第四章:

中纬度的经向环流系统(III)

- Ferrel cell, baroclinic eddies and the westerly jet

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Outline

- Observations
- The Ferrel Cell
- Baroclinic eddies
  - Review: baroclinic instability and baroclinic eddy life cycle
  - Eddy-mean flow interaction, E-P flux
  - Transformed Eulerian Mean equations
- Eddy-driven jet
- Energy cycle

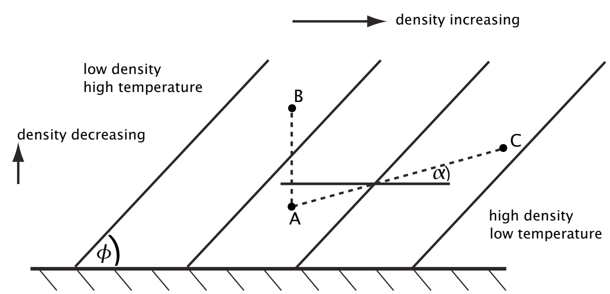
Baroclinic eddies

- baroclinic instability

Review

- **Baroclinic Instability** - "is an instability that arises in *rotating*, *stratified* fluids that are subject to a *horizontal temperature gradient*".

- Energetics: PE → KE



- Mathematics:
  - Linear Baroclinic Instability
    - Linear baroclinic system →
      - Eady's model (1949)
      - Charney's model (1947)

Baroclinic eddies

- linear baroclinic instability

Review

- Eady's model (1949)
- Charney's model (1947)

a) The basic zonal flow has uniform vertical shear,

$$U_o(Z) = \Lambda Z, \ \Lambda \text{ is a constant}$$

b) The fluid is uniformly stratified,

N^2 is a constant.

c) Two rigid lids at the top and bottom, flat horizontal surface, that is

$$\omega = 0 \text{ at } Z = 0 \text{ and } H.$$

d) The motion is on the **f -plane**, that is

$$\beta = 0$$

The most distinguished difference with Eady's model is that **beta effect** is considered.

Baroclinic eddies

- linear baroclinic instability

Small amplitude assumption
小扰动

Linear baroclinic system:
Eady model
Charney model

Normal mode assumption
标准波形

Obtain the solutions, e.g.
instability conditions
growth rate
most unstable mode

Variable = Basic state + Perturbation

$$u(\mathbf{x}, t) = U(z) + u'(\mathbf{x}, t)$$

$$u'(\mathbf{x}, t) \ll U(z)$$

Linearized PV equation (q=PV):

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) q' + \frac{\partial \psi}{\partial x}\frac{\partial \bar{q}}{\partial y} = 0$$

$$q' = \frac{\partial^2 \psi'}{\partial x^2} + \frac{\partial^2 \psi'}{\partial y^2} + \frac{f_o^2}{\rho_s}\frac{\partial}{\partial z}\left(\frac{\rho_s}{N^2}\frac{\partial \psi'}{\partial z}\right)$$

$$\bar{q} = \frac{\partial^2 \bar{\psi}}{\partial y^2} + \beta y + \frac{f_o^2}{\rho_s}\frac{\partial}{\partial z}\left(\frac{\rho_s}{N^2}\frac{\partial \bar{\psi}}{\partial z}\right)$$

标准波形法， 带入方程和边界条件：

$$\psi'(\mathbf{x}, t) = Ae^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$$

Find the conditions for non-trivial solutions and Ci >0

Baroclinic eddies

- linear baroclinic instability

Review

- Conclusions:

Necessary condition for baroclinic instability: PV gradient changes sign in the interior or boundaries (Charney-stern theory), according to which the midlatitude atmosphere is baroclinic unstable. Different models. i.e. Eady and Charney models have more rigorous conditions.

Growth rate: $\sigma = kc_i \approx 0.3\,\Lambda \dfrac{f_o}{N}$ in both Eady and Charney models!

Most unstable mode:

$$k_{\max}^{-1} \propto L_d = \left(\frac{NH}{f_o}\right) \qquad k_{\max}^{-1} \propto \Lambda \frac{f_o}{\beta N}$$

Eady | Charney

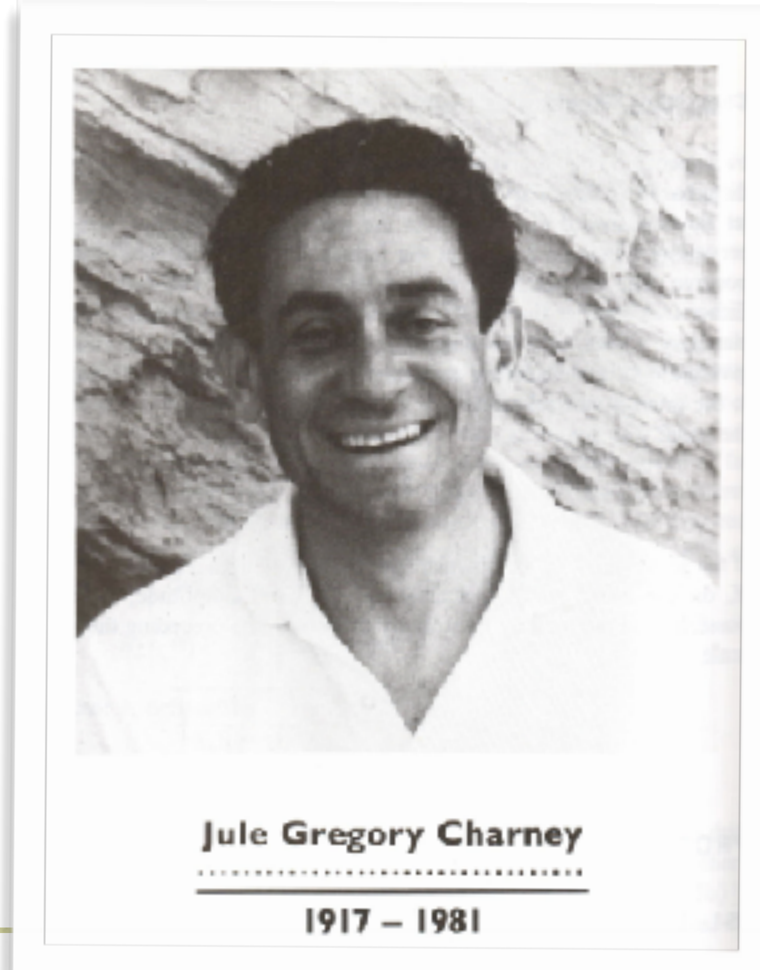
Baroclinic eddies

- linear baroclinic instability

“ **JULE CHARNEY** was one of the dominant figures in atmospheric science in the three decades following World War II. Much of the change in meteorology from an art to a science is due to his scientific vision and his thorough commitment to people and programs in this field.”

