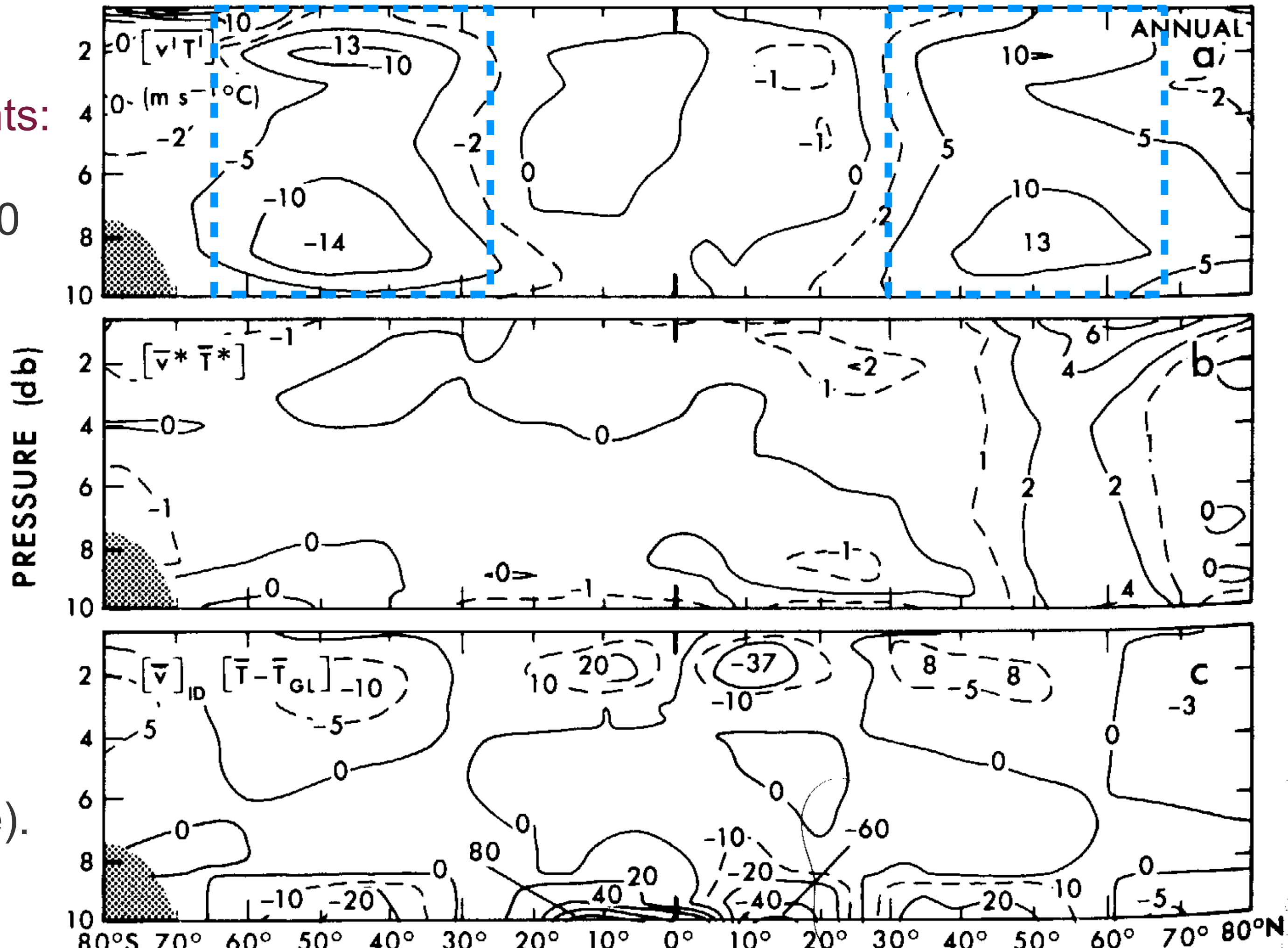


Observations

- Eddy fields

- Heat flux:

Transient components: two peaks in vertical direction (around 800 and 200 hPa).



Zonal-mean flow: two peaks in vertical direction (around 200 hPa and near surface).

Observations

- Summary:
  - Zonal-mean flow:
    - Ferrel Cell: an indirect cell centered at 40-60 degree, with strong seasonal variation in N.H.
    - Westerly jet: surface westerlies centered at 40-60 degree
  - Eddies: transient eddies are dominant with stationary eddies only obvious in N.H.
    - Kinetic energy
    - Momentum flux
    - Heat flux

The Ferrel Cell

eddy-zonal flow interaction (I)

- Start from the equations:
  - Momentum equation: $$\left(\frac{du}{dt}\right)_p - fv = -\left(\frac{\partial \Phi}{\partial x}\right)_p + F_x$$
  - Continuity equation: $$\nabla_p \cdot \boldsymbol{v} + \frac{\partial \omega}{\partial p} = 0$$
  - Thermodynamic equation: $$\left(\frac{d\ln\theta}{dt}\right)_p = \frac{Q}{c_p T}$$

$$\left(\frac{d}{dt}\right)_p = \left(\frac{\partial}{\partial t}\right)_p + u\left(\frac{\partial}{\partial x}\right)_p + v\left(\frac{\partial}{\partial y}\right)_p + \omega\frac{\partial}{\partial p}$$

Decompose into zonal mean and eddy components:

$$A = [A] + A^*$$

The Ferrel Cell
eddy-zonal flow interaction (I)

- Start from the equations:
  - Momentum equation:

$$\frac{\partial [u]}{\partial t}+\frac{\partial ([u][v])}{\partial y}+\frac{\partial ([u][\omega])}{\partial p}=-\frac{\partial ([u^*v^*])}{\partial y}-\frac{\partial ([u^*\omega^*])}{\partial p}+f[v]+[F_x]$$

  - Continuity equation:

$$\frac{\partial [v]}{\partial y}+\frac{\partial [\omega]}{\partial p}=0$$

  - Thermodynamic equation:

$$\frac{\partial [\theta]}{\partial t}+\frac{\partial ([v][\theta])}{\partial y}+\frac{\partial ([\omega][\theta])}{\partial p}=-\frac{\partial ([\theta^*v^*])}{\partial y}-\frac{\partial ([\theta^*\omega^*])}{\partial p}+\left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q]}{c_p}$$

$$\left(\frac{d}{dt}\right)_p=\left(\frac{\partial}{\partial t}\right)_p+u\left(\frac{\partial}{\partial x}\right)_p+v\left(\frac{\partial}{\partial y}\right)_p+\omega\frac{\partial}{\partial p}$$

Under the quasi-geostrophic approximation ($R_o \ll 1$), above equations can be simplified.

The Ferrel Cell

eddy-zonal flow interaction (I)

- Start from the equations:

$$\frac{\partial [u]}{\partial t} + \frac{\partial ([u][v])}{\partial y} + \frac{\partial ([u][\omega])}{\partial p} = -\frac{\partial ([u^* v^*])}{\partial y} - \frac{\partial ([u^* \omega^*])}{\partial p} + f[v] + [F_x]$$

$$\frac{\partial [\theta]}{\partial t} + \frac{\partial ([v][\theta])}{\partial y} + \frac{\partial ([\omega][\theta])}{\partial p} = -\frac{\partial ([\theta^* v^*])}{\partial y} - \frac{\partial ([\theta^* \omega^*])}{\partial p} + \left(\frac{p_o}{p}\right)^{R/c_p} \frac{[Q]}{c_p}$$

$$\frac{\partial [v]}{\partial y} + \frac{\partial [\omega]}{\partial p} = 0$$

- Simplification:
  - For midlatitude large scale flow, the eddy components of the meridional heat and momentum transports are dominant. (recall the observations)

$$\frac{\partial}{\partial y}[u^* v^*] \gg \frac{\partial}{\partial y}([u][v]) \qquad \frac{\partial}{\partial y}[\theta^* v^*] \gg \frac{\partial}{\partial y}([\theta][v])$$

