第四章:

中纬度的经向环流系统(III)

- Ferrel cell, baroclinic eddies and the westerly jet

授课教师：张洋

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Outline

- Observations
- The Ferrel Cell
- Baroclinic eddies
  - Review: baroclinic instability and baroclinic eddy life cycle
  - Eddy-mean flow interaction, E-P flux
  - Transformed Eulerian Mean equations
- Eddy-driven jet
- Energy cycle
Baroclinic Instability - “is an instability that arises in *rotating*, *stratified* fluids that are subject to a *horizontal temperature gradient*”.

- **Energetics:**
  - PE $\rightarrow$ KE

- **Mathematics:**
  - Linear Baroclinic Instability

  - Linear baroclinic system $\rightarrow$

  - Eady’s model (1949)
  - Charney’s model (1947)
Baroclinic eddies
- linear baroclinic instability

- Eady’s model (1949)
  a) The basic zonal flow has uniform vertical shear,
     \[ U_o(Z) = \Lambda Z, \quad \Lambda \text{ is a constant} \]
  b) The fluid is uniformly stratified, \( N^2 \) is a constant.
  c) Two rigid lids at the top and bottom, flat horizontal surface, that is
     \[ \omega = 0 \text{ at } Z = 0 \text{ and } H. \]
  d) The motion is on the \( f \)-plane, that is \( \beta = 0 \)

- Charney’s model (1947)

The most distinguished difference with Eady’s model is that beta effect is considered.
Baroclinic eddies
- linear baroclinic instability

Variable = Basic state + Perturbation
\[ u(x, t) = U(z) + u'(x, t) \]

\[ u'(x, t) \ll U(z) \]

Linearized PV equation (q=PV):
\[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) q' + \frac{\partial \psi}{\partial x} \frac{\partial \bar{q}}{\partial y} = 0 \]

\[ q' = \frac{\partial^2 \psi'}{\partial x^2} + \frac{\partial^2 \psi'}{\partial y^2} + \frac{f_0^2}{\rho_s} \frac{\partial}{\partial z} \left( \frac{\rho_s}{N^2} \frac{\partial \psi'}{\partial z} \right) \]

\[ \bar{q} = \frac{\partial^2 \bar{\psi}}{\partial y^2} + \beta y + \frac{f_0^2}{\rho_s} \frac{\partial}{\partial z} \left( \frac{\rho_s}{N^2} \frac{\partial \bar{\psi}}{\partial z} \right) \]

Find the conditions for non-trivial solutions and \( C_i > 0 \)

Small amplitude assumption

Linear baroclinic system:
- Eady model
- Charney model

Normal mode assumption

Obtain the solutions, e.g.
- Instability conditions
- Growth rate
- Most unstable mode

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Conclusions:

**Necessary condition for baroclinic instability:** PV gradient changes sign in the interior or boundaries (Charney-stern theory), according to which the midlatitude atmosphere is baroclinic unstable. Different models, i.e. Eady and Charney models have more rigorous conditions.

**Growth rate:** \( \sigma = k c_i \approx 0.3 \frac{\Lambda \frac{f_o}{N}}{\beta} \) in both Eady and Charney models!

**Most unstable mode:**

\[
\begin{align*}
    k_{\text{max}}^{-1} & \propto L_d = \left( \frac{NH}{f_o} \right) \\
    & \text{Eady} \\
    k_{\text{max}}^{-1} & \propto \frac{\Lambda \frac{f_o}{\beta N}}{\beta} \\
    & \text{Charney}
\end{align*}
\]
Baroclinic eddies
- linear baroclinic instability

“JULE CHARNEY was one of the dominant figures in atmospheric science in the three decades following World War II. Much of the change in meteorology from an art to a science is due to his scientific vision and his thorough commitment to people and programs in this field.”

-- by Norman Phillips
Baroclinic eddies

- From linear to nonlinear

Basic flow or Pre-existing flow (without zonal variation and baroclinic unstable)

Small perturbation

Perturbations grow with time (finite amplitude pert.)

Linear process

Reduce the zonal flow temperature gradient; stabilize the lower level stratification; enhance the westerly jet

Nonlinear interactions

Eddy-mean interactions (Adjust the zonal flow)

Equilibrated states between the adjusted zonal flow and baroclinic eddies
Baroclinic eddies

- From linear to nonlinear

Basic flow
or
Pre-existing flow
(without zonal variation and baroclinic unstable)

Small perturbation

Perturbations grow with time
(finite amplitude pert.)