-- *by Norman Phillips*

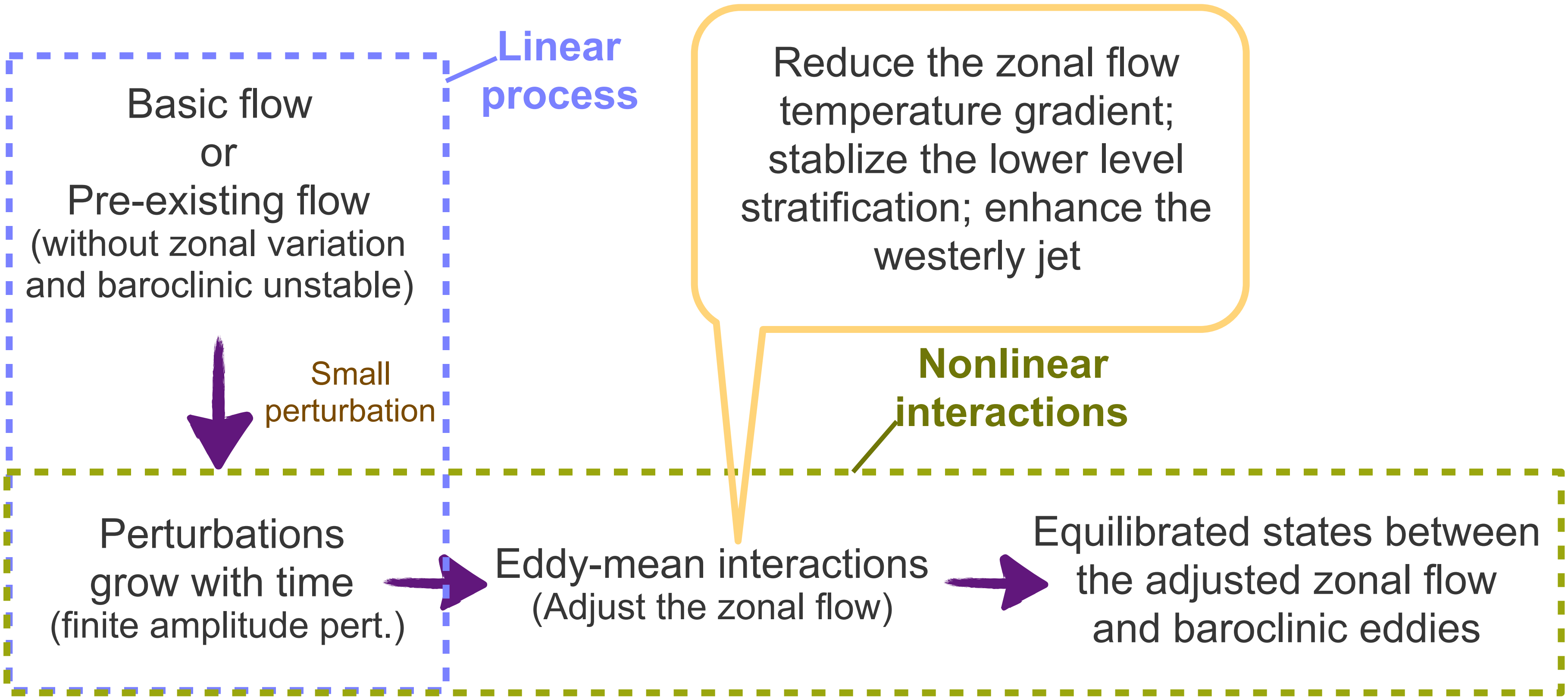
- Charney’s model



Baroclinic eddies

Review

- From linear to nonlinear



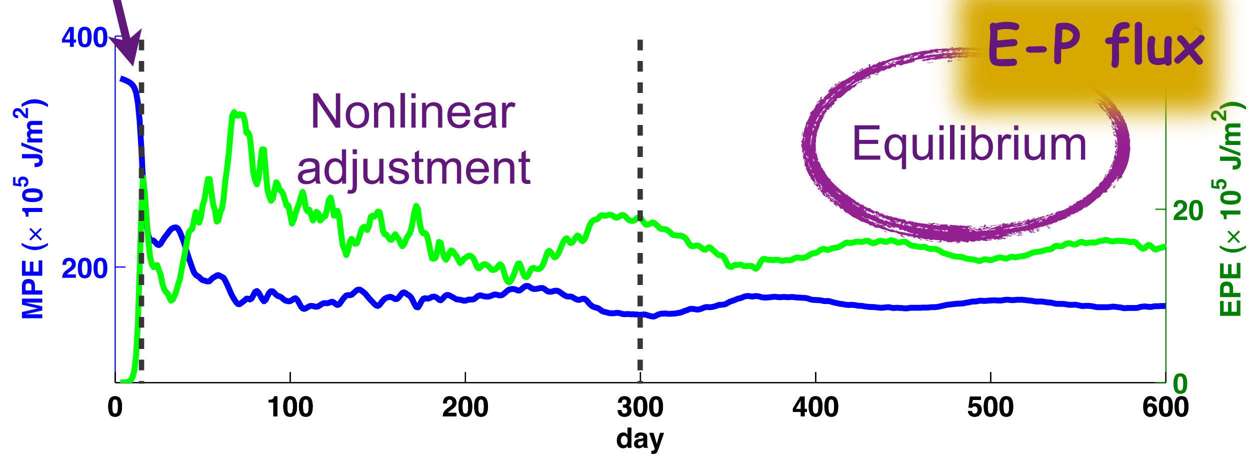
Baroclinic eddies

Review

- From linear to nonlinear

Numerical results from a QG model

(Zhang, 2009)



Basic flow
or
Pre-existing flow
(without zonal variation
and baroclinic unstable)

Small perturbation

Perturbations grow with time
(finite amplitude pert.)

→ Eddy-mean interactions
(Adjust the zonal flow)

→ Equilibrated states between the adjusted zonal flow and baroclinic eddies

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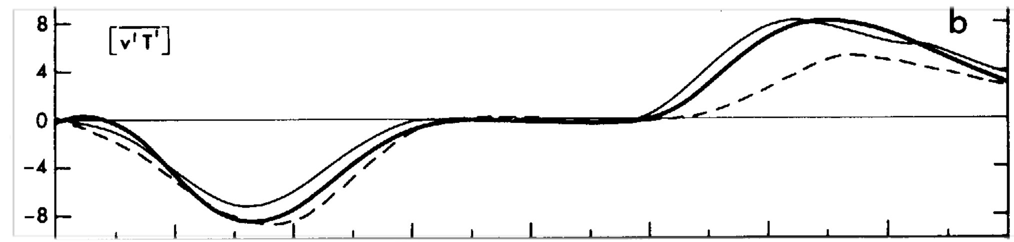
  - Transformed Eulerian Mean equations
- Eddy-driven jet
- The energy cycle

The Ferrel Cell

Review

- The balance equations



Tropopause

$$\frac{\partial [u]}{\partial t} = -\frac{\partial([u^* v^*])}{\partial y} + f[v] + [F_x]$$

$$\omega \frac{\partial \theta_s}{\partial p} \sim -\frac{\partial [\theta^* v^*]}{\partial y} < 0 \qquad \omega \frac{\partial \theta_s}{\partial p} \sim -\frac{\partial [\theta^* v^*]}{\partial y} > 0$$

$$\frac{\partial [\theta]}{\partial t} + [\omega]\frac{\partial \theta_s}{\partial p} = -\frac{\partial([\theta^* v^*])}{\partial y} + \left(\frac{p_o}{p}\right)^{R/c_p} \frac{[Q]}{c_p}$$

Boundary layer

Ground

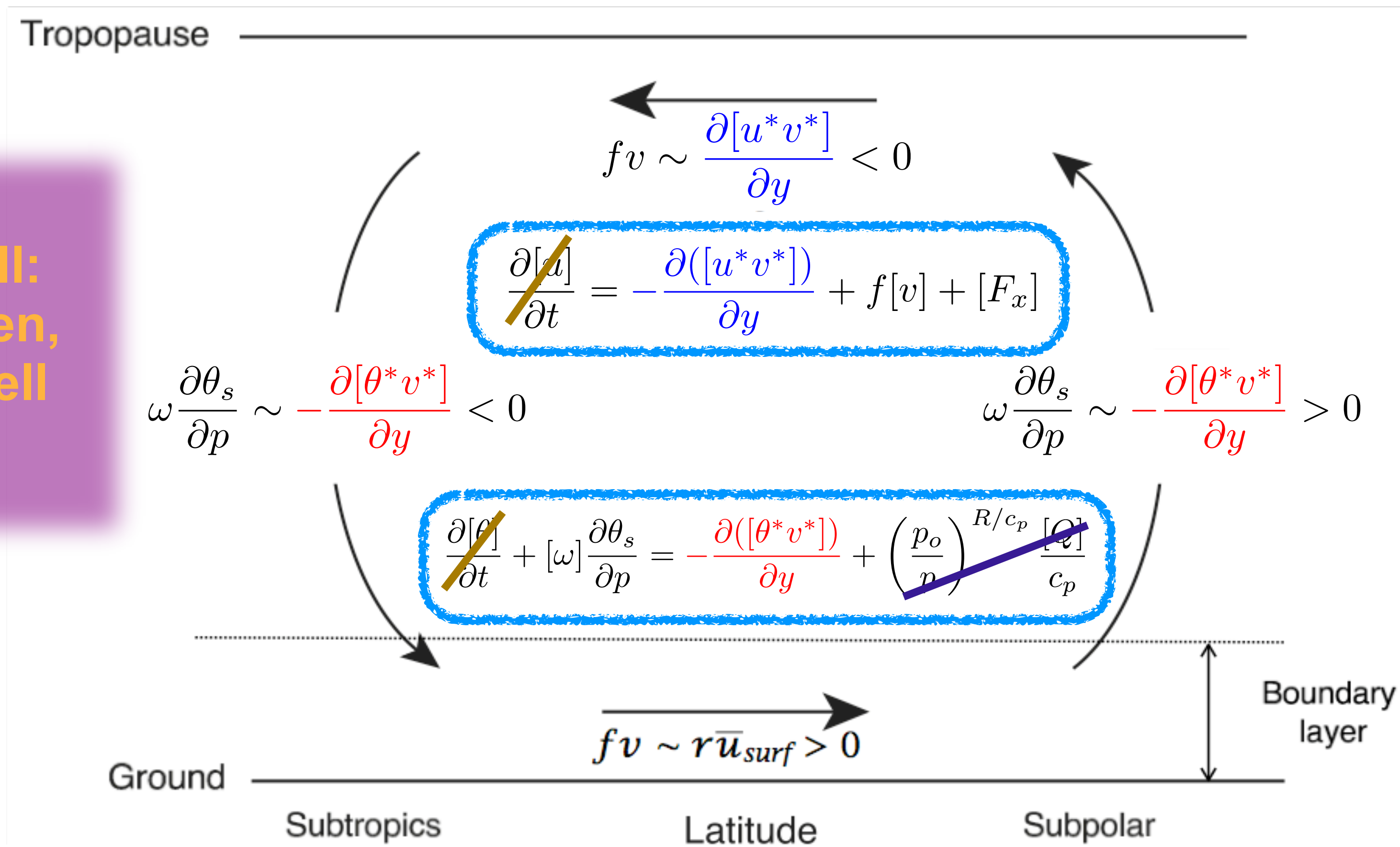
Subtropics Latitude Subpolar

The Ferrel Cell

Review

- The balance equations:

