第四章：中纬度的经向环流系统(IV)

授课教师：张洋

- Ferrel cell, baroclinic eddies and the westerly jet

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E-P flux, TEM and Residual Circulation

- Summary

**E-P flux:**

\[ F \equiv -[u^*v^*] j + f \left( \frac{v^*\theta^*}{\partial \theta_s / \partial p} \right) k \]

**In a steady, adiabatic and frictionless flow:**

\[ [v] = \frac{1}{f} \frac{\partial}{\partial y} ([u^*v^*]) \quad [\omega] = -\frac{\partial}{\partial y} \left( \frac{[\theta^*v^*]}{\partial \theta_s / \partial p} \right) \]

\[ \nabla \cdot F = 0 \]

**Residual mean circulations:**

\[ \tilde{\omega} = [\omega] + \frac{\partial}{\partial y} \left( \frac{[v^*\theta^*]}{\partial \theta_s / \partial p} \right), \quad \tilde{v} = [v] - \frac{\partial}{\partial p} \left( \frac{[v^*\theta^*]}{\partial \theta_s / \partial p} \right) \]

**TEM equations:**

\[ \frac{\partial [u]}{\partial t} = f[\tilde{v}] + \nabla \cdot F + [F_x], \quad \frac{\partial [\theta]}{\partial t} = -[\tilde{\omega}] \frac{\partial \theta_s}{\partial p} + \left( \frac{p_o}{p} \right)^{R/c_p} \frac{Q}{c_p} \]
E-P flux, TEM and Residual Circulation

- Summary

- **E-P flux:**
  \[ \mathcal{F} \equiv -[u^*v^*] \mathbf{j} + f \frac{[v^*\theta^*]}{\partial \theta_s / \partial p} \mathbf{k} \]

- In a **steady**, **adiabatic** and **frictionless** flow:
  \[ [v] = \frac{1}{f} \frac{\partial}{\partial y} ([u^*v^*]) \quad [\omega] = -\frac{\partial}{\partial y} \left( \frac{[\theta^*v^*]}{\partial \theta_s / \partial p} \right) \quad \nabla \cdot \mathcal{F} = 0 \]

  \[ [v^*q^*] = -\frac{\partial}{\partial y} [u^*v^*] + f_o \frac{\partial}{\partial p} \frac{[v^*\theta^*]}{\partial \theta_s / \partial p} \]

  \[ = \nabla \cdot \mathcal{F} \]

- **#1**
  \[ [v^*q^*] = -\frac{\partial}{\partial y} [u^*v^*] + f_o \frac{\partial}{\partial p} \frac{[v^*\theta^*]}{\partial \theta_s / \partial p} \]

- **#2**
  \[ \frac{\partial \mathcal{A}}{\partial t} + \nabla \cdot \mathcal{F} = 0 \]

- **#3**
  \[ \mathcal{F} = \mathbf{c}_g \mathcal{A} \]

- **Residual mean circulations:**
  \[ [\tilde{\omega}] = [\omega] + \frac{\partial}{\partial y} \left( \frac{[v^*\theta^*]}{\partial \theta_s / \partial p} \right), \quad [\tilde{v}] = [v] - \frac{\partial}{\partial p} \left( \frac{[v^*\theta^*]}{\partial \theta_s / \partial p} \right) \]

- **TEM equations:**
  \[ \frac{\partial [u]}{\partial t} = f[\tilde{v}] + \nabla \cdot \mathcal{F} + [F_x], \quad \frac{\partial [\theta]}{\partial t} = -[\tilde{\omega}] \frac{\partial \theta_s}{\partial p} + \left( \frac{p_o}{p} \right)^{R/c_p} \frac{[Q]}{c_p} \]
Outline

- Observations
- The Ferrel Cell
- Baroclinic eddies
  - Review: baroclinic instability and baroclinic eddy life cycle
  - Eddy-mean flow interaction, E-P flux
  - Transformed Eulerian Mean equations
- Eddy-driven jet
- The energy cycle
Baroclinic eddy life cycle
- An E-P flux view

\[ \mathcal{F} \equiv -[u^* v^*] \mathbf{j} + f \frac{[v^* \theta^*]}{\partial \theta_s / \partial p} \mathbf{k} \]

\[ \mathbf{\vec{F}} = \mathbf{c_g A} \]

Eddies: generate at lower level, propagate upwards and away from the eddy source region

Numerical results from Simmons and Hoskins, 1978, JAS

Review
E-P flux

\[ \mathcal{F} \equiv -[u^* v^*] \mathbf{j} + f \frac{[v^* \theta^*]}{\partial \theta_s/\partial p} \mathbf{k} \]

\[ \mathbf{\tilde{F}} = \mathbf{c}_g \mathbf{A} \]

\[ \frac{\partial [u]}{\partial t} = f[\tilde{v}] + \nabla \cdot \mathcal{F} + [\mathcal{F}_x] \]

In the vertical direction:

- Accelerating the lower jet
- Decelerating the upper jet
- Reduce the vertical shear of \( U \)

Convergence

Divergence

Wave energies: propagate \textbf{upwards} and \textbf{away} from the center of the jet
\[ \mathcal{F} \equiv -[u^* v^*] \mathbf{j} + f \frac{[v^* \theta^*]}{\partial \theta_s / \partial p} \mathbf{k} \]

Wave energies: propagate **upwards** and **away** from the center of the jet

\[ \mathbf{\bar{F}} = c_g A \]

\[ \frac{\partial}{\partial t} < [u] > = - \frac{\partial}{\partial y} < [u^* v^*] > - r [u_{surf}] \]

\(< > \) means vertical average
\[
\mathcal{F} \equiv -[u^* v^*] j + f \left[ \frac{v^* \theta^*}{\partial \theta_s / \partial p} \right] k
\]

In equilibrium:

\[
\vec{F} = c_g \vec{A}
\]

\[
\frac{\partial}{\partial t} < [u] > = -\frac{\partial}{\partial y} < [u^* v^*] > - r[u_{surf}]
\]

\(< > \) means vertical average
\[ \mathcal{F} \equiv -[u^* v^*] \mathbf{j} + f \frac{[v^* \theta^*]}{\partial s/\partial p} \mathbf{k} \]

In equilibrium:
\[ \vec{\mathcal{F}} = c_g \vec{A} \]
\[ r[u_{\text{surf}}] \sim -\frac{\partial}{\partial y} <[u^* v^*]> \]

Wave energies propagate **upwards and away** from the center of the jet.

There MUST be **surface westerlies** at midlatitudes.

\[ \frac{\partial}{\partial t} <[u]> = -\frac{\partial}{\partial y} <[u^* v^*]> - r[u_{\text{surf}}] \]

\(< >\) means vertical average
Eddy-driven jet:

\[ \mathcal{F} \equiv -[u^* v^*] \mathbf{j} + f \frac{[v^* \theta^*]}{\partial \theta_s / \partial p} \mathbf{k} \]

\[ \frac{\partial}{\partial t} < [u] > = - \frac{\partial}{\partial y} < [u^* v^*] > - r[u_{surf}] \]

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Wave energies: propagate **upwards** and **away** from the center of the jet

In equilibrium:

\[ \vec{\mathcal{F}} = c_g \mathcal{A} \]

\[ r[u_{surf}] \sim - \frac{\partial}{\partial y} < [u^* v^*] > \]

There MUST be **surface westerlies** at midlatitudes.