Eddy-mean interactions
(Adjust the zonal flow)

Equilibrated states between the adjusted zonal flow and baroclinic eddies

Numerical results from a QG model
(Zhang, 2009)

E-P flux

Equilibrium

Review
Outline

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  - Transformed Eulerian Mean equations
- Eddy-driven jet
- The energy cycle
The Ferrel Cell

The balance equations:

\[ \frac{\partial \theta}{\partial t} + \frac{\partial (\theta v)}{\partial y} = f v + [F_x] \]

\[ \omega \frac{\partial \theta_s}{\partial p} \sim - \frac{\partial [\theta^* v^*]}{\partial y} < 0 \]

\[ \omega \frac{\partial \theta_s}{\partial p} \sim - \frac{\partial [\theta^* v^*]}{\partial y} > 0 \]

\[ \frac{\partial \theta}{\partial t} + [\omega] \frac{\partial \theta_s}{\partial p} = - \frac{\partial ([\theta^* v^*])}{\partial y} + \left( \frac{p_o}{p} \right)^{R/c_p} (\theta) \]

- Eddy fields
- Heat flux:
- Transient components: strongest at 40-50 degree, with obvious seasonal variation in N.H.
- Stationary components: strongest at mid-latitude in N.H., whose directions are reversed from winter to summer.
- Zonal mean flow: centered in the tropics, whose directions are reversed from winter to summer.
The Ferrel Cell

The balance equations:

- Tropopause
  
  \[ f v \sim \frac{\partial[u^* v^*]}{\partial y} < 0 \]

- Ferrel Cell: eddy-driven, indirect cell
  
  \[ \frac{\partial \theta_s}{\partial p} \sim -\frac{\partial[\theta^* v^*]}{\partial y} < 0 \]

- Boundary layer
  
  \[ f v \sim r \vec{u}_{surf} > 0 \]
Baroclinic eddies

- E-P flux

In a QG, steady, adiabatic and frictionless flow:

- Momentum equation:

\[ f[v] - \frac{\partial ([u^*v^*])}{\partial y} = 0 \]

- Continuity equation:

\[ \frac{\partial [v]}{\partial y} + \frac{\partial [\omega]}{\partial p} = 0 \Rightarrow \nabla \cdot \mathbf{F} = 0 \]

- Thermodynamic equation:

\[ [\omega] \frac{\partial \theta_s}{\partial p} + \frac{\partial ([\theta^*v^*])}{\partial y} = 0 \]

Define Eliassen-Palm flux:

\[ \mathbf{F} \equiv -[u^*v^*] \mathbf{j} + f \frac{[v^*\theta^*]}{\partial \theta_s/\partial p} \mathbf{k} \]

Momentum equation:

\[ f[v] - \frac{\partial ([u^*v^*])}{\partial y} = 0 \]

Continuity equation:

\[ \frac{\partial [v]}{\partial y} + \frac{\partial [\omega]}{\partial p} = 0 \Rightarrow \nabla \cdot \mathbf{F} = 0 \]

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Baroclinic eddies

- E-P flux

\[ \mathcal{F} \equiv -[u^*v^*]j + f \frac{[v^*\theta^*]}{\partial \theta_s/\partial p} k \]

- In a QG, **steady**, adiabatic and **frictionless** flow:

\[ [v] = \frac{1}{f} \frac{\partial}{\partial y} ([u^*v^*]) \quad [\omega] = - \frac{\partial}{\partial y} \left( \frac{[\theta^*v^*]}{\partial \theta_s/\partial p} \right) \quad \nabla \cdot \mathcal{F} = 0 \]

- In a QG, **steady** flow:

\[ f[v] - \frac{\partial([u^*v^*])}{\partial y} + [F_x] = 0 \]

\[ [\omega] \frac{\partial \theta_s}{\partial p} + \frac{\partial([\theta^*v^*])}{\partial y} - \left( \frac{p_o}{p} \right)^{R/c_p} \frac{[Q]}{c_p} = 0 \]

The **meridional overturning flow**, in addition to the **eddy forcing**, has to balance the **external forcing**.
Baroclinic eddies

\[ \mathcal{F} \equiv -[u^*v^*] \mathbf{j} + f \frac{[v^*\theta^*]}{\partial s/\partial p} \mathbf{k} \]