Ferrel Cell: eddy-driven, indirect cell



Baroclinic eddies

- E-P flux

- In a QG, **steady**, **adiabatic** and **frictionless** flow:

- Momentum equation:

$$f[v] - \frac{\partial([u^*v^*])}{\partial y} = 0$$

- Continuity equation:

$$\frac{\partial [v]}{\partial y} + \frac{\partial [\omega]}{\partial p} = 0 \Rightarrow \nabla \cdot \mathcal{F} = 0$$

- Thermodynamic equation:

$$[\omega]\frac{\partial \theta_s}{\partial p} + \frac{\partial([\theta^* v^*])}{\partial y} = 0$$

$$[v] = \frac{1}{f}\frac{\partial}{\partial y}([u^*v^*])$$

$$[\omega] = -\frac{\partial}{\partial y}\left(\frac{[\theta^* v^*]}{\partial \theta_s/\partial p}\right)$$

Define *Eliassen-Palm flux:*

$$\mathcal{F} \equiv -[u^*v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

Baroclinic eddies

- E-P flux

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

- In a QG, **steady**, **adiabatic** and **frictionless** flow:

$$[v] = \frac{1}{f}\frac{\partial}{\partial y}([u^* v^*]) \qquad [\omega] = -\frac{\partial}{\partial y}\left(\frac{[\theta^* v^*]}{\partial\theta_s/\partial p}\right) \qquad \nabla\cdot\mathcal{F} = 0$$

- In a QG, **steady** flow:

$$f[v] - \frac{\partial([u^* v^*])}{\partial y} + [F_x] = 0$$

$$[\omega]\frac{\partial\theta_s}{\partial p} + \frac{\partial([\theta^* v^*])}{\partial y} - \left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q]}{c_p} = 0$$

The **meridional overturning flow**, in addition to the **eddy forcing**, has to balance the **external forcing**.

Baroclinic eddies

- E-P flux

$$\mathcal{F} \equiv -[u^*v^*]\,\mathbf{j} + f\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

- In a QG, **steady** flow:

$$f[v] - \frac{\partial([u^*v^*])}{\partial y} + [F_x] = 0$$

$$[\omega]\frac{\partial\theta_s}{\partial p} + \frac{\partial([\theta^*v^*])}{\partial y} - \left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q]}{c_p} = 0$$

$$\frac{\partial[\tilde{v}]}{\partial y} + \frac{\partial[\tilde{\omega}]}{\partial p} = 0$$

Define:

$$[\tilde{\omega}] = [\omega] + \frac{\partial}{\partial y}\left(\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\right)$$

$$[\tilde{v}] = [v] - \frac{\partial}{\partial p}\left(\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\right)$$

$$0 = -[\tilde{\omega}]\frac{\partial\theta_s}{\partial p} + \left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q]}{c_p}$$

$$0 = f[\tilde{v}] + \nabla\cdot\mathcal{F} + [F_x]$$

Residual mean meridional circulation

Baroclinic eddies

- E-P flux

$$\mathcal{F} \equiv -[u^*v^*]\,\mathbf{j} + f\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

- In a QG flow:

$$f[v] - \frac{\partial([u^*v^*])}{\partial y} + [F_x] = 0$$

$$[\omega]\frac{\partial\theta_s}{\partial p} + \frac{\partial([\theta^*v^*])}{\partial y} - \left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q]}{c_p} = 0$$

$$\frac{\partial[\tilde{v}]}{\partial y} + \frac{\partial[\tilde{\omega}]}{\partial p} = 0$$

Define:

$$[\tilde{\omega}] = [\omega] + \frac{\partial}{\partial y}\left(\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\right)$$

$$[\tilde{v}] = [v] - \frac{\partial}{\partial p}\left(\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\right)$$

Residual mean meridional circulation

$$0 = -[\tilde{\omega}]\frac{\partial\theta_s}{\partial p} + \left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q]}{c_p}$$

$$0 = f[\tilde{v}] + \nabla\cdot\mathcal{F} + [F_x]$$

(Quasi-geostrophic)
Transformed Eulerian mean equations

Baroclinic eddies

- TEM

$$\frac{\partial[\theta]}{\partial t} = -[\tilde{\omega}]\frac{\partial\theta_s}{\partial p} + \left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q]}{c_p}$$

$$\frac{\partial[u]}{\partial t} = f[\tilde{v}] + \nabla\cdot\mathcal{F} + [F_x]$$

$$\frac{\partial[\tilde{v}]}{\partial y} + \frac{\partial[\tilde{\omega}]}{\partial p} = 0$$

$$[\tilde{\omega}] = [\omega] + \frac{\partial}{\partial y}\left(\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\right)$$

$$[\tilde{v}] = [v] - \frac{\partial}{\partial p}\left(\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\right)$$

Recall the streamfunction for the zonal mean flow:

$$([v],[\omega]) = \left(-\frac{\partial\psi_m}{\partial p}, \frac{\partial\psi_m}{\partial y}\right)$$

Define a streamfunction for the residual mean circulation:

$$([\tilde{v}],[\tilde{\omega}]) = \left(-\frac{\partial\tilde{\psi}}{\partial p}, \frac{\partial\tilde{\psi}}{\partial y}\right)$$

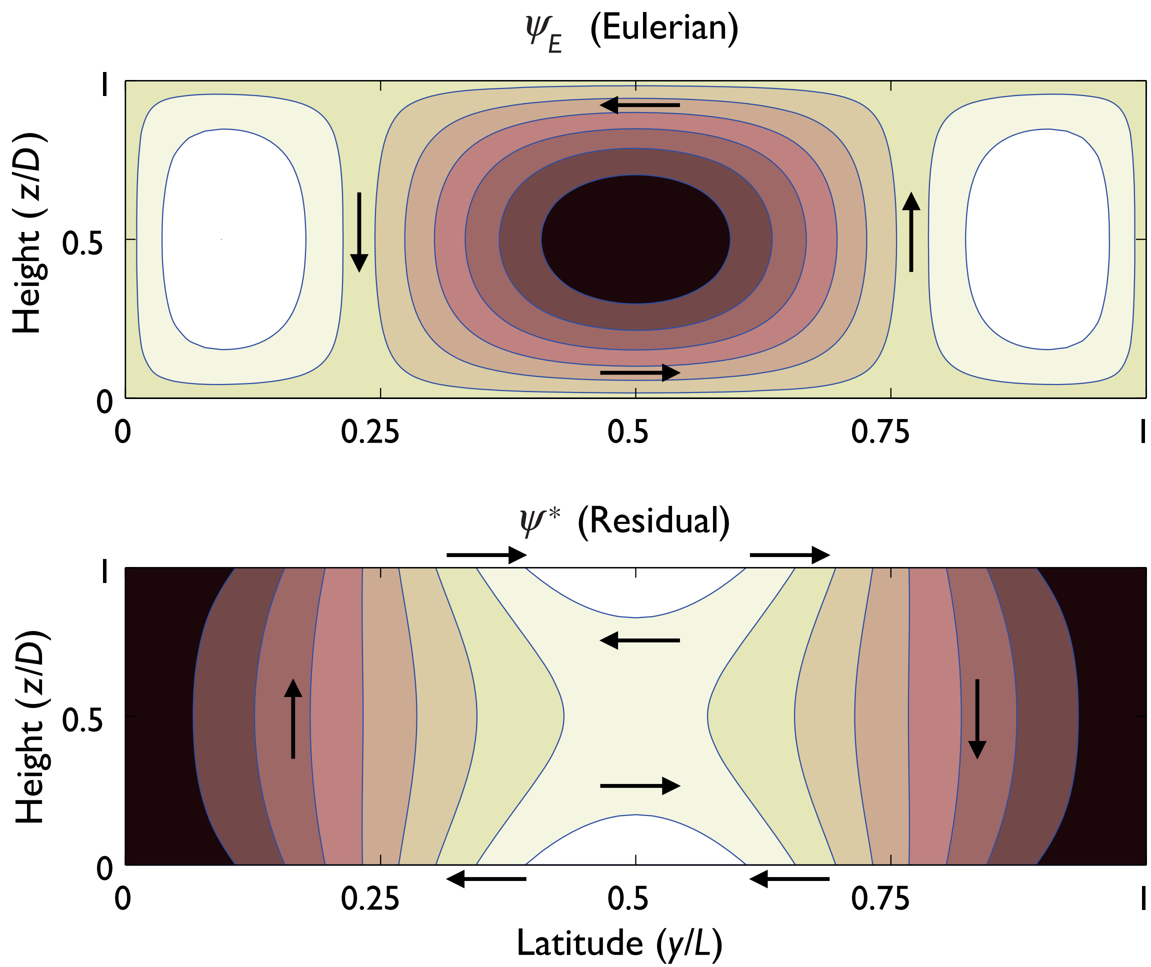
$$\tilde{\psi} = \psi_m + \frac{[v^*\theta^*]}{\partial\theta_s/\partial p}$$