  - From the QG approximation, $\frac{\partial \omega^*}{\partial p} \sim R_o \frac{\partial v^*}{\partial y} \rightarrow \frac{\partial}{\partial y}[u^* v^*] \gg \frac{\partial}{\partial p}[u^* \omega^*]$
  - Horizontal variation of the stratification is small: $\frac{\partial}{\partial p}([\theta][\omega]) \approx [\omega] \frac{\partial \theta_s}{\partial p}$

The Ferrel Cell
eddy-zonal flow interaction (I)

- The simplified equations:
  - Momentum equation:

$$\frac{\partial [u]}{\partial t} = -\frac{\partial ([u^* v^*])}{\partial y} + f[v] + [F_x]$$

  - Continuity equation:

$$\frac{\partial [v]}{\partial y} + \frac{\partial [\omega]}{\partial p} = 0$$

  - Thermodynamic equation:

$$\frac{\partial [\theta]}{\partial t} + [\omega]\frac{\partial \theta_s}{\partial p} = -\frac{\partial ([\theta^* v^*])}{\partial y} + \left(\frac{p_o}{p}\right)^{R/c_p} \frac{[Q]}{c_p}$$

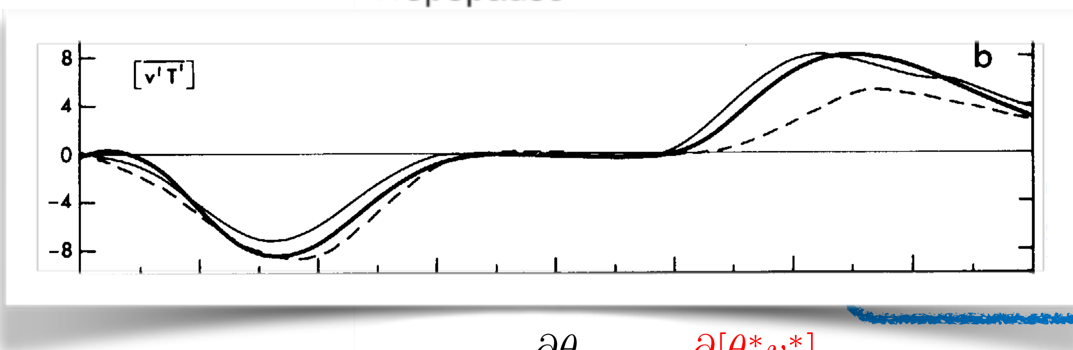
$$\left(\frac{d}{dt}\right)_p = \left(\frac{\partial}{\partial t}\right)_p + u\left(\frac{\partial}{\partial x}\right)_p + v\left(\frac{\partial}{\partial y}\right)_p + \omega\frac{\partial}{\partial p}$$

Under the quasi-geostrophic approximation $(R_o \ll 1)$

The Ferrel Cell

- The balance equations:

Tropopause



$$\ldots[u^*v^*]) \over \partial y} + f[v] + [F_x]$$

$$\omega \frac{\partial \theta_s}{\partial p} \sim -\frac{\partial [\theta^* v^*]}{\partial y} < 0 \qquad \omega \frac{\partial \theta_s}{\partial p} \sim -\frac{\partial [\theta^* v^*]}{\partial y} > 0$$

$$\frac{\partial [\theta]}{\partial t} + [\omega] \frac{\partial \theta_s}{\partial p} = -\frac{\partial ([\theta^* v^*])}{\partial y} + \left(\frac{p_o}{p}\right)^{R/c_p} \frac{[Q]}{c_p}$$

Boundary layer

Ground

Subtropics Latitude Subpolar

The Ferrel Cell

- The balance equations:

Tropopause

$$fv \sim \frac{\partial[u^*v^*]}{\partial y} < 0$$

$$\frac{\partial[u]}{\partial t} = -\frac{\partial([u^*v^*])}{\partial y} + f[v] + [F_x]$$

$$\omega\frac{\partial\theta_s}{\partial p} \sim -\frac{\partial[\theta^*v^*]}{\partial y} < 0 \qquad \omega\frac{\partial\theta_s}{\partial p} \sim -\frac{\partial[\theta^*v^*]}{\partial y} > 0$$

$$\frac{\partial[\theta]}{\partial t} + [\omega]\frac{\partial\theta_s}{\partial p} = -\frac{\partial([\theta^*v^*])}{\partial y} + \left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q]}{c_p}$$