From Vallis (2006)
\[ F \equiv -[u^* v^*] \mathbf{j} + f \frac{v^* \theta^*}{\partial \theta_s / \partial p} \mathbf{k} \]

\[ \vec{F} = c_g \mathbf{A} \]
E-P flux

- in the real atmosphere

\[ \mathcal{F} \equiv -[u^* v^*] j + f \left[ \frac{v^* \theta^*}{\partial \theta_s / \partial p} \right] k \]

\[ \vec{F} = c_g A \]
E-P flux

\[ \mathcal{F} \equiv -[u^* v^*] \mathbf{j} + f \left( \frac{v^* \theta^*}{\partial \theta_s / \partial p} \right) \mathbf{k} \]

\[ \mathbf{F} = c_g \mathbf{A} \]

Vertical component is dominant.
E-P flux

\[ F \equiv -[u^* v^*] j + f \left[ \frac{v^* \theta^*}{\partial \theta_s / \partial p} \right] k \]
\[ \vec{F} = c_g A \]

Vertical component is dominant.

EP divergence in the lower layers; convergence in the upper layers.
E-P flux and the eddy-driven jet

\[ \mathcal{F} \equiv -[u^* v^*] j + f \left[ \frac{v^* \theta^*}{\partial \theta_s/\partial p} \right] k \]

\[ \vec{\mathcal{F}} = \vec{c}_g A \]

\[ \frac{\partial [u]}{\partial t} = f[\tilde{v}] + \nabla \cdot \mathcal{F} + [F_x] \]
E-P flux and the eddy-driven jet

\[ \mathcal{F} \equiv -[u^* v^*] \mathbf{j} + f \left[ \frac{v^* \theta^*}{\partial \theta_s / \partial p} \right] \mathbf{k} \]

\[ \vec{\mathcal{F}} = \mathbf{c}_g \mathbf{A} \]

- Numerical results and observations: eddies generate in the lower level, propagate \textbf{upwards} and \textbf{away} from the eddy source region.

\[ \frac{\partial [u]}{\partial t} = f[\tilde{v}] + \nabla \cdot \mathcal{F} + [F_x] \]
E-P flux and the eddy-driven jet

\[ F \equiv -[u^* v^*] \mathbf{j} + f \left[ \frac{v^* \theta^*}{\partial \theta_s / \partial p} \right] \mathbf{k} \]

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- Numerical results and observations: eddies \textit{generate} in the lower level, propagate \textit{upwards} and \textit{away} from the eddy source region.

\[ \frac{\partial[u]}{\partial t} = f[\tilde{v}] + \nabla \cdot \mathbf{F} + [F_x] \]

- \textit{Accelerating} the lower jet, \textit{decelerating} the upper jet, reduce the vertical shear of U
E-P flux and the eddy-driven jet

- summary

\[ \mathcal{F} \equiv - [u^* v^*] \mathbf{j} + f \left[ \frac{v^* \theta^*}{\partial \theta_s / \partial p} \right] \mathbf{k} \]

\[ \mathbf{\tilde{F}} = \mathbf{c}_g \mathbf{A} \]

- Numerical results and observations: eddies generate in the lower level, propagate upwards and away from the eddy source region.

\[ \frac{\partial [u]}{\partial t} = f[\tilde{v}] + \nabla \cdot \mathcal{F} + [F_x] \]

- Accelerating the lower jet, decelerating the upper jet, reduce the vertical shear of U

- Momentum budget indicates that there MUST be surface westerlies in the eddy source latitude.
Outline

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Energy cycles
in the baroclinic eddy-mean flow interactions

- Basic forms of energy:
Basic forms of energy:

- Kinetic energy (动能):
  \[ K = \frac{1}{2}(u^2 + v^2 + w^2) \approx \frac{1}{2}(u^2 + v^2) \]
Basic forms of energy:

- Kinetic energy (动能):
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- Internal energy (内能):
  \[ I = c_v T \]
Basic forms of energy:

- Kinetic energy (动能): \[ K = \frac{1}{2} (u^2 + v^2 + w^2) \approx \frac{1}{2} (u^2 + v^2) \]
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- Gravitational-potential energy (位能): \[ \Phi = gz \]
Energy cycles
in the baroclinic eddy-mean flow interactions

- **Basic forms of energy:**
  - Kinetic energy (动能): \( K = \frac{1}{2} (u^2 + v^2 + w^2) \approx \frac{1}{2} (u^2 + v^2) \)
  - Internal energy (内能): \( I = c_v T \)
  - Gravitational-potential energy (位能): \( \Phi = gz \)
  - Latent energy (相变潜热能): \( LH = Lq \)
Energy cycles in the baroclinic eddy-mean flow interactions

- **Basic forms of energy:**
  - Kinetic energy (动能): \[ K = \frac{1}{2}(u^2 + v^2 + w^2) \approx \frac{1}{2}(u^2 + v^2) \]
  - Internal energy (内能): \[ I = c_v T \]
  - Gravitational-potential energy (位能): \[ \Phi = gz \]
  - Latent energy (相变潜热能): \[ \text{LH} = \text{Lq} \]
  - **Total energy:** \[ E = I + \Phi + \text{LH} + K \]
Basic forms of energy:

- Kinetic energy (动能): \[ K = \frac{1}{2} (u^2 + v^2 + w^2) \approx \frac{1}{2} (u^2 + v^2) \]
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Energy cycles in the baroclinic eddy-mean flow interactions

- **Basic forms of energy:**

  - **Kinetic energy (动能):** \( K = \frac{1}{2}(u^2 + v^2 + w^2) \approx \frac{1}{2}(u^2 + v^2) \)
  
  - **Internal energy (内能):** \( I = c_v T \)
  
  - **Gravitational-potential energy (位能):** \( \Phi = gz \)

- **Total potential energy:**

  \[
  \int_0^\infty \rho (I + \Phi) dz = \frac{1}{g} \int_0^{p_s} (c_v T + RT) dp = \frac{1}{g} \int_0^{p_s} c_p T dp
  \]
Basic forms of energy:

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### Energy cycles in the baroclinic eddy-mean flow interactions

#### Basic forms of energy:

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\]

*Figure by MIT OCW.*

As we see there are horizontal gradients and because of this PE can be released by the motions in spite of the stable stratification. Consider for example a displacement like that shown below.

From Stone’s class notes
Energy cycles in the baroclinic eddy-mean flow interactions

- Basic forms of energy:
  - Kinetic energy (动能):
    \[ K = \frac{1}{2} (u^2 + v^2 + w^2) \approx \frac{1}{2} (u^2 + v^2) \]
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    \int_0^\infty \rho(I + \Phi)dz = \frac{1}{g} \int_0^{P_s} (c_v T + RT)dp = \frac{1}{g} \int_0^{P_s} c_p T dp
    \]

- Fig. 6.9: A steady basic state giving rise to baroclinic instability. Potential density decreases upwards and equatorwards, and the associated horizontal pressure gradient is balanced by the Coriolis force. Parcel 'A' is heavier than 'C', and so statically stable, but it is lighter than 'B'. Hence, if 'A' and 'B' are interchanged there is a release of potential energy.

From Stone’s class notes
Energy cycles
in the baroclinic eddy-mean flow interactions

Basic forms of energy:

- Kinetic energy (动能):
  \[ K = \frac{1}{2} (u^2 + v^2 + w^2) \approx \frac{1}{2} (u^2 + v^2) \]

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From Stone’s class notes
Basic forms of energy:

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**Energy cycles**
in the baroclinic eddy-mean flow interactions

- **Basic forms of energy:**
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    \[ K = \frac{1}{2}(u^2 + v^2 + w^2) \approx \frac{1}{2}(u^2 + v^2) \]
    
    \[ \int_{0}^{\infty} \rho (I + \Phi) dz = \frac{1}{g} \int_{0}^{P_s} (c_v T + RT) dp = \frac{1}{g} \int_{0}^{P_s} c_p T dp \]

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**Energy cycles**
in the baroclinic eddy-mean flow interactions

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**Energy cycles**
in the baroclinic eddy-mean flow interactions

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  - Kinetic energy (动能):  
    \[ K = \frac{1}{2}(u^2 + v^2 + w^2) \approx \frac{1}{2}(u^2 + v^2) \]
    
    \[ \int_{0}^{\infty} \rho (I + \Phi) dz = \frac{1}{g} \int_{0}^{P_s} (c_v T + RT) dp = \frac{1}{g} \int_{0}^{P_s} c_p T dp \]
In this new state, since \( z > 0 \), there are no displacements which can release PE, because now for any displacement, \( w' < 0 \). Thus the PE of this state is unavailable.