Baroclinic eddies

- TEM

$$\tilde{\psi} = \psi_m + \frac{[v^*\theta^*]}{\partial\theta_s/\partial p}$$

Case 1: The EADY model



The residual mean circulation's direction can be **opposite** to the Eulerian mean circulation.

- In **isentropic** coordinate

$$(x, y, z) \Leftrightarrow (x, y, \theta)$$

$$\frac{D\theta}{Dt} = \dot{\theta}$$

$$\begin{aligned} \frac{D}{Dt} &= \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla_\theta + \frac{D\theta}{Dt}\frac{\partial}{\partial \theta} \\ &= \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla_\theta + \dot{\theta}\frac{\partial}{\partial \theta} \end{aligned}$$

zero for adiabatic flow

Isentrope: An isopleth of entropy. In meteorology it is usually identified with an isopleth of potential temperature.

Baroclinic eddies

- TEM

$$\tilde{\psi} = \psi_m + \frac{[v^*\theta^*]}{\partial\theta_s/\partial p}$$

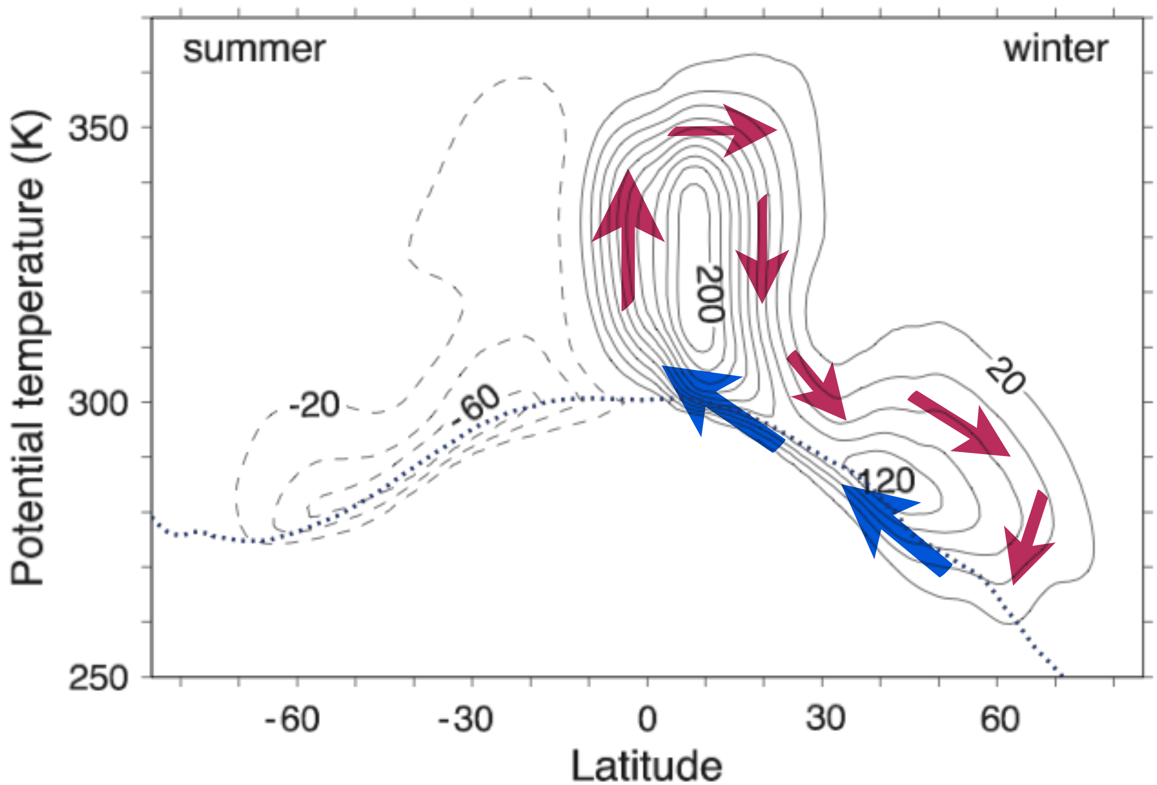
- In **isentropic** coordinate

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla_\theta + \frac{D\theta}{Dt}\frac{\partial}{\partial\theta} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla_\theta + \dot{\theta}\frac{\partial}{\partial\theta}$$

zero for adiabatic flow

The Ferrel cell in the isentropic coordinate is essentially reflect the *Residual Mean Circulation*.

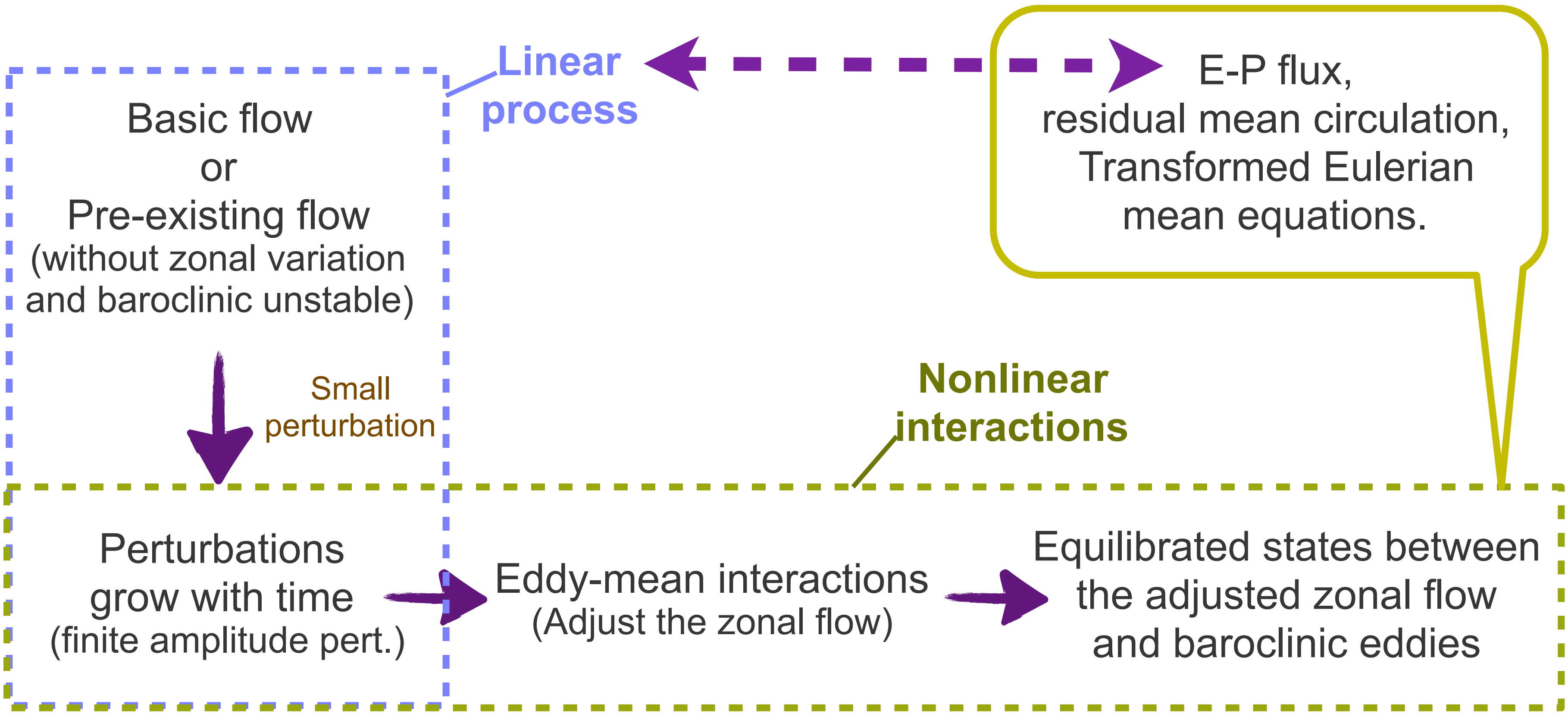
Case 2: Observed circulation



(Fig.11.4, Vallis, 2006)