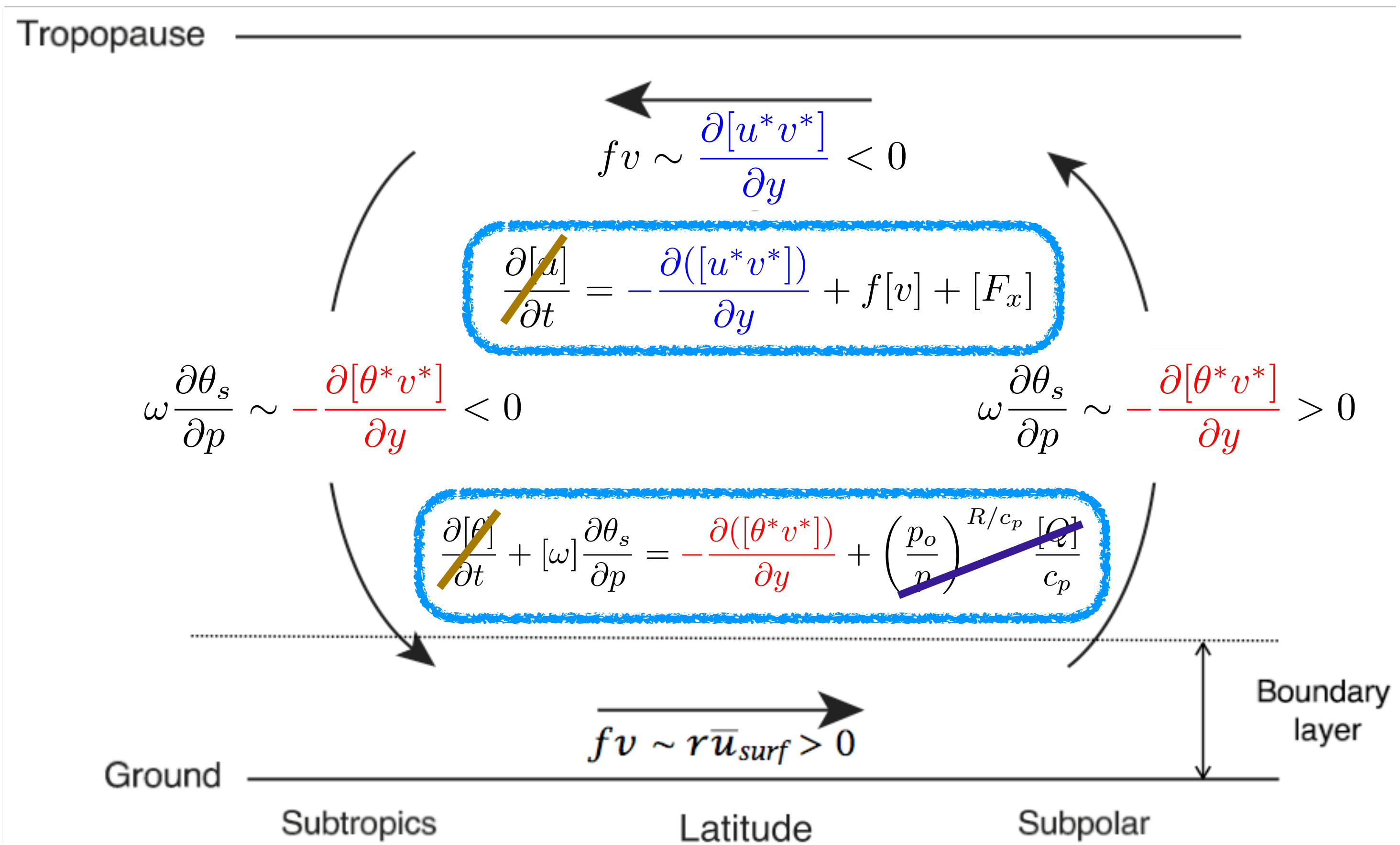
Boundary layer

Ground

Subtropics | Latitude | Subpolar

The Ferrel Cell

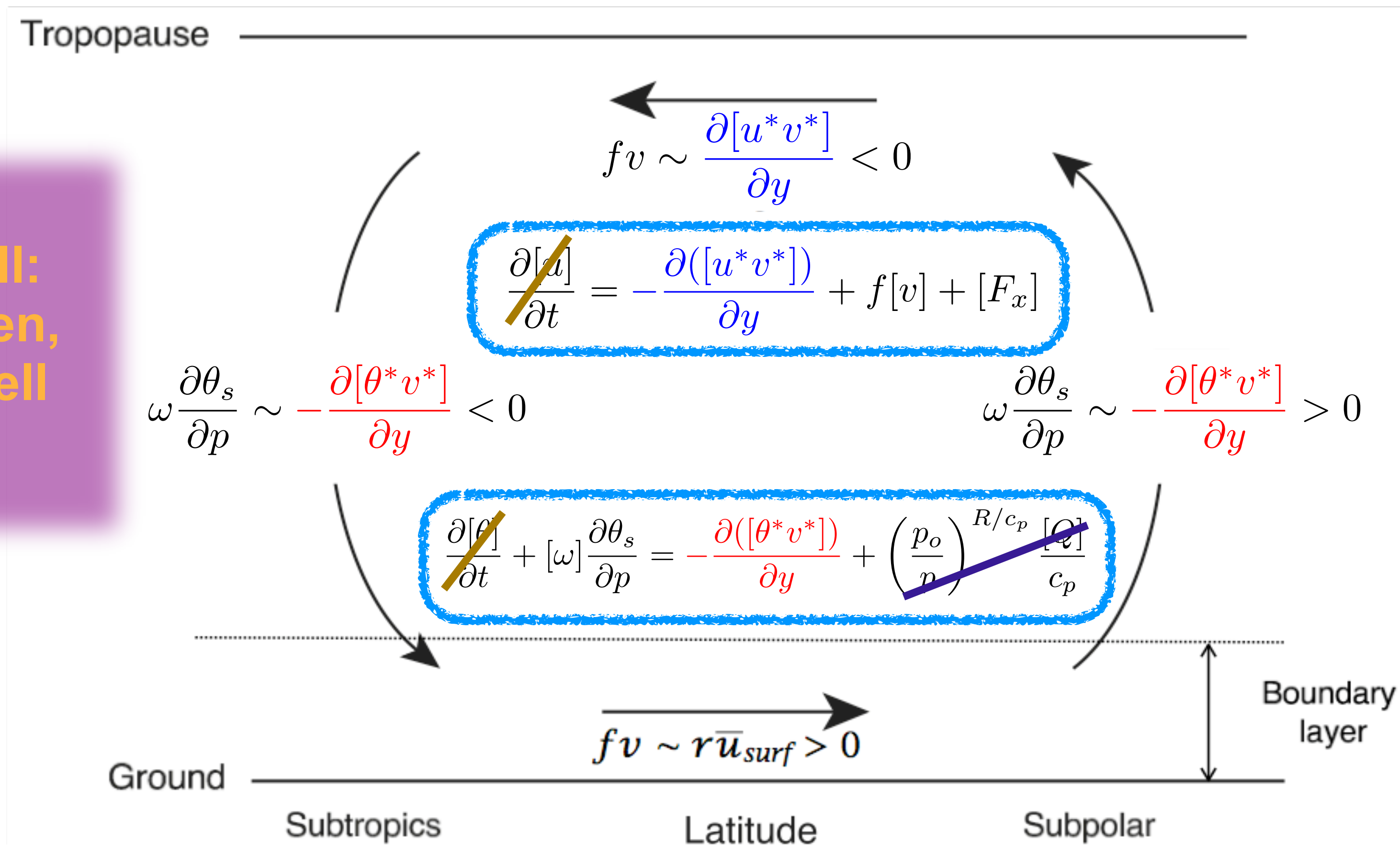
- The balance equations:



The Ferrel Cell

- The balance equations:

Ferrel Cell: eddy-driven, indirect cell



The Ferrel Cell

- In **isentropic** coordinate

$$(x, y, z) \Leftrightarrow (x, y, \theta)$$

$$\frac{D\theta}{Dt} = \dot{\theta}$$

$$\begin{aligned}\frac{D}{Dt} &= \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla_\theta + \frac{D\theta}{Dt}\frac{\partial}{\partial \theta} \\ &= \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla_\theta + \dot{\theta}\frac{\partial}{\partial \theta}\end{aligned}$$

zero for adiabatic flow

Isentrope: An isopleth of entropy. In meteorology it is usually identified with an isopleth of potential temperature.

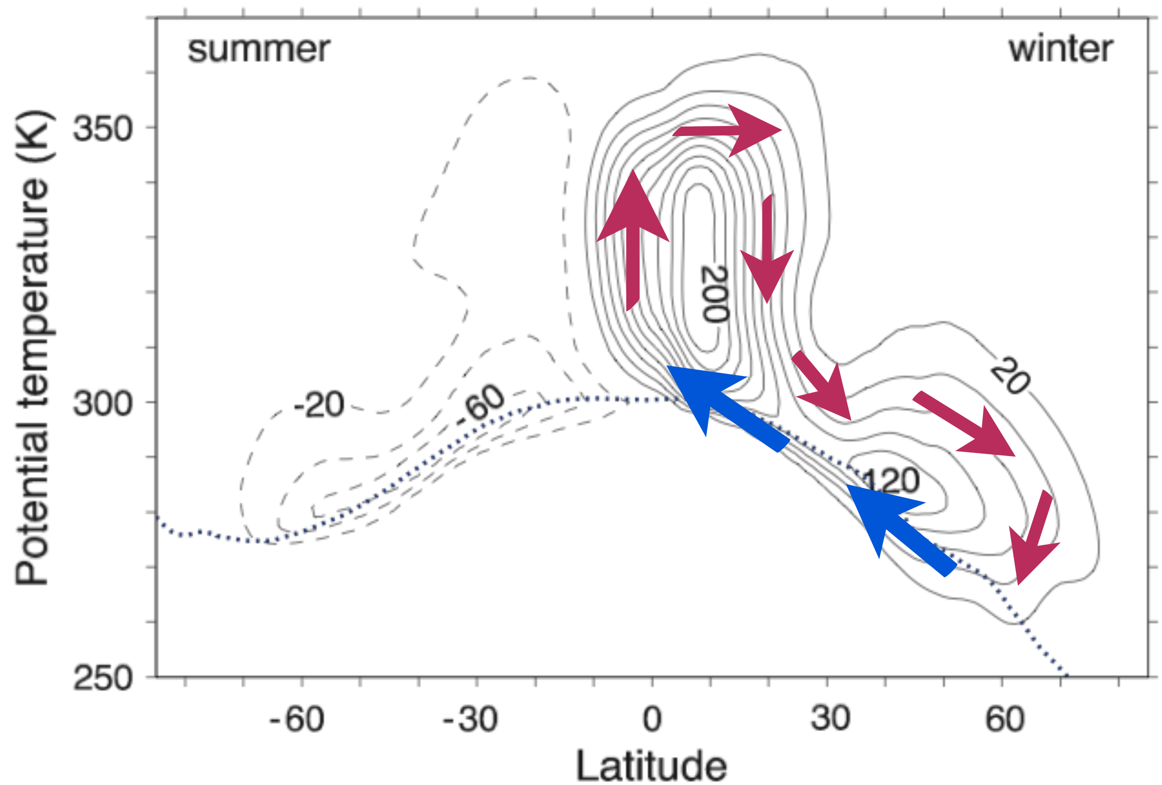
The Ferrel Cell

- In **isentropic** coordinate

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla_\theta + \frac{D\theta}{Dt}\frac{\partial}{\partial\theta}$$
$$= \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla_\theta + \dot{\theta}\frac{\partial}{\partial\theta}$$

The **direction** of Ferrel cell is **reversed** in the isentropic coordinate.