This led Lorenz (1955) to define the available potential energy as the difference between the TPE in the actual atmosphere and that in the adjusted state just described. If we call this \( P \), then for a given system or region,

\[
P = \left( C_P \right)_{p_0} \int_0^\infty \rho (I + \Phi) dz = \frac{1}{g} \int_0^{p_s} \left( c_v T + RT \right) dp = \frac{1}{g} \int_0^{p_s} c_p T dp
\]

Since \( p' \) is already integrated over the area, we can rewrite the integral as

\[
\int_0^\infty \rho (I + \Phi) dz = \frac{1}{g} \int_0^{p_s} \left( c_v T + RT \right) dp = \frac{1}{g} \int_0^{p_s} c_p T dp
\]

This is the so-called exact formula.

Note that, since \( \rho (I + \Phi) \) is already integrated over the area, we can rewrite the integral as

\[
\int_0^\infty \rho (I + \Phi) dz = \frac{1}{g} \int_0^{p_s} \left( c_v T + RT \right) dp = \frac{1}{g} \int_0^{p_s} c_p T dp
\]

Lorenz derived an approximation for \( P \) that is commonly used. Let \( p = p_s + p' \), and similarly for other variables:

\[
\int_0^\infty \rho (I + \Phi) dz = \frac{1}{g} \int_0^{p_s} \left( c_v T + RT \right) dp = \frac{1}{g} \int_0^{p_s} c_p T dp
\]
In this new state, since \( z > 0 \), there are no displacements which can release PE, because now for any displacement, \( w' < 0 \). Thus the PE of this state is unavailable.

This led Lorenz (1955) to define the available potential energy as the difference between the TPE in the actual atmosphere and that in the adjusted state just described. If we call this \( P \), then for a given system or region,

\[
P = \int_{0}^{\infty} \int_{0}^{P_s} \rho(I + \Phi)dz = \frac{1}{g} \int_{0}^{P_s} (c_v T + RT)dp = \frac{1}{g} \int_{0}^{P_s} c_p Tdp
\]

Since \( p' \) is already integrated over the area, we can rewrite the integral as

\[
\int_{0}^{\infty} \int_{0}^{P_s} (\rho(I + \Phi)dz = \frac{1}{g} \int_{0}^{P_s} c_p Tdp
\]

This is the so-called exact formula. Note that, since \( (p') > 0 \), \( (p') > 0 \), i.e. \( P \) is according to our definition always positive.

Lorenz derived an approximation for \( P \) that is commonly used. Let \( p' = p + p' \), and similarly for other variables:
Basic forms of energy:

- Kinetic energy (动能): 
  \[ K = \frac{1}{2} (u^2 + v^2 + w^2) \approx \frac{1}{2} (u^2 + v^2) \]

\[ \int_{0}^{\infty} \rho (I + \Phi) \, dz = \frac{1}{g} \int_{0}^{P_s} (c_v T + RT) \, dp = \frac{1}{g} \int_{0}^{P_s} c_p T \, dp \]

State A  \quad State B \quad = \text{Available potential energy}
Basic forms of energy:

- Kinetic energy (动能): \[ K = \frac{1}{2} (u^2 + v^2 + w^2) \approx \frac{1}{2} (u^2 + v^2) \]

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State A \rightarrow State B = Available potential energy

- Available potential energy (有效位能):

\[ P = \frac{1}{2} \int_0^{P_s} \frac{T_s}{\gamma_d - \gamma_s} \left( \frac{T - T_s}{T_s} \right)^2 dp = \frac{c_p}{2g} \int_0^{P_s} \Gamma (T - T_s)^2 dp \]

\[ \Gamma = -\frac{R}{c_p p} \left( \frac{p_s}{p} \right)^{\frac{R}{c_p}} \left( \frac{\partial \theta_s}{\partial p} \right)^{-1} \]

\[ = (\gamma_d/T_s) (\gamma_d - \gamma_s)^{-1} \]
**Energy cycles in the baroclinic eddy-mean flow interactions**

**Basic forms of energy:**

- **Kinetic energy (动能):**
  
  \[ K = \frac{1}{2} (u^2 + v^2 + w^2) \approx \frac{1}{2} (u^2 + v^2) \]

  \[ \int_0^\infty \rho(I + \Phi) \, dz = \frac{1}{g} \int_0^{P_s} (c_v T + RT) \, dp = \frac{1}{g} \int_0^{P_s} c_p T \, dp \]

State A \[\rightarrow\] State B = **Available potential energy**

- **Available potential energy (有效位能):**

  \[ P = \frac{1}{2} \int_0^{P_s} \frac{T_s}{\gamma_d - \gamma_s} \left( \frac{T - T_s}{T_s} \right)^2 \, dp = \frac{c_p}{2g} \int_0^{P_s} \Gamma (T - T_s)^2 \, dp \]

  From the “approximate” expression of Lorenz (1955)

  \[ \Gamma = -\frac{R}{c_p p} \left( \frac{p_s}{p} \right)^{\frac{R}{c_p}} \left( \frac{\partial \theta_s}{\partial p} \right)^{-1} \]

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Energy cycles in the baroclinic eddy-mean flow interactions

Basic forms of energy:

- Kinetic energy (动能): \( K = \frac{1}{2} (u^2 + v^2 + w^2) \approx \frac{1}{2} (u^2 + v^2) \)

- Available potential energy (有效位能):

\[
P = \frac{1}{2} \int_0^{p_s} \frac{T_s}{\gamma_d - \gamma_s} \left( \frac{T - T_s}{T_s} \right)^2 dp = \frac{c_p}{2g} \int_0^{p_s} \Gamma (T - T_s)^2 dp
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Tendency equations:
Energy cycles in the baroclinic eddy-mean flow interactions

- **Basic forms of energy:**
  - Kinetic energy (动能): \( K = \frac{1}{2}(u^2 + v^2 + w^2) \approx \frac{1}{2}(u^2 + v^2) \)
  - Available potential energy (有效位能):
    \[
P = \frac{1}{2} \int_0^{P_s} \frac{T_s}{\gamma_d - \gamma_s} \left( \frac{T - T_s}{T_s} \right)^2 \, dp = \frac{c_p}{2g} \int_0^{P_s} \Gamma (T - T_s)^2 \, dp
    \]

- **Tendency equations:**
  \[
  \frac{\partial}{\partial t} \int K \, dm = -R \int \frac{\omega T}{p} \, dm + \int (uF_x + vF_y) \, dm
  \]
  \[
  \frac{\partial}{\partial t} \int P \, dm = R \int \frac{\omega T}{p} \, dm + \int \Gamma (T - T_s)(Q - Q_s) \, dm
  \]
  \[\text{Q - diabatic heating}\]
Energy cycles
in the baroclinic eddy-mean flow interactions

- **Kinetic energy (动能):**

- **Available potential energy (有效位能):**

- **Tendency equations:**

\[
\frac{\partial}{\partial t} \int K dm = -R \int \frac{\omega T}{p} dm + \int (uF_x + vF_y) dm
\]

\[
\frac{\partial}{\partial t} \int P dm = R \int \frac{\omega T}{p} dm + \int \Gamma(T - T_s)(Q - Q_s) dm
\]

Q - diabatic heating
Energy cycles in the baroclinic eddy-mean flow interactions

- Zonal mean and eddy components:
  - Kinetic energy (动能):
  - Available potential energy (有效位能):

- Tendency equations:

\[
\frac{\partial}{\partial t} \int K \, dm = -R \int \frac{\omega T}{p} \, dm + \int (uF_x + vF_y) \, dm
\]

\[
\frac{\partial}{\partial t} \int P \, dm = R \int \frac{\omega T}{p} \, dm + \int \Gamma(T - T_s)(Q - Q_s) \, dm
\]