In a QG, **steady** flow:

Define:

\[ \tilde{\omega} = [\omega] + \frac{\partial}{\partial y} \left( \frac{[v^*\theta^*]}{\partial s/\partial p} \right) \]

\[ \tilde{v} = [v] - \frac{\partial}{\partial p} \left( \frac{[v^*\theta^*]}{\partial s/\partial p} \right) \]

*Residual mean meridional circulation*

\[
\begin{align*}
    f[v] - \frac{\partial([u^*v^*])}{\partial y} + [F_x] &= 0 \\
    [\omega] \left( \frac{\partial \theta_s}{\partial p} + \frac{\partial([\theta^*v^*])}{\partial y} - \left( \frac{p_o}{p} \right)^{R/c_p} [Q] \right) c_p &= 0 \\
    \frac{\partial \tilde{v}}{\partial y} + \frac{\partial \tilde{\omega}}{\partial p} &= 0 \\
    0 &= -[\tilde{\omega}] \frac{\partial \theta_s}{\partial p} + \left( \frac{p_o}{p} \right)^{R/c_p} [Q] c_p \\
    0 &= f[\tilde{v}] + \nabla \cdot \mathcal{F} + [F_x]
\end{align*}
\]
Baroclinic eddies

\[ \mathcal{F} \equiv -[u^*v^*] \hat{j} + f \frac{[v^*\theta^*]}{\partial \theta_s/\partial p} \hat{k} \]

- E-P flux

**In a QG flow:**

Define:

\[ [\tilde{\omega}] = [\omega] + \frac{\partial}{\partial y} \left( \frac{[v^*\theta^*]}{\partial \theta_s/\partial p} \right) \]

\[ [\tilde{v}] = [v] - \frac{\partial}{\partial p} \left( \frac{[v^*\theta^*]}{\partial \theta_s/\partial p} \right) \]

Residual mean meridional circulation

\[ f[v] - \frac{\partial([u^*v^*])}{\partial y} + [F_x] = 0 \]

\[ [\omega] \frac{\partial \theta_s}{\partial p} + \frac{\partial([\theta^*v^*])}{\partial y} - \left( \frac{p_o}{p} \right)^{R/c_p} \frac{[Q]}{c_p} = 0 \]

\[ \frac{\partial[\tilde{v}]}{\partial y} + \frac{\partial[\tilde{\omega}]}{\partial p} = 0 \]

\[ 0 = -[\tilde{\omega}] \frac{\partial \theta_s}{\partial p} + \left( \frac{p_o}{p} \right)^{R/c_p} \frac{[Q]}{c_p} \]

\[ 0 = f[\tilde{v}] + \nabla \cdot \mathcal{F} + [F_x] \]

(Quasi-geostrophic)

Transformed Eulerian mean equations

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Recall the streamfunction for the zonal mean flow:

$$([v], [\omega]) = \left(- \frac{\partial \psi_m}{\partial p}, \frac{\partial \psi_m}{\partial y}\right)$$

Define a streamfunction for the residual mean circulation:

$$([\tilde{v}], [\tilde{\omega}]) = \left(- \frac{\partial \tilde{\psi}}{\partial p}, \frac{\partial \tilde{\psi}}{\partial y}\right)$$

$$\tilde{\psi} = \psi_m + \left[\frac{v^* \theta^*}{\partial \theta_s / \partial p}\right]$$
Baroclinic eddies

\[ \tilde{\psi} = \psi_m + \frac{[\nu^* \theta^*]}{\partial \theta_s / \partial p} \]

Case 1: The EADY model

The residual mean circulation’s direction can be **opposite** to the Eulerian mean circulation.
Case 2: Observed circulation

The Ferrel Cell

- In isentropic coordinate

\[(x, y, z) \Leftrightarrow (x, y, \theta)\]

\[
\frac{D\theta}{Dt} = \dot{\theta}
\]

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \theta + \frac{D\theta}{Dt} \frac{\partial}{\partial \theta}
\]

\[
= \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \theta + \dot{\theta} \frac{\partial}{\partial \theta}
\]

Isentrope: An isopleth of entropy. In meteorology it is usually identified with an isopleth of potential temperature.

zero for adiabatic flow
Baroclinic eddies

\[
\tilde{\psi} = \psi_m + \frac{[v^* \theta^*]}{\partial \theta_s / \partial \rho}
\]

- In isentropic coordinate

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \theta + \frac{D \theta}{Dt} \frac{\partial}{\partial \theta}
\]

\[
= \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \theta + \dot{\theta} \frac{\partial}{\partial \theta}
\]

zero for adiabatic flow

The Ferrel cell in the isentropic coordinate is essentially reflect the Residual Mean Circulation.