Baroclinic eddies

- From linear to nonlinear



Baroclinic eddies

- E-P flux: a second view

- E-P flux and the Quasi-geostrophic potential vorticity

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

From the definition of QG potential vorticity:

$$\bar{q} = \frac{\partial^2\bar{\psi}}{\partial y^2} + \beta y + f_o^2\frac{\partial}{\partial p}\left(\frac{1}{s}\frac{\partial\bar{\psi}}{\partial p}\right)$$

$$q' = \frac{\partial^2\psi'}{\partial x^2} + \frac{\partial^2\psi'}{\partial y^2} + f_o^2\frac{\partial}{\partial p}\left(\frac{1}{s}\frac{\partial\psi'}{\partial p}\right)$$

$$\zeta' \qquad\qquad f_o\frac{\partial}{\partial p}\left(\frac{\theta'}{\partial\theta_s/\partial p}\right)$$

$$(u,v) = \left(-\frac{\partial\psi}{\partial y}, \frac{\partial\psi}{\partial x}\right) \qquad \theta = -\kappa f_o\frac{\partial\psi}{\partial p}$$

$$s = -\frac{1}{\kappa}\frac{\partial\theta_s}{\partial p} \qquad \kappa = \frac{p}{R}\left(\frac{p_o}{p}\right)^{R/c_p}$$

PV flux:

$$v'q' = v'\zeta' + \frac{f_o}{\partial\theta_s/\partial p}v'\frac{\partial\theta'}{\partial p}$$

$$v\zeta = v\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = -\frac{\partial}{\partial y}uv + \frac{1}{2}\frac{\partial}{\partial x}(v^2 - u^2)$$

$$v\frac{\partial\theta}{\partial p} = \frac{\partial}{\partial p}v\theta - \theta\frac{\partial v}{\partial p}$$

thermal wind relation for meridional wind

$$+\frac{\theta}{\kappa f_o}\frac{\partial\theta}{\partial x}$$

$$v\frac{\partial\theta}{\partial p} = \frac{\partial}{\partial p}v\theta + \frac{1}{2\kappa f_o}\frac{\partial}{\partial x}\theta^2$$

Note: ‘ denotes small perturbation

Baroclinic eddies
- E-P flux: a second view

- E-P flux and the Quasi-geostrophic potential vorticity

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

From the definition of QG potential vorticity:

$$
\begin{aligned}
v'q' &= v'\zeta' + \frac{f_o}{\partial\theta_s/\partial p} v' \frac{\partial\theta'}{\partial p} \\
&= \frac{1}{2}\frac{\partial}{\partial x}\left(v'^2 - u'^2 + \frac{1}{\kappa}\frac{\theta'^2}{\partial\theta_s/\partial p}\right) \\
&\quad - \frac{\partial}{\partial y}u'v' \\
&\quad + f_o\frac{\partial}{\partial p}\frac{v'\theta'}{\partial\theta_s/\partial p}
\end{aligned}
$$

$$
\begin{aligned}
v\zeta &= v\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \\
&= -\frac{\partial}{\partial y}uv + \frac{1}{2}\frac{\partial}{\partial x}(v^2 - u^2)
\end{aligned}
$$

$$v\frac{\partial\theta}{\partial p} = \frac{\partial}{\partial p}v\theta - \theta\frac{\partial v}{\partial p}$$

thermal wind relation for meridional wind

$$+\frac{\theta}{\kappa f_o}\frac{\partial\theta}{\partial x}$$

$$v\frac{\partial\theta}{\partial p} = \frac{\partial}{\partial p}v\theta + \frac{1}{2\kappa f_o}\frac{\partial}{\partial x}\theta^2$$

Baroclinic eddies

- E-P flux: a second view

- E-P flux and the Quasi-geostrophic potential vorticity

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

From the definition of QG potential vorticity:

$$\begin{aligned} v'q' &= v'\zeta' + \frac{f_o}{\partial\theta_s/\partial p} v' \frac{\partial\theta'}{\partial p} \\ &= \frac{1}{2}\frac{\partial}{\partial x}\left(v'^2 - u'^2 + \frac{1}{\kappa}\frac{\theta'^2}{\partial\theta_s/\partial p}\right) \\ &\quad - \frac{\partial}{\partial y} u'v' \\ &\quad + f_o \frac{\partial}{\partial p}\frac{v'\theta'}{\partial\theta_s/\partial p} \end{aligned}$$

#1

Zonally averaged PV flux by eddies:

$$\begin{aligned} [v^* q^*] &= - \frac{\partial}{\partial y}[u^* v^*] + f_o \frac{\partial}{\partial p}\frac{[v^*\theta^*]}{\partial\theta_s/\partial p} \\ &= \nabla\cdot\mathcal{F} \end{aligned}$$

Baroclinic eddies
- E-P flux: a second view

- E-P flux and the Eliassen-Palm relation

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

Linearized PV equation (q=PV):

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) q' + \frac{\partial\psi'}{\partial x}\frac{\partial\bar{q}}{\partial y} = 0$$

Multiplying by q' and zonally average:

$$\frac{1}{2}\frac{\partial}{\partial t}[q'^2] + [v'q']\frac{\partial\bar{q}}{\partial y} = 0$$

Define *wave activity density:*

$$\mathcal{A} = \frac{[q'^2]}{2\partial\bar{q}/\partial y}$$

#2

$$\frac{\partial\mathcal{A}}{\partial t} + \nabla\cdot\mathcal{F} = 0$$

Eliassen-Palm relation

Baroclinic eddies

- E-P flux: a second view

- E-P flux and the Rossby waves

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

Linearized PV equation (q=PV):

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) q' + \frac{\partial \psi'}{\partial x}\frac{\partial \bar{q}}{\partial y} = 0$$

Assume U is fixed, and $\frac{\partial \bar{q}}{\partial y} = \beta$

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left[\nabla^2\psi' + f_o^2\frac{\partial}{\partial p}\left(\frac{1}{s}\frac{\partial \psi'}{\partial p}\right)\right] + \beta\frac{\partial \psi'}{\partial x} = 0$$

Exist solutions of the form

$$\psi' = \mathrm{Re}\Psi e^{i(kx+ly+mp-\omega t)}$$

Dispersion relation of Rossby waves:

$$\omega = Uk - \frac{\beta k}{K^2}$$

$$K^2 = k^2 + l^2 + m^2 f_o^2/s$$

with group velocity

$$c_{gy} = \frac{2\beta kl}{K^4} \quad c_{gp} = \frac{2\beta k m f_o^2/s}{K^4}$$

$$A' = \mathrm{Re}\hat{A}e^{i(kx+ly+mp-\omega t)}$$

$$\hat{u} = -\mathrm{Re}il\Psi,\ \hat{v} = \mathrm{Re}ik\Psi$$

$$\hat{\theta} = -\mathrm{Re}im\kappa f_o\Psi,\ \hat{q} = -\mathrm{Re}K^2\Psi$$

Baroclinic eddies

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- E-P flux and the Rossby waves

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

Linearized PV equation (q=PV):

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) q' + \frac{\partial\psi'}{\partial x}\frac{\partial\bar{q}}{\partial y} = 0$$

with group velocity

$$c_{gy} = \frac{2\beta kl}{K^4} \quad c_{gp} = \frac{2\beta km f_o^2/s}{K^4}$$

$$K^2 = k^2 + l^2 + m^2 f_o^2/s$$

$$\hat{u} = -\mathrm{Re}\, il\Psi,\ \hat{v} = \mathrm{Re}\, ik\Psi$$

$$\hat{\theta} = -\mathrm{Re}\, im\kappa f_o\Psi,\ \hat{q} = -\mathrm{Re}\, K^2\Psi$$

Wave activity density:

$$\mathcal{A} = \frac{[q'^2]}{2\beta} = \frac{K^4}{4\beta}|\Psi^2|$$

$$-[u'v'] = \frac{1}{2}kl|\Psi^2| = c_{gy}\mathcal{A}$$

$$f_o\frac{[v'\theta']}{\partial\theta_s/\partial p} = \frac{f_o^2}{2s}km|\Psi^2| = c_{gz}\mathcal{A}$$