Q - diabatic heating
Energy cycles in the baroclinic eddy-mean flow interactions

- Zonal mean and eddy components:
  - Kinetic energy (动能): \( K_M = \frac{1}{2} \left( [u]^2 + [v]^2 \right) \quad K_E = \frac{1}{2} \left( [u^*]^2 + [v^*]^2 \right) \)
  - Available potential energy (有效位能):

- Tendency equations:
  \[
  \frac{\partial}{\partial t} \int K \, dm = -R \int \frac{\omega T}{p} \, dm + \int (u F_x + v F_y) \, dm
  \]
  \[
  \frac{\partial}{\partial t} \int P \, dm = R \int \frac{\omega T}{p} \, dm + \int \Gamma (T - T_s) (Q - Q_s) \, dm
  \]
  \( Q \) - diabatic heating
Energy cycles
in the baroclinic eddy-mean flow interactions

- **Zonal mean and eddy components:**

  - **Kinetic energy (动能):** \( K_M = \frac{1}{2} ([u]^2 + [v]^2) \) \( K_E = \frac{1}{2} ([u^*]^2 + [v^*]^2) \)

  - **Available potential energy (有效位能):**
    \[
    P_M = \frac{c_p}{2} \Gamma ([T] - T_s)^2 \\
    P_E = \frac{c_p}{2} \Gamma [T^*]^2
    \]

- **Tendency equations:**

  \[
  \frac{\partial}{\partial t} \int K dm = -R \int \frac{\omega T}{p} dm + \int (u F_x + v F_y) dm \\
  \frac{\partial}{\partial t} \int P dm = R \int \frac{\omega T}{p} dm + \int \Gamma (T - T_s) (Q - Q_s) dm
  \]

  \( Q \) - diabatic heating
Energy cycles in the baroclinic eddy-mean flow interactions

\[
\frac{\partial}{\partial t} \int K \, dm = -R \int \frac{\omega T}{p} \, dm + \int (u F_x + v F_y) \, dm
\]

\[
\frac{\partial}{\partial t} \int P \, dm = R \int \frac{\omega T}{p} \, dm + \int \Gamma(T - T_s)(Q - Q_s) \, dm
\]
Energy cycles
in the baroclinic eddy-mean flow interactions

- Equations under the **Quasi-geostrophic** assumption:

\[
\frac{\partial}{\partial t} \int Kdm = -R \int \frac{\omega T}{p} dm + \int (uF_x + vF_y) dm
\]

\[
\frac{\partial}{\partial t} \int Pdm = R \int \frac{\omega T}{p} dm + \int \Gamma(T - T_s)(Q - Q_s) dm
\]
Energy cycles
in the baroclinic eddy-mean flow interactions

Equations under the **Quasi-geostrophic** assumption:

\[
\frac{\partial}{\partial t} \int K \, dm = -R \int \frac{\omega T}{p} \, dm + \int (uF_x + vF_y) \, dm
\]

\[
\frac{\partial}{\partial t} \int K_M \, dm = -R \int \frac{[\omega][T]}{p} \, dm + \int [u^*v^*] \frac{\partial [u]}{\partial y} \, dm + \int ([u][F_x] + [v][F_y]) \, dm
\]

\[
\frac{\partial}{\partial t} \int K_E \, dm = -R \int \frac{[\omega^*T^*]}{p} \, dm - \int [u^*v^*] \frac{\partial [u]}{\partial y} \, dm + \int ([u^*F_x^* + v^*F_y^*]) \, dm
\]

\[
\frac{\partial}{\partial t} \int P \, dm = R \int \frac{\omega T}{p} \, dm + \int \Gamma (T - T_s) (Q - Q_s) \, dm
\]
Energy cycles in the baroclinic eddy-mean flow interactions

Equations under the Quasi-geostrophic assumption:

\[
\frac{\partial}{\partial t} \int K dm = - R \int \frac{\omega T}{p} dm + \int (uF_x + vF_y) dm
\]

\[
\frac{\partial}{\partial t} \int K_M dm = - R \int \frac{[\omega][T]}{p} dm + \int [u^*v^*] \frac{\partial[u]}{\partial y} dm + \int ([u][F_x] + [v][F_y]) dm
\]

\[
\frac{\partial}{\partial t} \int K_E dm = - R \int \frac{[\omega^*T^*]}{p} dm - \int [u^*v^*] \frac{\partial[u]}{\partial y} dm + \int ([u^*F_x^* + v^*F_y^*]) dm
\]

\[
\frac{\partial}{\partial t} \int P dm = R \int \frac{\omega T}{p} dm + \int \Gamma(T - T_s)(Q - Q_s) dm
\]

\[
\frac{\partial}{\partial t} \int P_M dm = R \int \frac{[\omega][T]}{p} dm + c_p \int \Gamma[v^*T^*] \frac{\partial[T]}{\partial y} dm + \int \Gamma([T] - T_s)([Q] - Q_s) dm
\]

\[
\frac{\partial}{\partial t} \int P_E dm = R \int \frac{[\omega^*T^*]}{p} dm - c_p \int \Gamma[v^*T^*] \frac{\partial[T]}{\partial y} dm + \int \Gamma[T^*Q^*] dm
\]
Energy cycles in the baroclinic eddy-mean flow interactions

Equations under the **Quasi-geostrophic** assumption:

\[
\frac{\partial}{\partial t} \int K dm = -R \int \frac{\omega T}{p} \, dm + \int (uF_x + vF_y) \, dm
\]

\[
\frac{\partial}{\partial t} \int K_M dm = -R \int \frac{[\omega][T]}{p} \, dm + \int [u^*v^*] \frac{\partial [u]}{\partial y} \, dm + \int ([u][F_x] + [v][F_y]) \, dm
\]

\[
\frac{\partial}{\partial t} \int K_E dm = -R \int \frac{[\omega^*T^*]}{p} \, dm - \int [u^*v^*] \frac{\partial [u]}{\partial y} \, dm + \int ([u^*F_x^* + v^*F_y^*]) \, dm
\]

\[
\frac{\partial}{\partial t} \int P dm = R \int \frac{\omega T}{p} \, dm + \int \Gamma (T - T_s)(Q - Q_s) \, dm
\]

\[
\frac{\partial}{\partial t} \int P_M dm = R \int \frac{[\omega][T]}{p} \, dm + c_p \int \Gamma [v^*T^*] \frac{\partial [T]}{\partial y} \, dm + \int \Gamma ([T] - T_s)([Q] - Q_s) \, dm
\]

\[
\frac{\partial}{\partial t} \int P_E dm = R \int \frac{[\omega^*T^*]}{p} \, dm - c_p \int \Gamma [v^*T^*] \frac{\partial [T]}{\partial y} \, dm + \int \Gamma [T^*Q^*] \, dm
\]
Energy cycles in the baroclinic eddy-mean flow interactions

- Equations under the Quasi-geostrophic assumption:

\[
\frac{\partial}{\partial t} \int Kdm = -R \int \frac{\omega T}{p} dm + \int (uF_x + vF_y)dm
\]

\[
\frac{\partial}{\partial t} \int K_M dm = -R \int \left[ \frac{[\omega][T]}{p} \right] dm + \int [u^*v^*] \frac{\partial[u]}{\partial y} dm + \int ([u][F_x] + [v][F_y])dm
\]

\[
\frac{\partial}{\partial t} \int K_E dm = -R \int \left[ \frac{[\omega^*T^*]}{p} \right] dm - \int [u^*v^*] \frac{\partial[u]}{\partial y} dm + \int ([u^*F_x^* + v^*F_y^*])dm
\]

\[
\frac{\partial}{\partial t} \int P dm = R \int \frac{\omega T}{p} dm + \int \Gamma(T - T_s)(Q - Q_s)dm
\]

\[
\frac{\partial}{\partial t} \int P_M dm = R \int \left[ \frac{[\omega][T]}{p} \right] dm + c_p \int \Gamma[v^*T^*] \frac{\partial[T]}{\partial y} dm + \int \Gamma([T] - T_s)([Q] - Q_s)dm
\]

\[
\frac{\partial}{\partial t} \int P_E dm = R \int \left[ \frac{[\omega^*T^*]}{p} \right] dm - c_p \int \Gamma[v^*T^*] \frac{\partial[T]}{\partial y} dm + \int \Gamma[T^*Q^*]dm
\]
Energy cycles in the baroclinic eddy-mean flow interactions