Interactions between Hadley and Ferrel cells are expected.



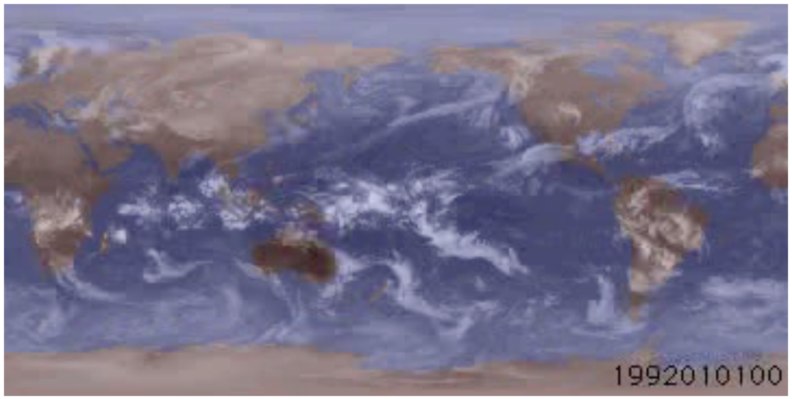
(Fig.11.4, Vallis, 2006)

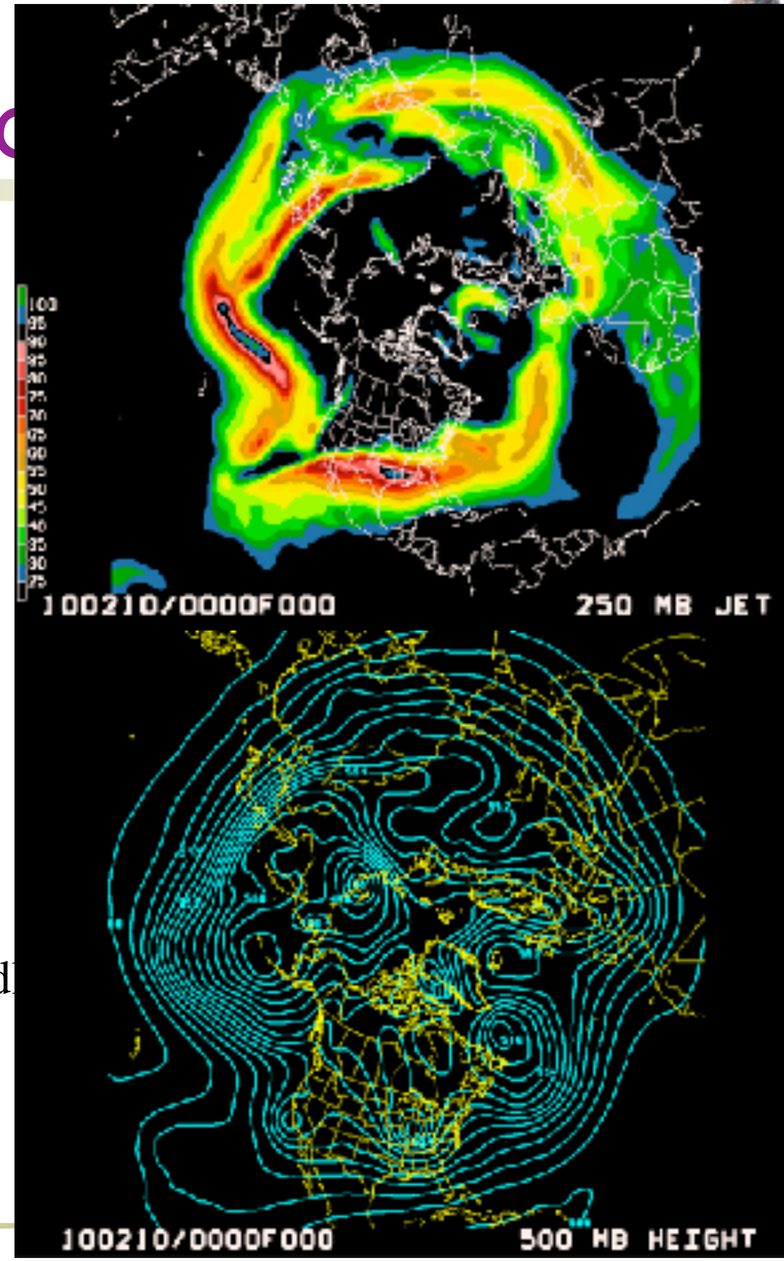
Outline

- Observations
- The Ferrel Cell
- **Baroclinic eddies**
  - Review: baroclinic instability and baroclinic eddy life cycle
  - Eddy-mean flow interaction
  - Transformed Eulerian Mean equation
- Eddy-driven jet
- The energy cycle

Observed

Baroclinic e…



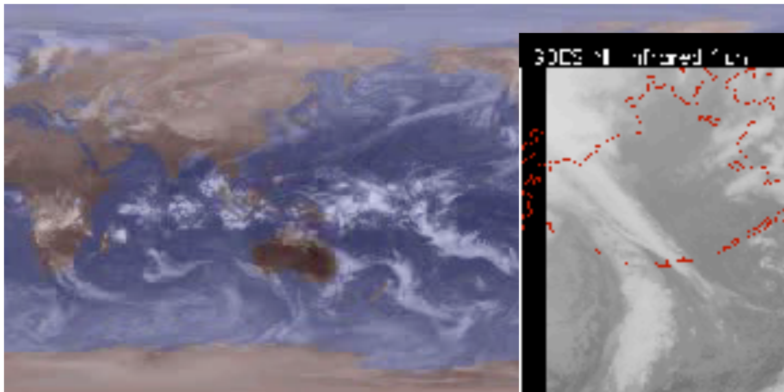


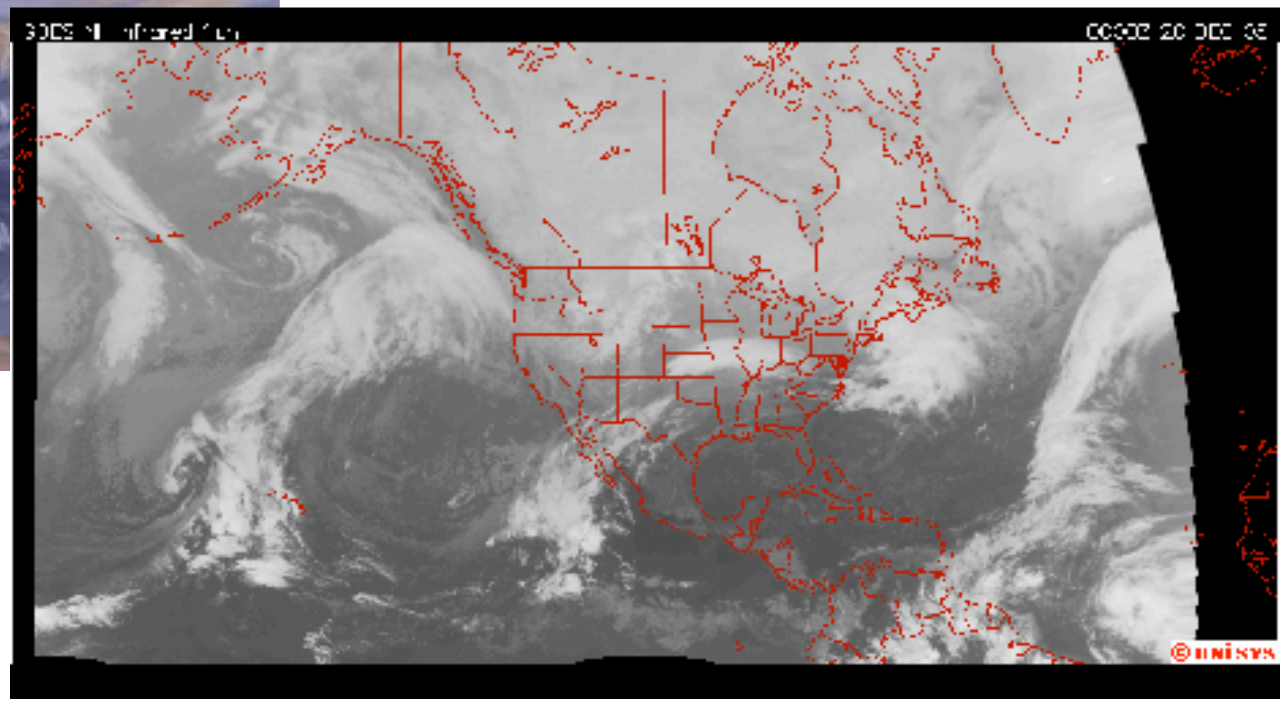
Strong **baroclinic eddy** activity in the mid…