(Fig.11.4, Vallis, 2006)
Baroclinic eddies

- From linear to nonlinear

1. Basic flow or Pre-existing flow (without zonal variation and baroclinic unstable)
   - Small perturbation

2. Perturbations grow with time (finite amplitude pert.)
   - Eddy-mean interactions (Adjust the zonal flow)

3. Linear process
   - E-P flux, residual mean circulation, Transformed Eulerian mean equations.

4. Nonlinear interactions
   - Equilibrated states between the adjusted zonal flow and baroclinic eddies
Baroclinic eddies

- E-P flux: a second view

**E-P flux and the Quasi-geostrophic potential vorticity**

\[ \mathcal{F} = -[u^* v^*] \mathbf{j} + f [v^* \theta^*] \frac{\partial \theta_s}{\partial p} \mathbf{k} \]

From the definition of QG potential vorticity:

\[ \bar{q} = \frac{\partial^2 \bar{\psi}}{\partial y^2} + \beta y + f_o \frac{\partial}{\partial p} \left( \frac{1}{s} \frac{\partial \bar{\psi}}{\partial p} \right) \]

\[ q' = \frac{\partial^2 \psi'}{\partial x^2} + \frac{\partial^2 \psi'}{\partial y^2} + f_o \frac{\partial}{\partial p} \left( \frac{1}{s} \frac{\partial \psi'}{\partial p} \right) \]

\[ \zeta' = f_o \frac{\partial}{\partial p} \left( \frac{\theta'}{\partial \theta_s/\partial p} \right) \]

\[ (u, v) = \left( -\frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial x} \right), \quad \theta = -\kappa f_o \frac{\partial \psi}{\partial p} \]

\[ s = -\frac{1}{\kappa} \frac{\partial \theta_s}{\partial p}, \quad \kappa = \frac{p}{R} \left( \frac{p_o}{p} \right)^{R/c_p} \]

**PV flux:**

\[ v' q' = v' \zeta' + \frac{f_o}{\partial \theta_s/\partial p} v' \frac{\partial \theta'}{\partial p} \]

\[ v \zeta = v \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \]

\[ = -\frac{\partial}{\partial y} uv + \frac{1}{2} \frac{\partial}{\partial x} (v^2 - u^2) \]

**thermal wind relation for meridional wind**

\[ \frac{v}{\partial p} = \frac{\partial}{\partial p} v \theta - \theta \frac{\partial v}{\partial p} + \frac{\theta \frac{\partial \theta}{\partial x}}{\kappa f_o} \frac{\partial}{\partial x} \]

\[ = \frac{\partial}{\partial p} v \theta + \frac{1}{2\kappa f_o} \frac{\partial}{\partial x} \theta^2 \]

Note: ‘ denotes small perturbation

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E-P flux and the Quasi-geostrophic potential vorticity

\[ \mathcal{F} \equiv -[u^* v^*] \mathbf{j} + f \left[ \frac{v^* \theta^*}{\partial \theta_s / \partial p} \right] \mathbf{k} \]

From the definition of QG potential vorticity:

\[ v' q' = v' \zeta' + \frac{f_o}{\partial \theta_s / \partial p} v' \frac{\partial \theta'}{\partial p} \]

\[ = \frac{1}{2} \frac{\partial}{\partial x} \left( v'^2 - u'^2 + \frac{1}{\kappa} \frac{\theta'^2}{\partial \theta_s / \partial p} \right) \]

\[ - \frac{\partial}{\partial y} u' v' \]

\[ + f_o \frac{\partial}{\partial p} \frac{\partial}{\partial \theta_s / \partial p} v' \theta' \]

\[ v \frac{\partial \theta}{\partial p} = \frac{\partial}{\partial p} v \theta - \theta \frac{\partial v}{\partial p} \]

\[ + \frac{\theta}{\kappa f_o} \frac{\partial \theta}{\partial x} \]

\[ v \frac{\partial \theta}{\partial p} = 1 \frac{\partial}{\partial p} v \theta + \frac{1}{2 \kappa f_o} \frac{\partial}{\partial x} \theta^2 \]
Baroclinic eddies
- E-P flux: a second view