Equations under the Quasi-geostrophic assumption:

\[
\frac{\partial}{\partial t} \int K \, dm = -R \int \frac{\omega T}{p} \, dm + \int (uF_x + vF_y) \, dm
\]

\[
\frac{\partial}{\partial t} \int K_M \, dm = -R \int \frac{[\omega][T]}{p} \, dm + \int [u^*v^*] \frac{\partial [u]}{\partial y} \, dm + \int ([u][F_x] + [v][F_y]) \, dm
\]

\[
\frac{\partial}{\partial t} \int K_E \, dm = -R \int \frac{[\omega^*T^*]}{p} \, dm + \int [u^*v^*] \frac{\partial [u]}{\partial y} \, dm + \int ([u^*F_x^* + v^*F_y^*]) \, dm
\]

\[
\frac{\partial}{\partial t} \int P \, dm = R \int \frac{\omega T}{p} \, dm + \int \Gamma(T - T_s)(Q - Q_s) \, dm
\]

\[
\frac{\partial}{\partial t} \int P_M \, dm = -R \int \frac{[\omega][T]}{p} \, dm + c_p \int \Gamma(v^*T^*) \frac{\partial [T]}{\partial y} \, dm + \int \Gamma([T] - T_s)([Q] - Q_s) \, dm
\]

\[
\frac{\partial}{\partial t} \int P_E \, dm = R \int \frac{[\omega^*T^*]}{p} \, dm - c_p \int \Gamma(v^*T^*) \frac{\partial [T]}{\partial y} \, dm + \int \Gamma[T^*Q^*] \, dm
\]
Energy cycles in the baroclinic eddy-mean flow interactions

Equations under the Quasi-geostrophic assumption:

\[ \frac{\partial}{\partial t} \int K dm = -R \int \frac{\omega T}{p} dm + \int (u F_x + v F_y) dm \]

\[ \frac{\partial}{\partial t} \int K_M dm = -R \left< \int \frac{[\omega][T]}{p} \right> + \int [u^* v^*] \frac{\partial[u]}{\partial y} dm + \int ([u][F_x] + [v][F_y]) dm \]

\[ \frac{\partial}{\partial t} \int K_E dm = -R \left< \int \frac{[\omega^* T^*]}{p} \right> - \int [u^* v^*] \frac{\partial[u]}{\partial y} dm + \int ([u^* F_x^* + v^* F_y^*]) dm \]

\[ \frac{\partial}{\partial t} \int P dm = R \int \frac{\omega T}{p} dm + \int \Gamma(T - T_s)(Q - Q_s) dm \]

\[ \frac{\partial}{\partial t} \int P_M dm = -R \left< \int \frac{[\omega][T]}{p} \right> + \left< \int P_E^* \frac{\partial[T]}{\partial y} \right> dm + \int \Gamma([T] - T_s)([Q] - Q_s) dm \]

\[ \frac{\partial}{\partial t} \int P_E dm = R \left< \int \frac{[\omega^* T^*]}{p} \right> - \left< \int P_E^* \frac{\partial[T]}{\partial y} \right> dm + \int \Gamma[T^* Q^*] dm \]
Energy cycles
in the baroclinic eddy-mean flow interactions

- Equations under the Quasi-geostrophic assumption:

\[
\frac{\partial}{\partial t} \int K \, dm = -R \int \frac{\omega T}{p} \, dm + \int (uF_x + vF_y) \, dm
\]

\[
\frac{\partial}{\partial t} \int K_M \, dm = -\rho \int \frac{\omega [T]}{p} \, dm + \int [u^*v^*] \frac{\partial [u]}{\partial y} \, dm + \int ([u][F_x] + [v][F_y]) \, dm
\]

\[
\frac{\partial}{\partial t} \int K_E \, dm = -\rho \int \frac{\omega^*[T^*]}{p} \, dm - \int [u^*v^*] \frac{\partial [u]}{\partial y} \, dm + \int ([u^*F_x^* + v^*F_y^*]) \, dm
\]

\[
\frac{\partial}{\partial t} \int P \, dm = R \int \frac{\omega T}{p} \, dm + \int \Gamma(T - T_s)(Q - Q_s) \, dm
\]

\[
\frac{\partial}{\partial t} \int P_M \, dm = \rho \int \frac{\omega [T]}{p} \, dm + c_p \int [P_E^*P_M] \frac{\partial [T]}{\partial y} \, dm + \int \Gamma(\Gamma(G(P_M))(Q - Q_s)) \, dm
\]

\[
\frac{\partial}{\partial t} \int P_E \, dm = \rho \int \frac{\omega^*[T^*]}{p} \, dm - c_p \int [P_E^*P_M] \frac{\partial [T]}{\partial y} \, dm + \int \Gamma(G(P_E)) \, dm
\]
Energy cycles in the baroclinic eddy-mean flow interactions

- Equations under the **Quasi-geostrophic** assumption:
  \[
  \frac{\partial}{\partial t} \int K dm = -R \int \frac{\omega T}{p} dm + \int (uF_x + vF_y) dm
  \]

  \[
  \frac{\partial}{\partial t} \int K_M dm = -P \int \frac{[\omega][T]}{p} dm + \int [K_E, K_M] \frac{\partial [n]}{\partial y} dm + \int ([u][F_x] + [v][F_y]) dm
  \]

  \[
  \frac{\partial}{\partial t} \int K_E dm = -P \int \frac{[\omega^*][T^*]}{p} dm + \int [K_E, K_M] \frac{\partial [n]}{\partial y} dm + \int ([u^* F_x^* + v^* F_y^*]) dm
  \]

  \[
  \frac{\partial}{\partial t} \int P dm = R \int \frac{\omega T}{p} dm + \int \Gamma(T - T_s)(Q - Q_s) dm
  \]

  \[
  \frac{\partial}{\partial t} \int P_M dm = -P \int \frac{[\omega][T]}{p} dm + c_p \int [P_E, P_M] \frac{\partial [T]}{\partial y} dm + \int \Gamma((G(P_M))[Q] - Q_s) dm
  \]

  \[
  \frac{\partial}{\partial t} \int P_E dm = -P \int \frac{[\omega^*][T^*]}{p} dm - c_p \int [P_E, P_M] \frac{\partial [T]}{\partial y} dm + \int \Gamma(G(P_E)) dm
  \]
Energy cycles in the baroclinic eddy-mean flow interactions