Synoptic time scale (2-8 days)

$L_R \sim O(1000km)$

Observed
Baroclinic eddies





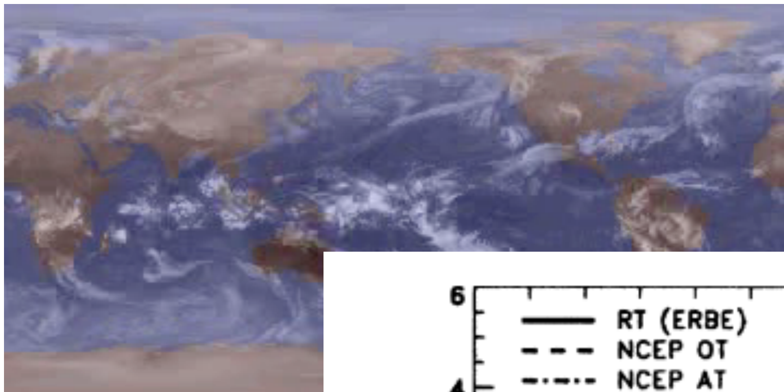
Strong **baroclinic eddy** activity in the midlatitudes

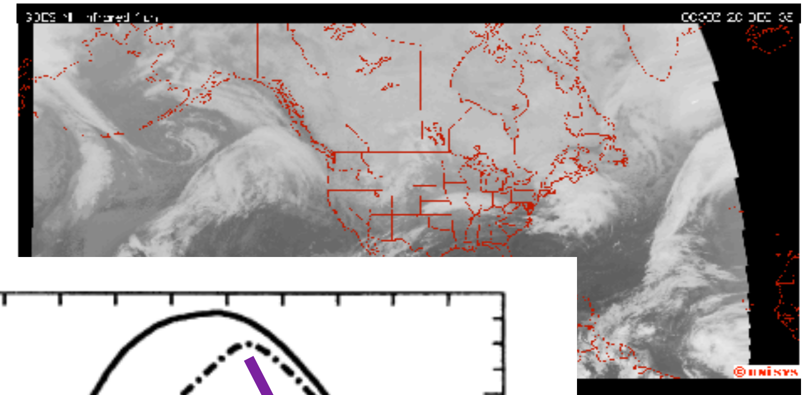
Synoptic time scale (2-8 days)