E-P flux and the Quasi-geostrophic potential vorticity

\[ \mathcal{F} \equiv -[u^* v^*] \mathbf{j} + f \left( \frac{v^* \theta^*}{\partial \theta_s / \partial p} \right) \mathbf{k} \]

From the definition of QG potential vorticity:

\[ v' q' = v' \zeta' + \frac{f_o}{\partial \theta_s / \partial p} v' \frac{\partial \theta'}{\partial p} \]

\[ = \frac{1}{2} \frac{\partial}{\partial x} \left( v'^2 - u'^2 + \frac{1}{\kappa} \frac{\theta'^2}{\partial \theta_s / \partial p} \right) \]

\[ - \frac{\partial}{\partial y} u' v' \]

\[ + f_o \frac{\partial}{\partial p} \frac{v' \theta'}{\partial \theta_s / \partial p} \]

Zonally averaged PV flux by eddies:

\[ [v^* q^*] = - \frac{\partial}{\partial y} [u^* v^*] + f_o \frac{\partial}{\partial p} \left[ \frac{v^* \theta^*}{\partial \theta_s / \partial p} \right] \]

\[ = \nabla \cdot \mathcal{F} \]
Baroclinic eddies
- E-P flux: a second view

- E-P flux and the Eliassen-Palm relation

\[ \mathcal{F} \equiv -[u^*v^*] j + f \frac{[v^*\theta^*]}{\partial \theta_s / \partial p} \mathbf{k} \]

Linearized PV equation (q=PV):

\[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) q' + \frac{\partial \psi'}{\partial x} \frac{\partial q}{\partial y} = 0 \]

Multiplying by \( q' \) and zonally average:

\[ \frac{1}{2} \frac{\partial}{\partial t} [q'^2] + [v'q'] \frac{\partial q}{\partial y} = 0 \]

Define wave activity density:

\[ \mathcal{A} = \frac{[q'^2]}{2\partial q/\partial y} \]

\[ \frac{\partial \mathcal{A}}{\partial t} + \nabla \cdot \mathcal{F} = 0 \]

Eliassen-Palm relation
Baroclinic eddies
- E-P flux: a second view

E-P flux and the Rossby waves

\[ \mathcal{F} \equiv -[u^* \nu^*] \mathbf{j} + f \frac{[\nu^* \theta^*]}{\partial \theta_s / \partial p} \mathbf{k} \]

Linearized PV equation (q=PV):

\[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) q' + \frac{\partial \psi'}{\partial x} \frac{\partial \bar{q}}{\partial y} = 0 \]

Assume \( U \) is fixed, and \( \frac{\partial \bar{q}}{\partial y} = \beta \)

\[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \left[ \nabla^2 \psi' + f^2 \frac{\partial}{\partial p} \left( \frac{1}{s} \frac{\partial \psi'}{\partial p} \right) \right] + \beta \frac{\partial \psi'}{\partial x} = 0 \]

Exist solutions of the form

\[ \psi' = \text{Re} \Psi e^{i(kx + ly + mp - \omega t)} \]

Dispersion relation of Rossby waves:

\[ \omega = Uk - \frac{\beta k}{K^2} \]

\[ K^2 = k^2 + l^2 + m^2 f_o^2 / s \]

with group velocity

\[ c_{gy} = \frac{2 \beta kl}{K^4} \quad c_{gp} = \frac{2 \beta km f_o^2 / s}{K^4} \]

\[ A' = \text{Re} \hat{\Psi} e^{i(kx + ly + mp - \omega t)} \]

\[ \hat{u} = -\text{Re}i \Psi, \quad \hat{v} = \text{Re}k \Psi \]

\[ \hat{\theta} = -\text{Re}im \kappa f_o \Psi, \quad \hat{q} = -\text{Re}K^2 \Psi \]
### E-P flux and the Rossby waves

\[
\mathcal{F} \equiv -[u^*v^*] \mathbf{j} + f \left[ \frac{v^* \theta^*}{\partial \theta_s / \partial p} \right] \mathbf{k}
\]