- Equations under the **Quasi-geostrophic** assumption:

\[
\frac{\partial}{\partial t} \int K dm = -R \int \frac{\omega T}{p} dm + \int (uF_x + vF_y) dm
\]

\[
\frac{\partial}{\partial t} \int K_M dm = -R \langle P_M, K_M \rangle + \frac{\partial}{\partial y} \langle K_E, K_M \rangle + \int (u) [F_x D(K_M)] dm
\]

\[
\frac{\partial}{\partial t} \int K_E dm = -R \langle P_E, K_E \rangle - \frac{\partial}{\partial y} \langle K_E, K_M \rangle + \int (u^* F_x D(K_E)) dm
\]

\[
\frac{\partial}{\partial t} \int P dm = R \int \frac{\omega T}{p} dm + \int \Gamma(T - T_s)(Q - Q_s) dm
\]

\[
\frac{\partial}{\partial t} \int P_M dm = -R \langle P_M, K_M \rangle + c_p \frac{\partial}{\partial y} \langle P_E, P_M \rangle + \int \Gamma([G(P_M)](Q - Q_s) dm
\]

\[
\frac{\partial}{\partial t} \int P_E dm = R \langle P_E, K_E \rangle - c_p \frac{\partial}{\partial y} \langle P_E, P_M \rangle + \int IG(P_E) dm
\]
Energy cycles in the baroclinic eddy-mean flow interactions
Energy cycles in the baroclinic eddy-mean flow interactions

\[ G(P_M) \]

\[ \frac{\partial P_M}{\partial t} \]

\[ \frac{\partial K_M}{\partial t} \]

\[ D(K_M) \]

\[ G(P_E) \]

\[ \frac{\partial P_E}{\partial t} \]

\[ \frac{\partial K_E}{\partial t} \]

\[ D(K_E) \]

\[ < P_M, K_M > \]

\[ < K_E, K_M > \]

\[ < P_E, P_M > \]

\[ < P_E, K_E > \]

\[ R[\omega][T] \]

\[ c_p \Gamma[v^*T^*] \frac{\partial}[T] \]

\[ \frac{\partial}[u] \]

\[ \frac{\partial}[u^*v^*] \]

\[ R[\omega^*T^*] \]
Energy cycles in the baroclinic eddy-mean flow interactions

\[ G(P_M) \]

\[ \frac{\partial P_M}{\partial t} \]

\[ \langle P_M, K_M \rangle \]

\[ \frac{\partial K_M}{\partial t} \]

\[ D(K_M) \]

\[ \frac{R[\omega][T]}{p} \]

\[ G(P_E) \]

\[ \frac{\partial P_E}{\partial t} \]

\[ \langle P_E, K_E \rangle \]

\[ \frac{\partial K_E}{\partial t} \]

\[ D(K_E) \]

\[ c_p \Gamma[v^*T^*] \frac{\partial [T]}{\partial y} \]

\[ \frac{\partial [u]}{\partial y} [u^*v^*] \]
Energy cycles in the baroclinic eddy-mean flow interactions

- Energy cycles in eddy life cycle:

\[ G(P_M) \]

\[ \frac{\partial P_M}{\partial t} \]

\[ \frac{R[\omega][T]}{p} \]

\[ \langle P_M, K_M \rangle \]

\[ \frac{\partial K_M}{\partial t} \]

\[ D(K_M) \]

\[ G(P_E) \]

\[ \frac{\partial P_E}{\partial t} \]

\[ \langle P_E, P_M \rangle \]

\[ c_p \Gamma[v^*T^*] \frac{\partial [T]}{\partial y} \]

\[ \langle K_E, K_M \rangle \]

\[ \frac{\partial [u]}{\partial y} [u^*v^*] \]

\[ G(P_E) \]

\[ \frac{\partial P_E}{\partial t} \]

\[ \langle P_E, K_E \rangle \]

\[ \frac{R[\omega^*][T^*]}{p} \]

\[ \langle K_E, P_M \rangle \]

\[ \frac{\partial K_E}{\partial t} \]

\[ D(K_E) \]
Energy cycles in the baroclinic eddy-mean flow interactions

- Energy cycles in eddy life cycle:

\[ G(P_M) \]
\[ \frac{\partial P_M}{\partial t} < P_M, K_M > \]
\[ \frac{\partial K_M}{\partial t} D(K_M) \]

\[ \frac{R[\omega][T]}{p} \]

\[ G(P_E) \]
\[ \frac{\partial P_E}{\partial t} < P_E, K_M > \]
\[ \frac{\partial K_E}{\partial t} D(K_E) \]

\[ c_p \Gamma[v^*T^*] \frac{\partial [T]}{\partial y} \]

\[ \frac{\partial[u]}{\partial y}[u^*v^*] \]
Energy cycles in the baroclinic eddy-mean flow interactions

- Energy cycles in eddy life cycle:

\[ G(P_M) \]
\[ \frac{\partial P_M}{\partial t} \]
\[ < P_M, K_M > \]
\[ \frac{\partial K_M}{\partial t} \]
\[ D(K_M) \]

\[ G(P_E) \]
\[ \frac{\partial P_E}{\partial t} \]
\[ < P_E, K_E > \]
\[ \frac{\partial K_E}{\partial t} \]
\[ D(K_E) \]

\[ R[\omega][T] \]
\[ p \]

\[ \rho c_0\Gamma[v^*T^*] \frac{\partial [T]}{\partial y} \]

\[ \frac{\partial [u]}{\partial y}[u^*v^*] \]
Energy cycles in the baroclinic eddy-mean flow interactions

- Energy cycles in eddy life cycle: $\text{PE} \rightarrow \text{KE}$

\[
G(P_M) \quad \frac{\partial P_M}{\partial t} < P_M, K_M > \quad \frac{\partial K_M}{\partial t} \quad D(K_M)
\]

\[
\frac{R[\omega][T]}{p} \quad \frac{\partial P}{\partial t} < P_E, P_M > \quad \frac{\partial P_E}{\partial t} < P_E, K_E > \quad \frac{\partial K_E}{\partial t} \quad D(K_E)
\]

\[
e_p \Gamma [v^* T^*] \frac{\partial [T]}{\partial y} \quad \frac{\partial [u]}{\partial y} [u^* v^*]
\]
Energy cycles in the baroclinic eddy-mean flow interactions

- Energy cycles in eddy life cycle:

\[ G(P_M) \]
\[ \frac{\partial P_M}{\partial t} \]
\[ < P_M, K_M > \]
\[ \frac{\partial K_M}{\partial t} \]
\[ D(K_M) \]

\[ G(P_E) \]
\[ \frac{\partial P_E}{\partial t} \]
\[ < P_E, K_E > \]
\[ \frac{\partial K_E}{\partial t} \]
\[ D(K_E) \]

\[ \frac{R[\omega][T]}{p} \]
\[ c_p \Gamma [u^*T^*] \frac{\partial [T]}{\partial y} \]
\[ \frac{\partial [u]}{\partial y} [u^*v^*] \]

授课教师：张洋
Baroclinic eddy life cycle
- An E-P flux view

\[ \mathcal{F} \equiv -[u^* \mathcal{V}] \mathbf{j} + f \frac{[v^* \theta^*]}{\partial \theta_s / \partial p} \mathbf{k} \]

\[ \mathbf{\vec{F}} = \mathbf{c} \mathbf{g} \mathbf{A} \]

Eddies: generate at lower level, propagate **upwards** and **away** from the eddy source region

*Numerical results from Simmons and Hoskins, 1978, JAS*
Baroclinic eddies
- baroclinic eddy life cycle

- Westerly jet and energy cycle:

\[
\frac{\partial P_M}{\partial t}, \quad \frac{\partial K_M}{\partial t}, \quad \frac{\partial P_E}{\partial t}, \quad \frac{\partial K_E}{\partial t}
\]

\[< P_E, P_M >, \quad c_p \Gamma [v^* T^*] \frac{\partial [T]}{\partial y}, \quad < K_E, K_M >, \quad \frac{\partial [u]}{\partial y}[u^* v^*], \quad < P_E, K_E >, \quad \frac{R [\omega^* T^*]}{p} \]

\[< P_E, P_M >, \quad \frac{\partial P_E}{\partial t}, \quad \frac{\partial K_E}{\partial t}, \quad TOTAL \ E-P \ FLUX \ DIVERGENCE \ \text{DAY} \ 1.00, \quad TOTAL \ E-P \ FLUX \ DIVERGENCE \ \text{DAY} \ 5.00, \quad TOTAL \ E-P \ FLUX \ DIVERGENCE \ \text{DAY} \ 8.00 \]
Baroclinic eddies
- baroclinic eddy life cycle