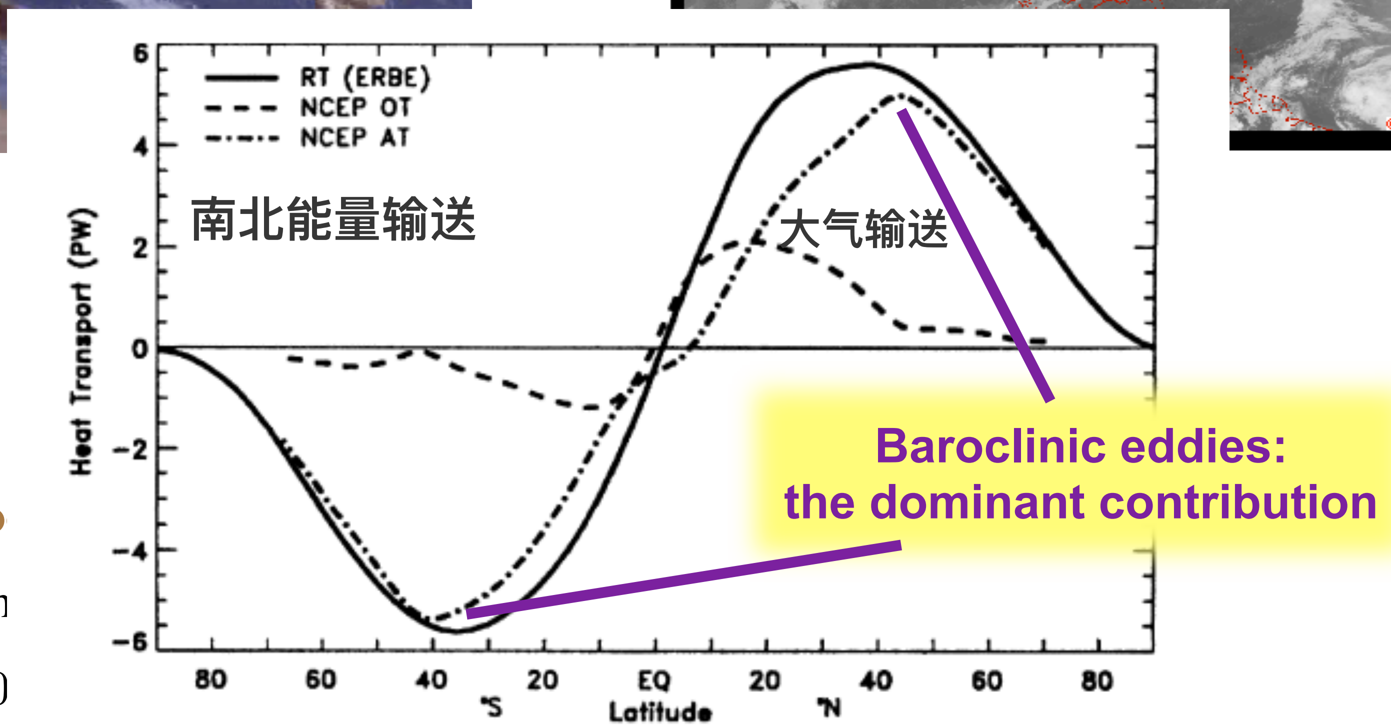
$L_R \sim O(1000km)$

Observed

Baroclinic eddies







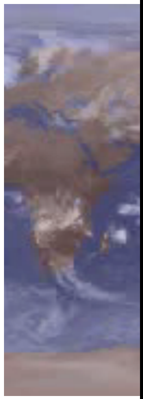
Strong **baro**

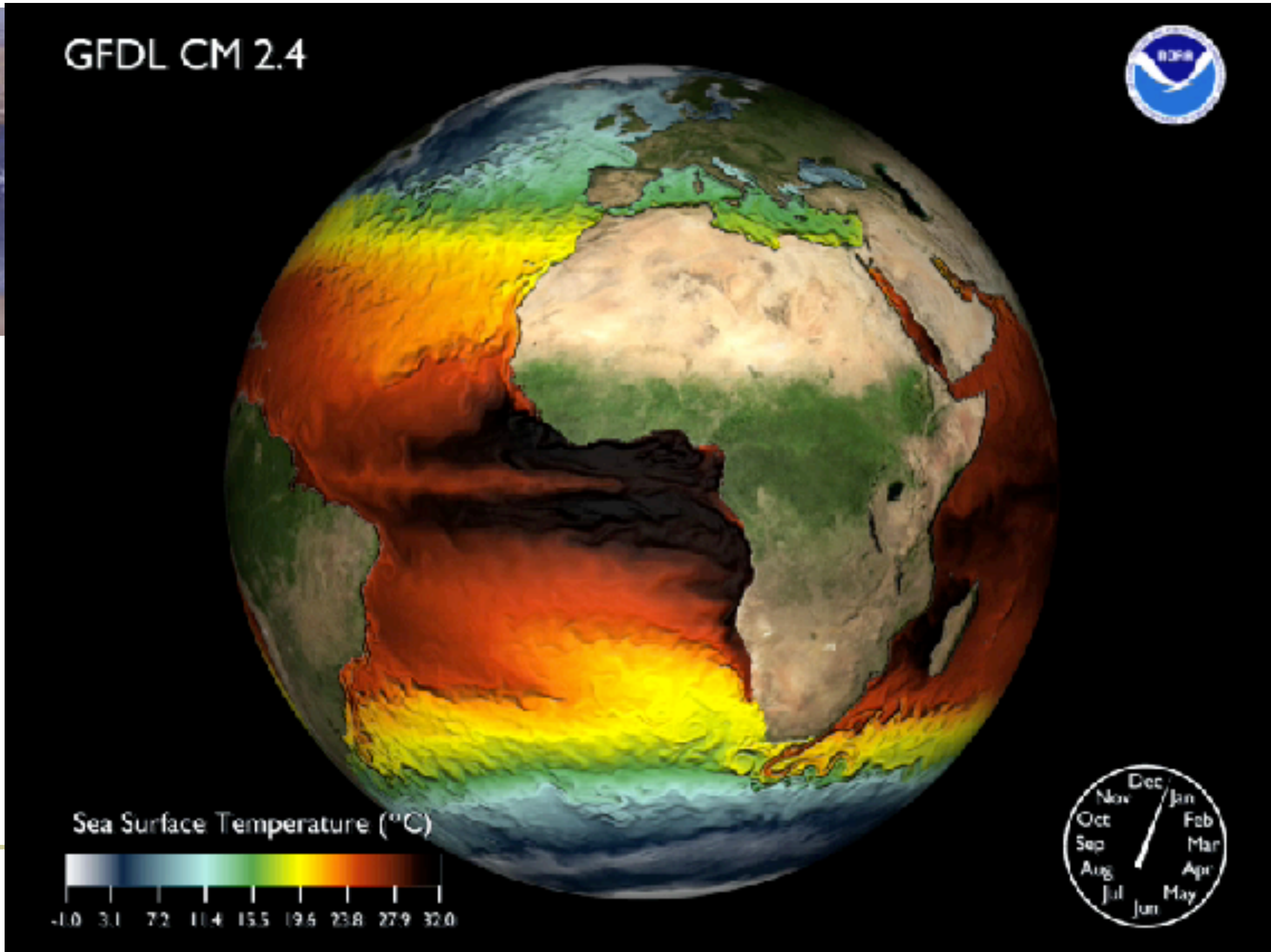
Synoptic tin

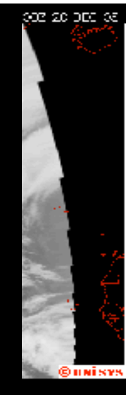
$L_R \sim O(10$

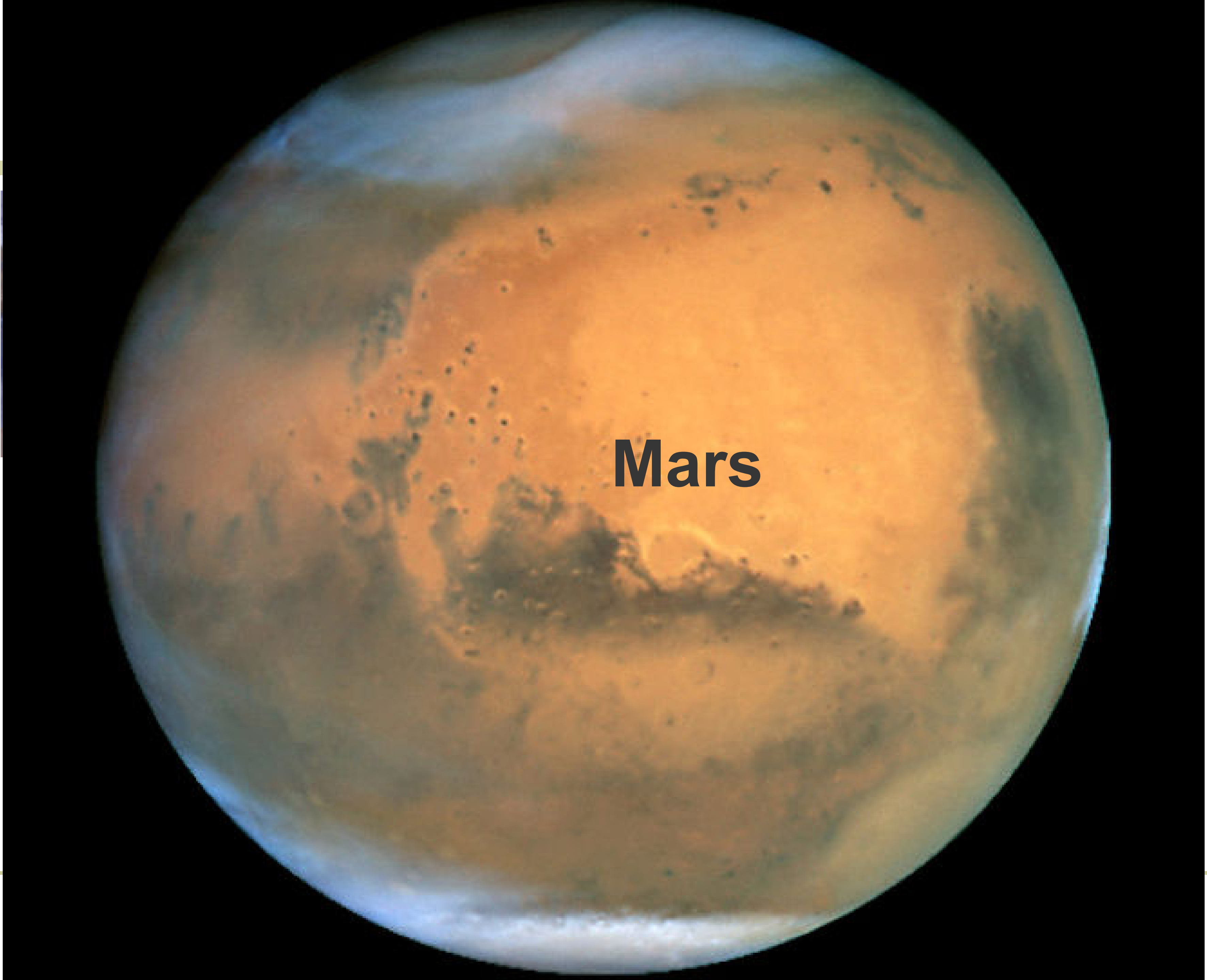
Observed

Baroclinic eddies

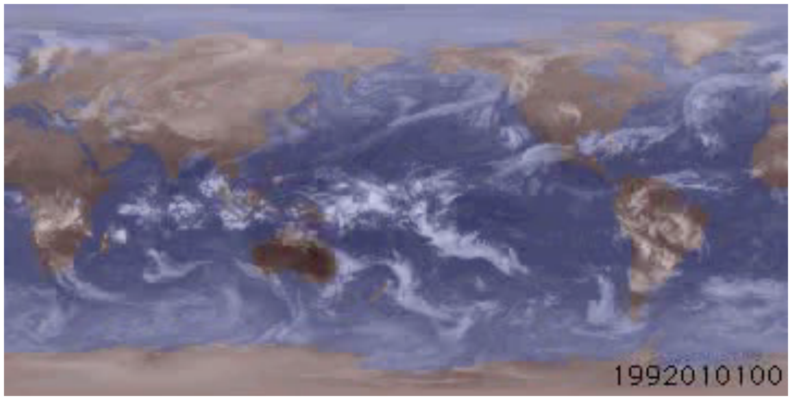


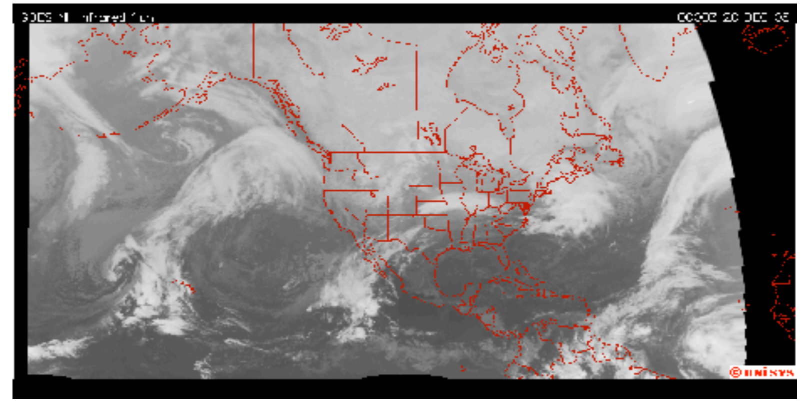




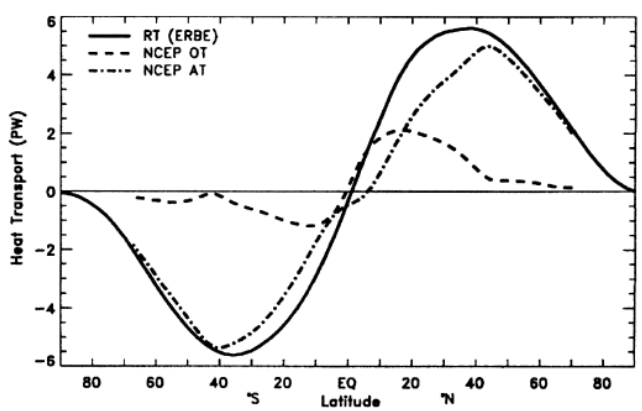

Mars

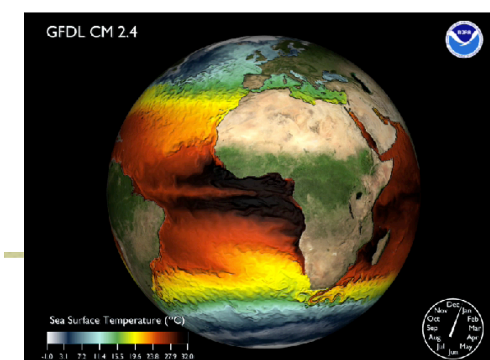
Observed
Baroclinic eddies





斜压扰动是中纬度各种日常天气现象背后主要的物理过程，更是维持大气环流和地球大气能量南北输送以及现在的地球气候状态的最主要动力机制，此外斜压扰动也是海洋和其他一些行星大气中的主要动力过程。





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  - Review: baroclinic instability and baroclinic eddy life cycle
  - Eddy-mean flow interaction, E-P flux
  - Transformed Eulerian Mean equations
- Eddy-driven jet