Linearized PV equation (q=PV):

\[
\left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) q' + \frac{\partial \psi'}{\partial x} \frac{\partial q}{\partial y} = 0
\]

with group velocity

\[
c_{gy} = \frac{2 \beta kl}{K^4} \quad c_{gp} = \frac{2 \beta km f_o^2 / s}{K^4}
\]

\[
K^2 = k^2 + l^2 + m^2 f_o^2 / s
\]

\[
\hat{u} = -\text{Re}i \ell \Psi, \quad \hat{v} = \text{Re} i k \Psi
\]

\[
\hat{\theta} = -\text{Re} i m k f_o \Psi, \quad \hat{q} = -\text{Re} K^2 \Psi
\]

**Wave activity density:**

\[
\mathcal{A} = \frac{[q'^2]}{2 \beta} = \frac{K^4}{4 \beta} |\Psi^2|
\]

\[
-[u' v'] = \frac{1}{2} k l |\Psi^2| = c_{gy} \mathcal{A}
\]

\[
f_o \frac{[v' \theta']}{\partial \theta_s / \partial p} = \frac{f_o^2}{2 s} k m |\Psi^2| = c_{gz} \mathcal{A}
\]

\[
\vec{\mathcal{F}} = \mathbf{c}_g \mathcal{A}
\]
E-P flux, TEM and Residual Circulation
- Summary

- **E-P flux:**
  \[ \mathcal{F} \equiv -[u^*v^*] \mathbf{j} + f \frac{[v^*\theta^*]}{\partial \theta_s/\partial p} \mathbf{k} \]

- In a **steady**, **adiabatic** and **frictionless** flow:
  \[ [v] = \frac{1}{f} \frac{\partial}{\partial y} ([u^*v^*]) \quad [\omega] = -f \frac{\partial}{\partial y} \left( \frac{[\theta^*v^*]}{\partial \theta_s/\partial p} \right) \quad \nabla \cdot \mathcal{F} = 0 \]

  \[ [v^*q^*] = -\frac{\partial}{\partial y} [u^*v^*] + f_o \frac{\partial}{\partial p} \frac{[v^*\theta^*]}{\partial \theta_s/\partial p} \]
  \[ = \nabla \cdot \mathcal{F} \]

  \[ \mathcal{F} = \mathbf{c}_g \mathcal{A} \]

- **Residual mean circulations:**
  \[ [\tilde{\omega}] = [\omega] + \frac{\partial}{\partial y} \left( \frac{[v^*\theta^*]}{\partial \theta_s/\partial p} \right), \quad [\tilde{v}] = [v] - \frac{\partial}{\partial p} \left( \frac{[v^*\theta^*]}{\partial \theta_s/\partial p} \right) \]

- **TEM equations:**
  \[ \frac{\partial [u]}{\partial t} = f[\tilde{v}] + \nabla \cdot \mathcal{F} + [F_x], \quad \frac{\partial [\theta]}{\partial t} = -[\tilde{\omega}] \frac{\partial \theta_s}{\partial p} + \left( \frac{p_o}{p} \right)^{R/c_p} \frac{[Q]}{c_p} \]

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- The energy cycle
Baroclinic eddy life cycle  
- An E-P flux view

\[ \mathcal{F} \equiv -[u^* v^*] \mathbf{j} + f \frac{[v^* \theta^*]}{\partial \theta_s / \partial p} \mathbf{k} \]

\[ \vec{\mathcal{F}} = c_g \mathbf{A} \]

\[ \frac{\partial [u]}{\partial t} = f[\tilde{v}] + \nabla \cdot \mathcal{F} + [F_x] \]

\[ \frac{\partial [\theta]}{\partial t} = -[\tilde{\omega}] \frac{\partial \theta_s}{\partial p} + \left( \frac{p_o}{p} \right)^{R/c_p} \]

Numerical results from Simmons and Hoskins, 1978, JAS

Initial state:
Baroclinic eddies
- baroclinic eddy life cycle

- Eddies’ development

Small amplitude perturbations

Finite amplitude perturbations

Wave breaking

(Thorncroft et al, 1993, Q.J.R.)
Baroclinic eddy life cycle
- An E-P flux view

$$\mathcal{F} \equiv -[u^* v^*] \mathbf{j} + f \left[ \frac{v^* \theta^*}{\partial \theta_s / \partial p} \right] \mathbf{k}$$

$$\vec{\mathcal{F}} = \mathbf{c}_g \mathbf{A}$$

*Eddies: generate at lower level, propagate upwards and away from the eddy source region*