- Westerly jet and energy cycle:

\[
\begin{align*}
\frac{\partial P_M}{\partial t} &\quad < P_E, P_M > \\
\frac{\partial P_E}{\partial t} &\quad c_p \Gamma \left[ v^* T^* \right] \frac{\partial [T]}{\partial y} \\
\frac{\partial K_M}{\partial t} &\quad < K_E, K_M > \\
\frac{\partial K_E}{\partial t} &\quad \frac{\partial [u]}{\partial y} [u^* v^*] \\
\end{align*}
\]
Baroclinic eddies
- baroclinic eddy life cycle

- Westerly jet and energy cycle:

\[
\begin{align*}
\frac{\partial P_M}{\partial t} & \quad \frac{\partial K_M}{\partial t} \\
\frac{\partial P_E}{\partial t} & \quad \frac{\partial K_E}{\partial t}
\end{align*}
\]

\[
\begin{align*}
< P_E, P_M > & \quad c_p \Gamma [v^* T^*] \frac{\partial T}{\partial y} \\
< K_E, K_M > & \quad \frac{\partial [u]}{\partial y} [u^* v^*] \\
< P_E, K_E > & \quad \frac{\partial P_M}{\partial t} \\
< K_E, K_M > & \quad \frac{\partial K_M}{\partial t}
\end{align*}
\]

Numerical results from
Simmons and Hoskins, 1978, JAS

Strengthen of westerly jet
Baroclinic eddies
- baroclinic eddy life cycle

- Westerly jet and energy cycle:

\[
\begin{align*}
\frac{\partial P_M}{\partial t} & \downarrow \\
\frac{\partial K_M}{\partial t} & \uparrow \\
\frac{\partial P_E}{\partial t} & \downarrow \\
\frac{\partial K_E}{\partial t} & \uparrow \\
\end{align*}
\]

\[
\begin{align*}
\langle P_E, P_M \rangle & \downarrow \\
\langle K_E, K_M \rangle & \uparrow \\
\langle P_E, K_E \rangle & \downarrow \\
\langle K_E, K_M \rangle & \uparrow \\
\end{align*}
\]

Eddy momentum flux grows, which extracts kinetic energy from the eddies to the zonal mean flow, then the growth of the eddy energy ceases.

Numerical results from Simmons and Hoskins, 1978, JAS

From Vallis (2006)
Energy cycles in the baroclinic eddy-mean flow interactions

\[
G(P_M) \quad \frac{\partial P_M}{\partial t} \quad \frac{R[\omega[T]}{p} \quad \frac{\partial K_M}{\partial t} \quad D(K_M)
\]

\[
G(P_E) \quad \frac{\partial P_E}{\partial t} \quad \frac{\partial K_E}{\partial t} \quad D(K_E)
\]

\[
< P_E, P_M > \quad c_p\Gamma[v^*T^*] \frac{\partial[T]}{\partial y} \quad < K_E, K_M > \quad \frac{\partial[u]}{\partial y}[u^*v^*]
\]
Energy cycles in the baroclinic eddy-mean flow interactions

- Energy cycles in equilibrium:

\[ G(P_M) \]

\[ \frac{\partial P_M}{\partial t} \]

\[ < P_M, K_M > \]

\[ \frac{\partial K_M}{\partial t} \]

\[ R \left[ \omega [T] \right] \]

\[ p \]

\[ D(K_M) \]

\[ c_p \Gamma[v^* T^*] \frac{\partial [T]}{\partial y} \]

\[ < P_E, P_M > \]

\[ G(P_E) \]

\[ \frac{\partial P_E}{\partial t} \]

\[ < P_E, K_E > \]

\[ \frac{\partial K_E}{\partial t} \]

\[ R \left[ \omega^* T^* \right] \]

\[ p \]

\[ < K_E, K_M > \]

\[ \frac{\partial [u]}{\partial y} [u^* v^*] \]

\[ D(K_E) \]
Energy cycles in the baroclinic eddy-mean flow interactions

**Energy cycles in equilibrium:**

\[ G(P_M) \rightarrow \frac{\partial P_M}{\partial t} \rightarrow \frac{<P_M,K_M>}{p} \rightarrow \frac{\partial K_M}{\partial t} \rightarrow D(K_M) \]

\[ G(P_E) \rightarrow \frac{\partial P_E}{\partial t} \rightarrow \frac{<P_E,K_E>}{p} \rightarrow \frac{\partial K_E}{\partial t} \rightarrow D(K_E) \]

- \( G(P_M) \)
- \( \partial P_M / \partial t \)
- \( <P_M,K_M> / p \)
- \( \partial K_M / \partial t \)
- \( D(K_M) \)
- \( G(P_E) \)
- \( \partial P_E / \partial t \)
- \( <P_E,K_E> / p \)
- \( \partial K_E / \partial t \)
- \( D(K_E) \)

- Energy cycles in equilibrium:

\[ R[\omega[T]] \]

\[ c_p \Gamma[v^* T^*] \frac{\partial[T]}{\partial y} \]

\[ \frac{\partial[u]}{\partial y} [u^* v^*] \]

\[ R[\omega^*[T^*]] \]

\[ R[\omega[T^*]] \]
Energy cycles in the baroclinic eddy-mean flow interactions

Energy cycles in equilibrium:

\[ G(P_M) \rightarrow \frac{\partial P_M}{\partial t} \rightarrow \frac{R[\omega][T]}{p} \rightarrow \frac{\partial K_M}{\partial t} \rightarrow D(K_M) \]

\[ G(P_E) \rightarrow \frac{\partial P_E}{\partial t} \rightarrow \frac{c_p \Gamma[v^*T^*]}{\partial y} \frac{\partial[T]}{\partial y} \rightarrow \frac{\partial K_E}{\partial t} \rightarrow D(K_E) \]
Energy cycles in equilibrium:

\[ G(P_M) \rightarrow \frac{\partial P_M}{\partial t} < P_M, K_M > \frac{\partial K_M}{\partial t} D(K_M) \]

\[ G(P_E) \rightarrow \frac{\partial P_E}{\partial t} < P_E, K_E > \frac{\partial K_E}{\partial t} D(K_E) \]

Energy cycles in the baroclinic eddy-mean flow interactions
Energy cycles in the baroclinic eddy-mean flow interactions

- Energy cycles in equilibrium:

\[
G(P_M) \rightarrow \frac{\partial P_M}{\partial t} \rightarrow \frac{\partial K_M}{\partial t} \rightarrow D(K_M)
\]

\[
G(P_E) \leftarrow \frac{\partial P_E}{\partial t} \leftarrow \frac{\partial K_E}{\partial t} \leftarrow D(K_E)
\]

\[
G(P_M) = \frac{R[\omega][T]}{p} \quad <P_M, K_M>
\]

\[
G(P_E) = \frac{\partial P_E}{\partial t} \leftarrow <P_E, K_E>
\]

\[
D(K_M) = \frac{\partial K_M}{\partial t} \rightarrow <K_E, K_M>
\]

\[
D(K_E) = \frac{\partial K_E}{\partial t} \rightarrow <K_E, K_M>
\]

\[
c_p \Gamma[v^*T^*] \frac{\partial[T]}{\partial y} \quad <P_E, P_M>
\]

\[
\frac{\partial[u]}{\partial y}[u^*v^*] \quad <K_E, K_M>
\]
Energy cycles in the baroclinic eddy-mean flow interactions

Energy cycles in equilibrium:

\[ G(P_M) \rightarrow \frac{\partial P_M}{\partial t} \rightarrow \frac{\partial K_M}{\partial t} \rightarrow D(K_M) \]

\[ \frac{\partial P_E}{\partial t} \leftarrow \frac{\partial K_E}{\partial t} \rightarrow D(K_E) \]

\[ <P_{E}, P_{M}> \]

\[ e_{p} \Gamma [v^{*}T^{*}] \frac{\partial [T]}{\partial y} \]

\[ <K_{E}, K_{M}> \]

\[ \frac{\partial [u]}{\partial y} [u^{*}v^{*}] \]

\[ R[\omega[T]]_{P} \]

\[ <K_{E}, P_{E}> \]

\[ R[\omega[T^{*}]]_{P} \]
Energy cycles in the baroclinic eddy-mean flow interactions