#3

$$\vec{\mathcal{F}} = \vec{\mathbf{c}_\mathbf{g}}\mathcal{A}$$

E-P flux, TEM and Residual Circulation - Summary

- E-P flux: $$\mathcal{F} \equiv -[u^*v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

- In a **steady**, **adiabatic** and **frictionless** flow:

$$[v] = \frac{1}{f}\frac{\partial}{\partial y}([u^*v^*]) \qquad [\omega] = -\frac{\partial}{\partial y}\left(\frac{[\theta^*v^*]}{\partial\theta_s/\partial p}\right) \qquad \nabla\cdot\mathcal{F} = 0$$

#1
$$[v^*q^*] = -\frac{\partial}{\partial y}[u^*v^*] + f_o\frac{\partial}{\partial p}\frac{[v^*\theta^*]}{\partial\theta_s/\partial p} = \nabla\cdot\mathcal{F}$$

#2
$$\frac{\partial\mathcal{A}}{\partial t} + \nabla\cdot\mathcal{F} = 0$$

#3
$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A} \quad \longrightarrow \quad \frac{\partial\mathcal{A}}{\partial t} + \nabla\cdot(\mathcal{A}\vec{\mathbf{c_g}}) = 0$$

- Residual mean circulations:

$$[\tilde{\omega}] = [\omega] + \frac{\partial}{\partial y}\left(\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\right), \qquad [\tilde{v}] = [v] - \frac{\partial}{\partial p}\left(\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\right)$$

- TEM equations: $$\frac{\partial[u]}{\partial t} = f[\tilde{v}] + \nabla\cdot\mathcal{F} + [F_x], \qquad \frac{\partial[\theta]}{\partial t} = -[\tilde{\omega}]\frac{\partial\theta_s}{\partial p} + \left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q]}{c_p}$$

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Baroclinic eddy life cycle

- An E-P flux view

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^* \theta^*]}{\partial \theta_s / \partial p}\,\mathbf{k}$$

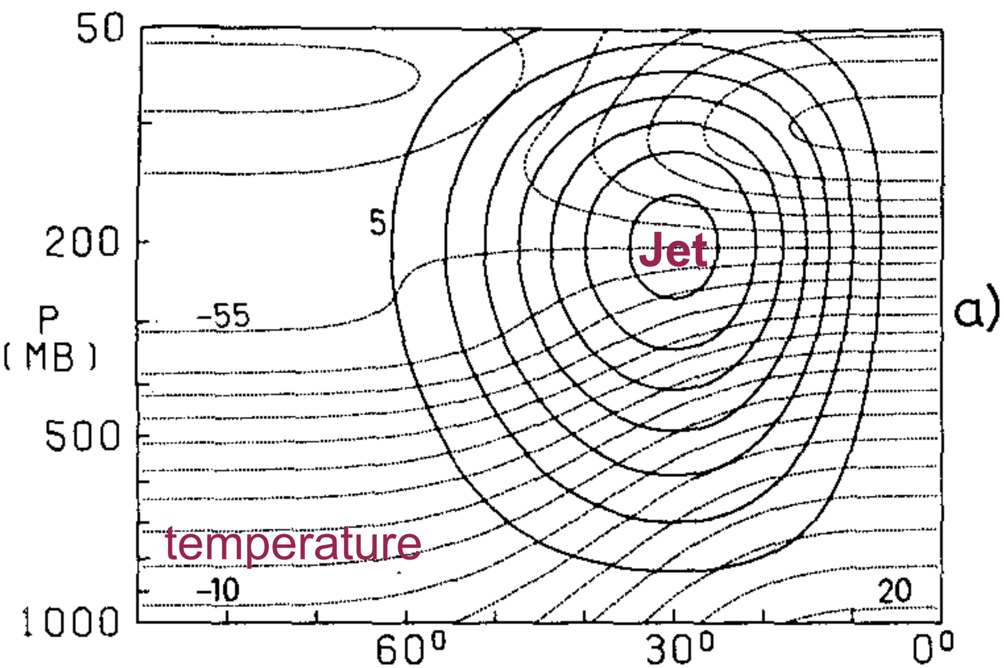
$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

$$\frac{\partial [u]}{\partial t} = f[\tilde{v}] + \nabla \cdot \mathcal{F} + [F_x]$$

$$\frac{\partial [\theta]}{\partial t} = -[\tilde{\omega}]\frac{\partial \theta_s}{\partial p} + \left(\frac{p_o}{p}\right)^{R/c_p}$$

Numerical results from
Simmons and Hoskins, 1978, JAS

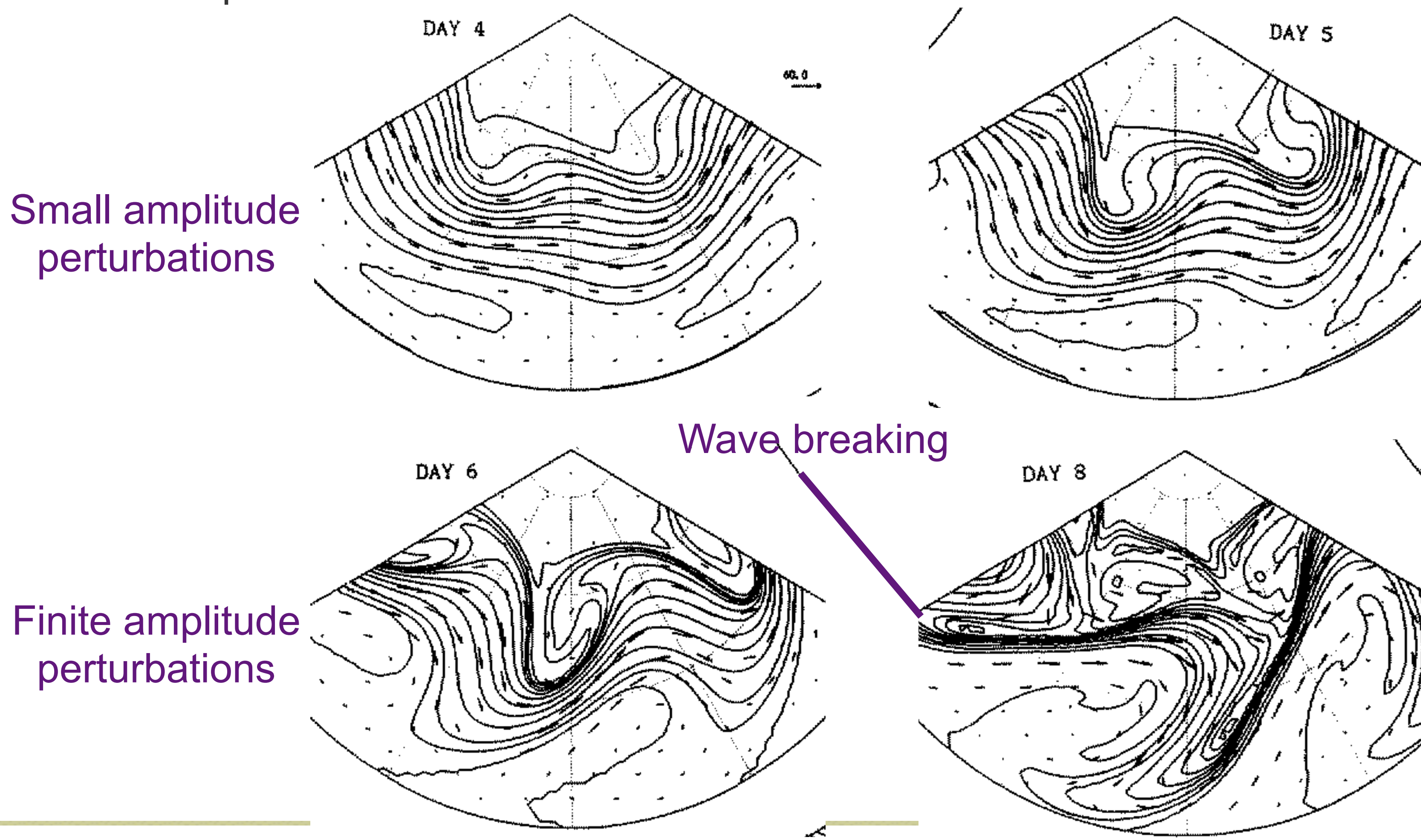
Initial state:



Baroclinic eddies

- baroclinic eddy life cycle

- Eddies' development



(Thorncroft et al, 1993, Q.J.R.)