- Energy cycle

Baroclinic eddies

- baroclinic instability

- Instability:

  - Phenomenon: Given a *basic flow* with *perturbations* at the initial moment, if the perturbation *grows with time*, the basic flow is always taken *unstable*.

  - Mathematics: $P \propto Ae^{\alpha t}, \exists\, \alpha > 0$ (相对于波动解:$P \propto Ae^{i\omega t}$)

  - Energy: 能量源 → 扰动动能

- Linear Instability: the instability that arises in *a linear system*.

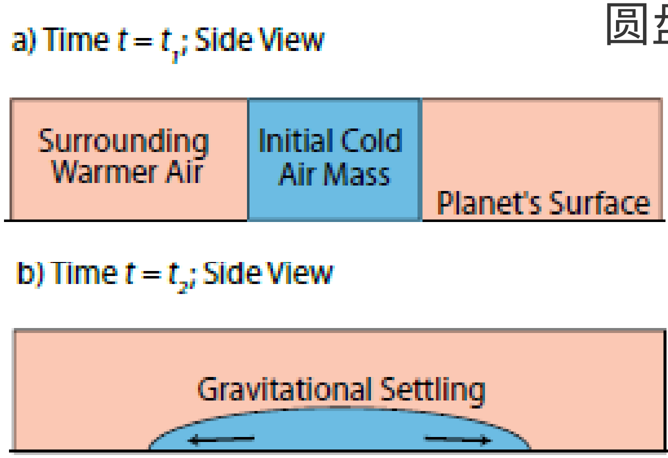
Baroclinic eddies

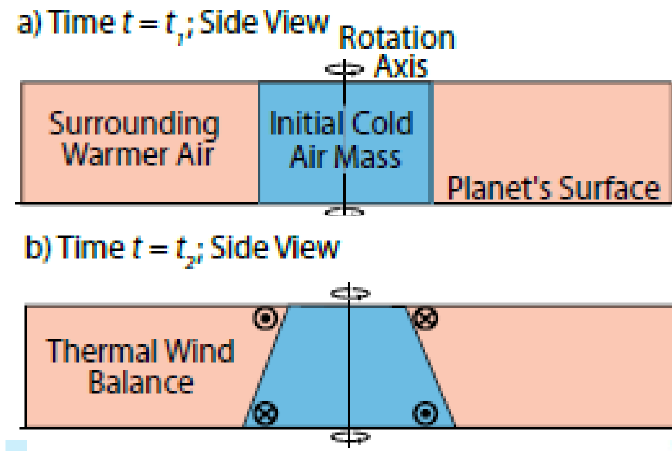
- baroclinic instability

- **Baroclinic Instability** - “is an instability that arises in *rotating*, *stratified* fluids that are subject to a *horizontal temperature gradient*”.

■ Baroclinic Instability - "is an instability that arises in *rotating*, *stratified* fluids that are subject to a *horizontal temperature gradient*".