Numerical results from Simmons and Hoskins, 1978, JAS
E-P flux

- In the equilibrium state

\[ \mathcal{F} \equiv -[u^* v^*] \mathbf{j} + f \frac{[v^* \theta^*]}{\partial \theta_s / \partial p} \mathbf{k} \]

\[ \vec{\mathcal{F}} = c_g \mathbf{A} \]

Wave energies: propagate \textbf{upwards} and \textbf{away} from the center of the jet

Numerical results from idealized model with pure midlatitude jet (Vallis, 2006)
The Eliassen–Palm flux in an idealized primitive equation of the atmosphere. (a) The EP flux (arrows) and its divergence (contours, with intervals of 2 ms$^{-1}$ day$^{-1}$). The solid contours denote flux divergence, a positive PV flux, and eastward flow acceleration; the dashed contours denote flux convergence and deceleration. (b) The EP flux (arrows) and the time and zonally averaged zonal wind (contours). See the appendix for details of plotting EP fluxes.

From Vallis (2006)

\[ \mathbf{F} \equiv -[u^* v^*] \mathbf{j} + f \left[ \frac{v^* \theta^*}{\partial \theta^s / \partial \theta^s} \right] \mathbf{k} \]

\[ \mathbf{F} \rightleftharpoons \mathbf{c}_g \mathbf{A} \]

\[ \frac{\partial [u]}{\partial t} = f[\tilde{v}] + \nabla \cdot \mathbf{F} + [F_x] \]

In the vertical direction:

- Accelerating the lower jet
- Decelerating the upper jet
- Reduce the vertical shear of $U$

Wave energies: propagate \textbf{upwards} and \textbf{away} from the center of the jet.
E-P flux

\[ \mathcal{F} \equiv -[u^* v^*] \mathbf{j} + f \frac{[v^* \theta^*]}{\partial \theta_s / \partial p} \mathbf{k} \]

\[ \vec{\mathcal{F}} = \mathbf{c_g A} \]

\[ \frac{\partial [u]}{\partial t} = f[\tilde{v}] + \nabla \cdot \mathcal{F} + [F_x] \]

Integrate vertically:

\[ \frac{1}{g} \int_0^{p_s} \mathrm{d} p \]

\[ \frac{\partial}{\partial t} < [u] > = - \frac{\partial}{\partial y} < [u^* v^*] > - r[u_{\text{surf}}] \]

\(< >\) means vertical average
Eddy-driven jet:
- the momentum budget

\[ \mathcal{F} \equiv -\mathbf{[u^* v^*]} \mathbf{j} + f \left[ \frac{v^* \theta^*}{\partial \theta_s / \partial p} \right] \mathbf{k} \]

Wave energies: propagate upwards and away from the center of the jet

In equilibrium:

\[ \vec{\mathcal{F}} = \mathbf{c_g A} \]

\[ r[u_{surf}] \sim -\frac{\partial}{\partial y} <[u^* v^*]> - r[u_{surf}] \]

<> means vertical average

Stirring

Rossby waves break & dissipate

Momentum divergence

Momentum convergence

Rossby waves break & dissipate

From Vallis (2006)

Eddy-driven jet:
- the momentum budget

In equilibrium:

\[ \vec{\mathcal{F}} = \mathbf{c_g A} \]

\[ r[u_{surf}] \sim -\frac{\partial}{\partial y} <[u^* v^*]> - r[u_{surf}] \]

<> means vertical average

There MUST be surface westerlies at midlatitudes.
E-P flux

- in the real atmosphere

\[ F \equiv -[u^*v^*]j + f \frac{v^* \theta^*}{\partial \theta_s / \partial p} k \]

\[ \vec{F} = c_g \mathcal{A} \]

Vertical component is dominant.

EP divergence in the lower layers; convergence in the upper layers.
E-P flux and the eddy-driven jet

- Summary

\[ \mathcal{F} \equiv -[u^* v^*] \mathbf{j} + f \frac{[v^* \theta^*]}{\partial \theta_s / \partial p} \mathbf{k} \]

\[ \mathcal{F} = c_g A \]

- Numerical results and observations: eddies generate in the lower level, propagate upwards and away from the eddy source region.

\[ \frac{\partial [u]}{\partial t} = f [\tilde{v}] + \nabla \cdot \mathcal{F} + [F_x] \]

- Accelerating the lower jet, decelerating the upper jet, reduce the vertical shear of U

- Momentum budget indicates that there MUST be surface westerlies in the eddy source latitude.