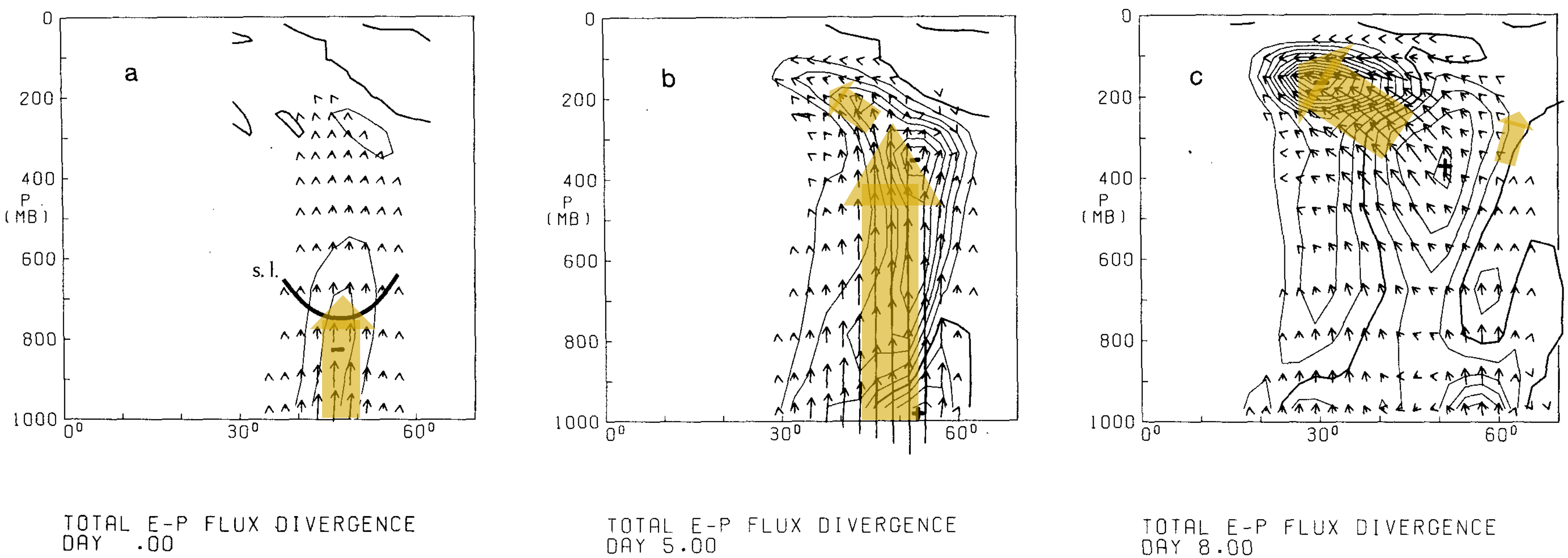
Baroclinic eddy life cycle
- An E-P flux view

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

Numerical results from
Simmons and Hoskins, 1978, JAS

Eddies: generate at lower level, propagate **upwards** and **away** from the eddy source region



E-P flux

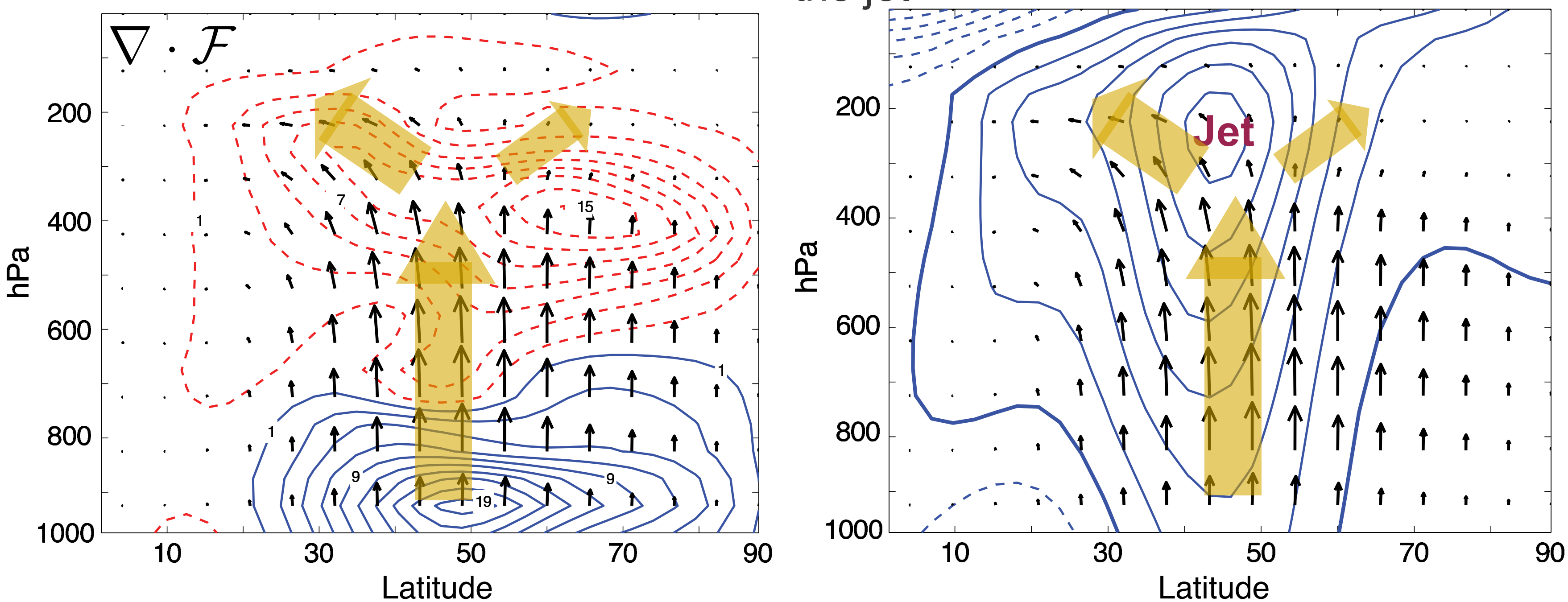
- In the equilibrium state

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

Wave energies: propagate **upwards** and **away** from the center of the jet

Numerical results from idealized model with pure midlatitude jet (Vallis, 2006)



E-P flux

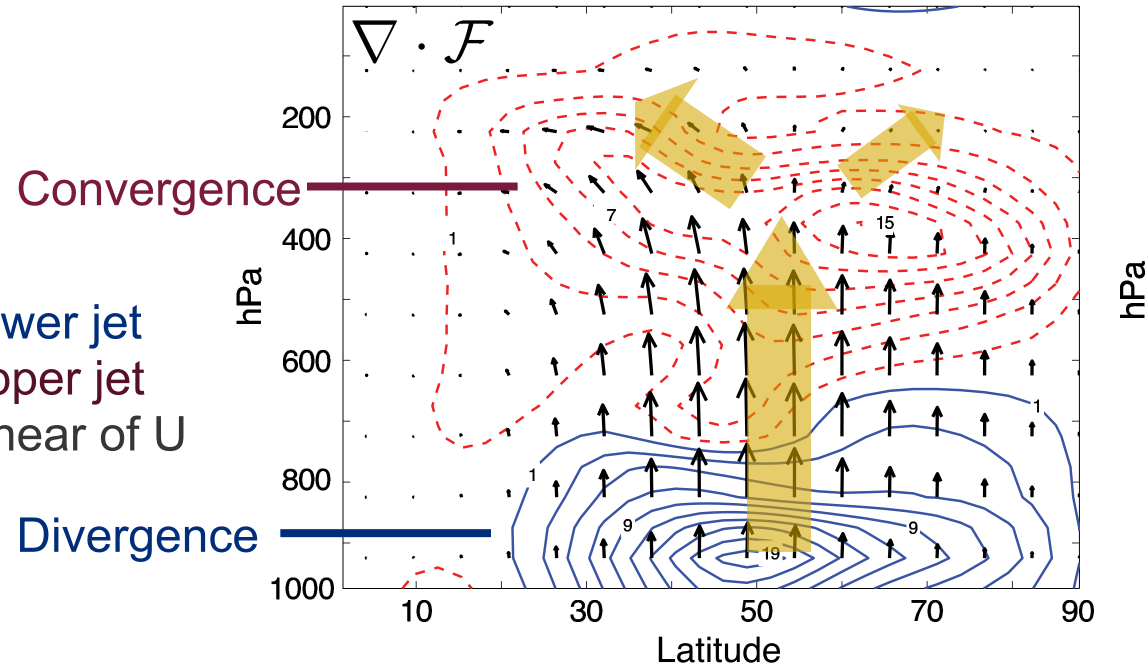
- The westerly jet

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

$$\frac{\partial [u]}{\partial t} = f[\tilde{v}] + \nabla\cdot\mathcal{F} + [F_x]$$

Wave energies: propagate **upwards** and **away** from the center of the jet



In the vertical direction:

Accelerating the lower jet
decelerating the upper jet
reduce the vertical shear of U