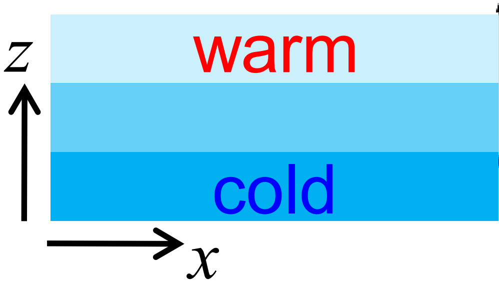
圆盘实验

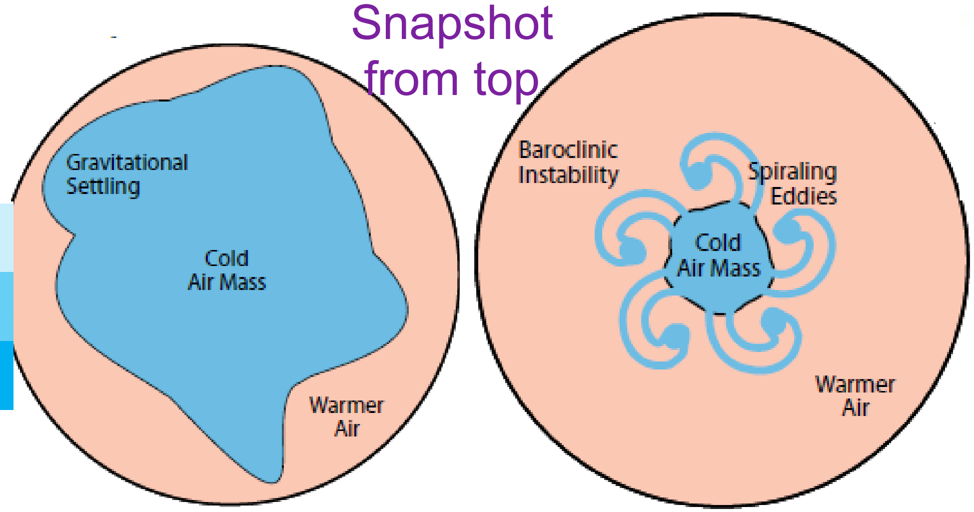


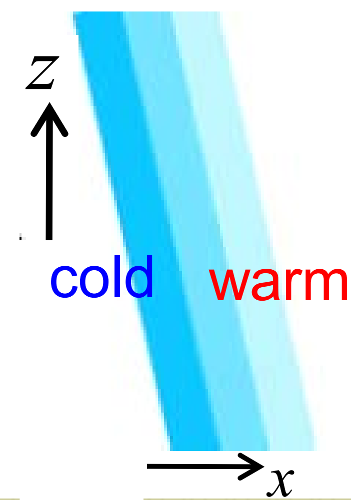


Snapshot from top

In equilibrium

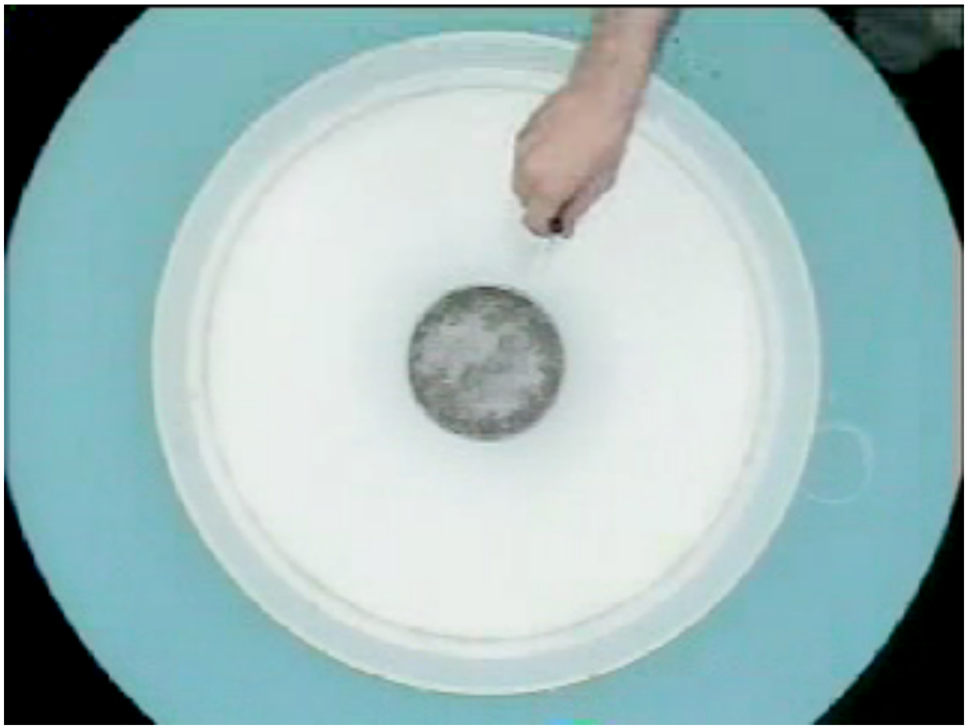


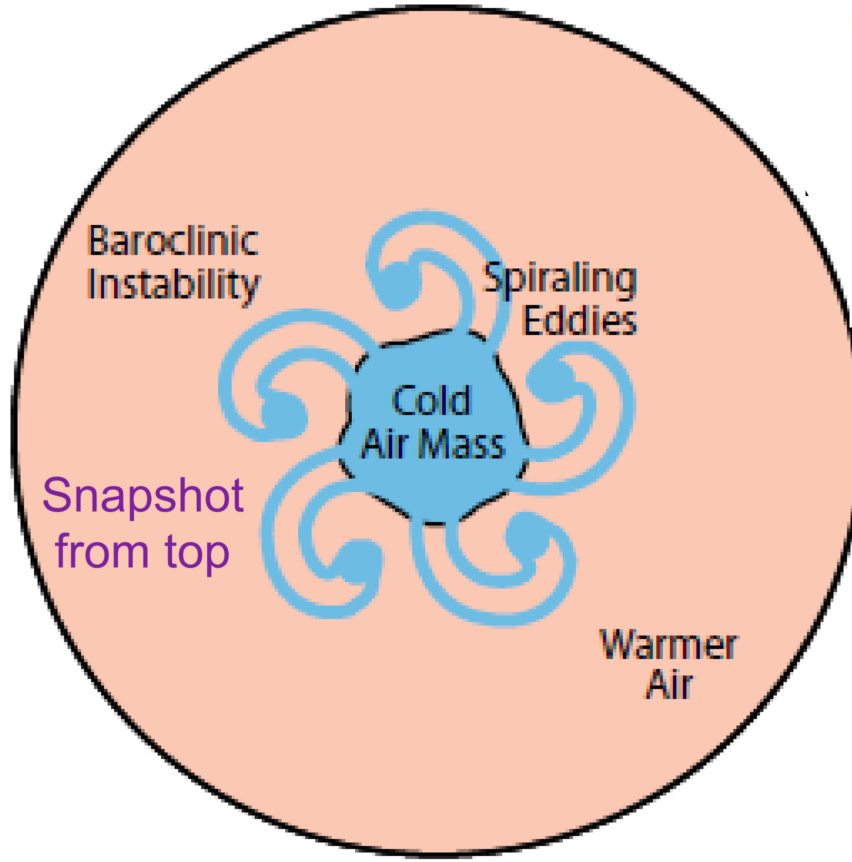




Above from Prof. Fang Juan's class slides

- **Baroclinic Instability** - "is an instability that arises in *rotating*, *stratified* fluids that are subject to a *horizontal temperature gradient*".





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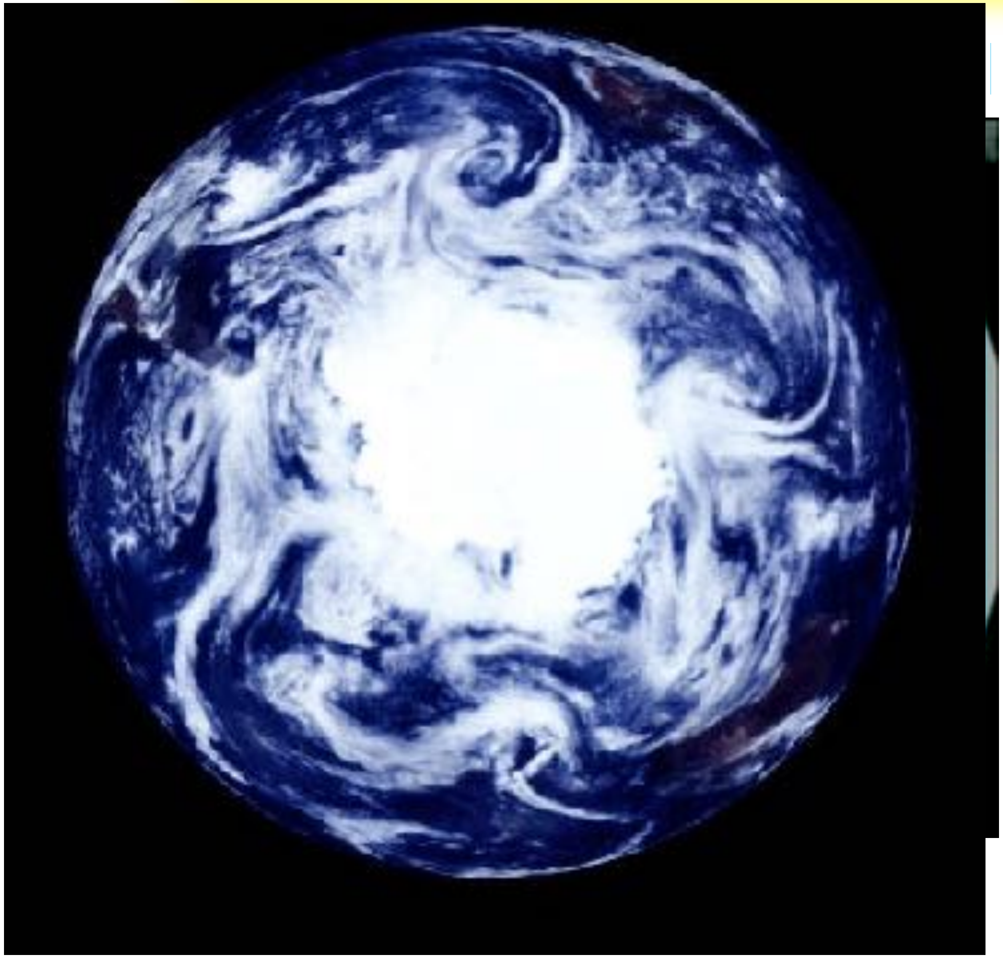

Satellite view at south pole, from NASA.