E-P flux

- The westerly jet

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

$$\frac{\partial [u]}{\partial t} = f[\tilde{v}] + \nabla\cdot\mathcal{F} + [F_x]$$

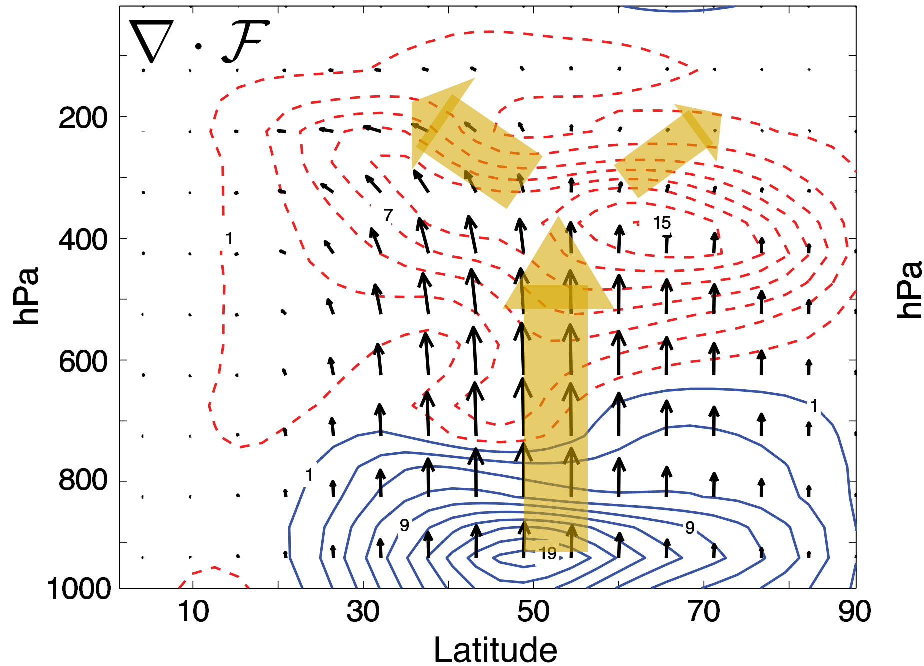
Integrate vertically:

$$\frac{1}{g}\int_0^{p_s} \mathrm{d}p$$

$$\frac{\partial}{\partial t} < [u] >= -\frac{\partial}{\partial y} < [u^* v^*] > -r[u_{\mathrm{surf}}]$$

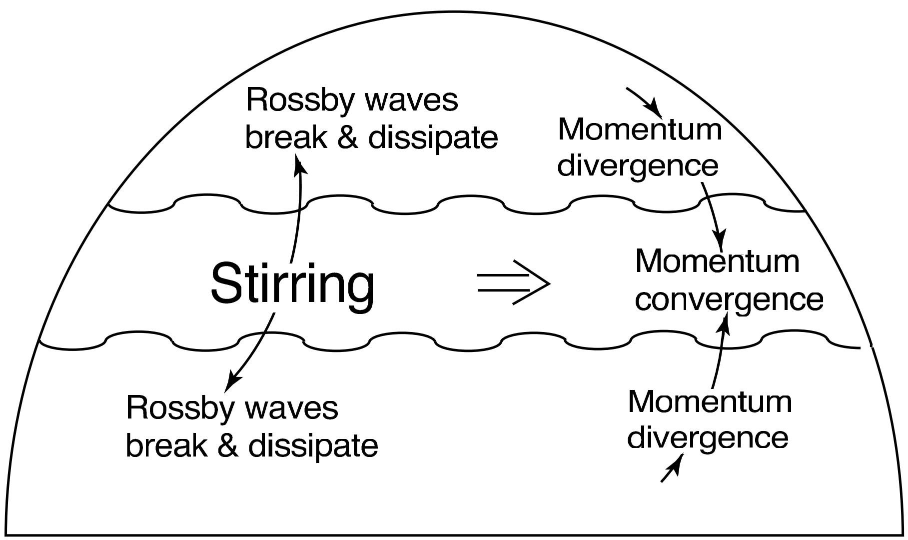
< > means vertical average

Wave energies: propagate **upwards** and **away** from the center of the jet



Eddy-driven jet:
- the momentum budget

$$\mathcal{F} \equiv -[u^*v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$



$$\frac{\partial}{\partial t} < [u] > = -\frac{\partial}{\partial y} < [u^*v^*] > -r[u_{\text{surf}}]$$

< > means vertical average

Wave energies: propagate **upwards** and **away** from the center of the jet

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

In equilibrium:

$$r[u_{\text{surf}}] \sim -\frac{\partial}{\partial y} < [u^*v^*] >$$

There MUST be **surface westerlies** at midlatitudes.

E-P flux

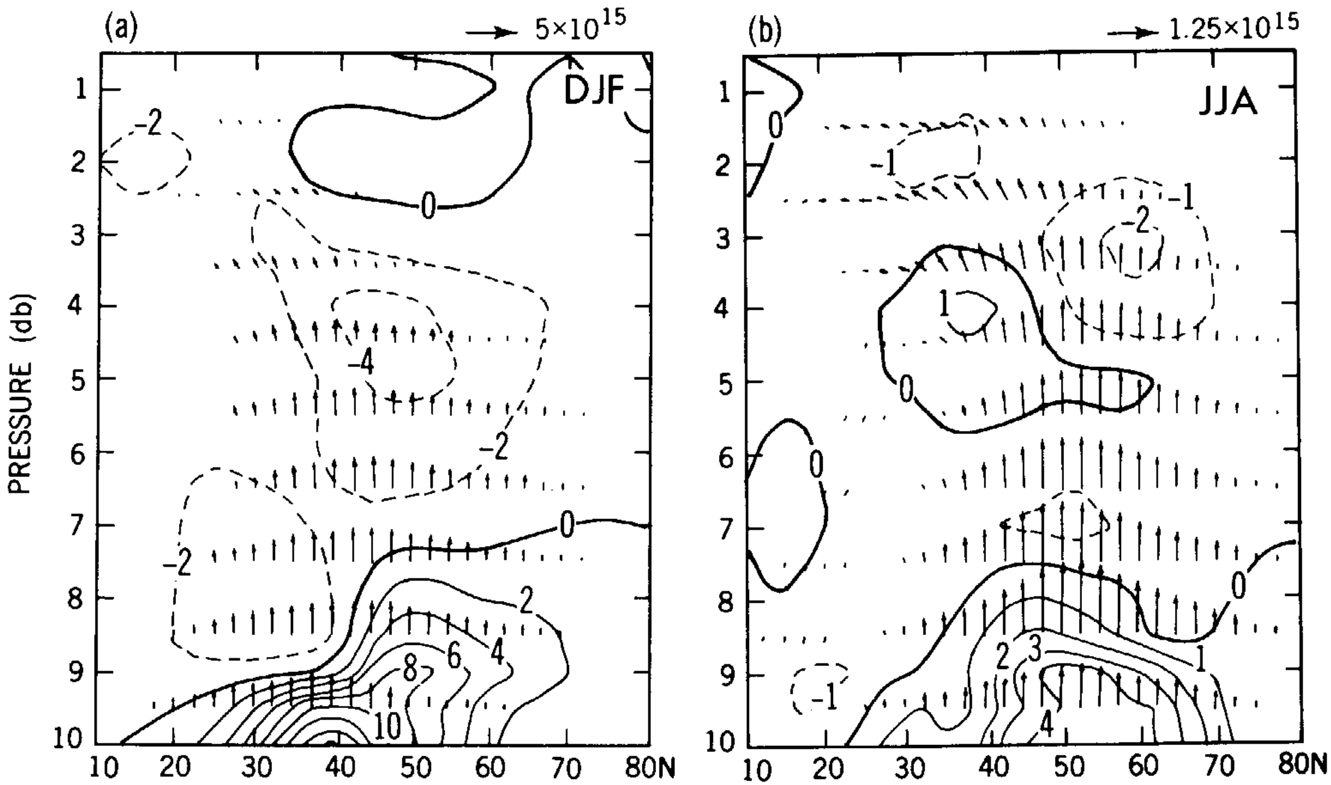
- in the real atmosphere

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

Vertical component is dominant.

EP divergence in the lower layers; convergence in the upper layers.



E-P flux and the eddy-driven jet

-summary

$$\mathcal{F} \equiv -[u^*v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

- Numerical results and observations: eddies generate in the lower level, propagate **upwards** and **away** from the eddy source region.

$$\frac{\partial[u]}{\partial t} = f[\tilde{v}] + \nabla\cdot\mathcal{F} + [F_x]$$

- Accelerating the lower jet, decelerating the upper jet, reduce the vertical shear of U

- Momentum budget indicates that there MUST be surface westerlies in the eddy source latitude.