- Energy cycles in equilibrium:

\[
\begin{align*}
G(P_M) \rightarrow & \quad \frac{\partial P_M}{\partial t} \left< P_M, K_M \right> \leftarrow G(P_E) \\
& \quad R[\omega][T]_p \\
& \quad c_p \Gamma[v^*T^*] \frac{\partial[T]}{\partial y} \\
& \quad <P_E, P_M> \\
& \quad e_p \Gamma[v^*T^*] \frac{\partial[T]}{\partial y} [u^*v^*] \\
& \quad <P_E, K_M> \\
\end{align*}
\]

\[
\begin{align*}
& \rightarrow \quad \frac{\partial K_M}{\partial t} \rightarrow \quad D(K_M) \\
& \left< K_E, K_M \right> \\
& \frac{\partial[u]}{\partial y} [u^*v^*] \\
\end{align*}
\]

\[
\begin{align*}
G(P_E) \rightarrow & \quad \frac{\partial P_E}{\partial t} \rightarrow D(K_E) \\
& \left< P_E, K_E \right> \\
& \left< K_E, P_E \right> \\
\end{align*}
\]
Energy cycles in the baroclinic eddy-mean flow interactions

- Energy cycles in equilibrium:
  \[ G(P_M) \rightarrow \frac{\partial P_M}{\partial t} \rightarrow <P_M, K_M> \rightarrow \frac{\partial K_M}{\partial t} \rightarrow D(K_M) \]
  \[ G(P_E) \leftarrow \frac{\partial P_E}{\partial t} \leftarrow <P_E, K_E> \rightarrow \frac{\partial K_E}{\partial t} \rightarrow D(K_E) \]

\[ R[\omega[T] \rightarrow_p \frac{\partial [T]}{\partial y} \rightarrow <K_E, K_M> \rightarrow \frac{\partial[u]{y}}{\partial y} [u^* v^*] \]

\[ c_p \Gamma[v* T^*] \frac{\partial[T]}{\partial y} \leftarrow <P_E, P_M> \]

授课教师：张洋
Energy cycles in the baroclinic eddy-mean flow interactions

Energy cycles in equilibrium:

\[ G(P_M) \rightarrow \frac{\partial P_M}{\partial t} \rightarrow R^p[\omega[T] \frac{\partial[T]}{\partial y} \frac{\partial P_M}{\partial t} \rightarrow \frac{\partial K_M}{\partial t} \rightarrow D(K_M) \]

\[ G(P_E) \leftarrow \frac{\partial P_E}{\partial t} \leftarrow c_p \Gamma [v^* T^*] \frac{\partial[T]}{\partial y} \frac{\partial P_E}{\partial t} \rightarrow \frac{\partial K_E}{\partial t} \rightarrow D(K_E) \]

Lorenz energy cycle

\[ \langle P_E, P_M \rangle \]

\[ \langle P_E, K_E \rangle \]

\[ \langle K_E, K_M \rangle \]

\[ \langle K_E, K_M \rangle \]

\[ \langle K_E, K_M \rangle \]

\[ \langle K_E, K_M \rangle \]
Energy cycles in Hadley Cell
Energy cycles in Hadley Cell

\[ G(P_M) \rightarrow \frac{\partial P_M}{\partial t} \rightarrow <P_M, K_M> \rightarrow \frac{\partial K_M}{\partial t} \rightarrow D(K_M) \]

If assume no eddies.
Energy cycles in the baroclinic eddy-mean flow interactions

\[
G(P_M) \xrightarrow{\frac{\partial P_M}{\partial t}} \langle P_M, K_M \rangle \xleftarrow{\langle K_E, K_M \rangle} \frac{\partial K_M}{\partial t} \xrightarrow{D(K_M)} \\
G(P_E) \xrightarrow{\frac{\partial P_E}{\partial t}} \langle P_E, P_M \rangle \xleftarrow{\langle P_E, K_E \rangle} \frac{\partial K_E}{\partial t} \xrightarrow{D(K_E)}
\]
Energy cycles in the baroclinic eddy-mean flow interactions

- Energy cycles in real atmosphere:

\[
G(P_M) \quad \frac{\partial P_M}{\partial t} \quad \langle P_M, K_M \rangle \quad \frac{\partial K_M}{\partial t} \quad D(K_M)
\]

\[
G(P_E) \quad \frac{\partial P_E}{\partial t} \quad \langle P_E, K_E \rangle \quad \frac{\partial K_E}{\partial t} \quad D(K_E)
\]
Energy cycles in the baroclinic eddy-mean flow interactions

- Energy cycles in real atmosphere:

\[
G(P_M) \xrightarrow{\frac{\partial P_M}{\partial t}} \langle P_M, K_M \rangle \xrightarrow{\frac{\partial K_M}{\partial t}} D(K_M)
\]

\[
G(P_E) \xrightarrow{\frac{\partial P_E}{\partial t}} \langle P_E, K_E \rangle \xrightarrow{\frac{\partial K_E}{\partial t}} D(K_E)
\]
Energy cycles in the baroclinic eddy-mean flow interactions

- Energy cycles in real atmosphere:

\[
\begin{align*}
G(P_M) & \rightarrow \frac{\partial P_M}{\partial t} & \langle P_M, K_M \rangle & \leftarrow \quad \frac{\partial K_M}{\partial t} & \rightarrow D(K_M) \\
G(P_E) & \rightarrow \frac{\partial P_E}{\partial t} & \langle P_E, P_M \rangle & \downarrow \text{GLOBAL ANNUAL} & \langle K_E, K_M \rangle & \uparrow \\
& & & \langle P_E, K_E \rangle & \rightarrow \frac{\partial K_E}{\partial t} & \rightarrow D(K_E)
\end{align*}
\]
Energy cycles in the baroclinic eddy-mean flow interactions

- Energy cycles in real atmosphere:

\[ G(P_M) \rightarrow \frac{\partial P_M}{\partial t} \left\langle P_M, K_M \right\rangle \rightarrow \frac{\partial K_M}{\partial t} \rightarrow D(K_M) \]

\[ G(P_E) \rightarrow \frac{\partial P_E}{\partial t} \left\langle P_E, P_M \right\rangle \rightarrow \frac{\partial K_E}{\partial t} \left\langle P_E, K_E \right\rangle \rightarrow D(K_E) \]


Energy: \(10^5 \text{ Jm}^{-2}\)

Global annual energy: 33.3

1.1

4.5

7.3
Energy cycles in the baroclinic eddy-mean flow interactions

- Energy cycles in real atmosphere:

\[
G(P_M) \quad \frac{\partial P_M}{\partial t} \quad \frac{\partial K_M}{\partial t} \quad D(K_M)
\]

\[
G(P_E) \quad \frac{\partial P_E}{\partial t} \quad \frac{\partial K_E}{\partial t} \quad D(K_E)
\]

Energy: \(10^5 \text{Jm}^{-2}\)
Conversion: \(Wm^{-2}\)
Edward N. Lorenz, a Meteorologist and a Father of Chaos Theory, Dies at 90

By KENNETH CHANG
Published: April 17, 2008

Edward N. Lorenz, a meteorologist who tried to predict the weather with computers but instead gave rise to the modern field of chaos theory, died Wednesday at his home in Cambridge, Mass. He was 90.

The cause was cancer, said his daughter Cheryl Lorenz.

In discovering “deterministic chaos,” Dr. Lorenz established a principle that “profoundly influenced a wide range of basic sciences and brought about one of the most dramatic changes in mankind’s view of nature since Sir Isaac Newton,” said a committee that awarded him the 1991 Kyoto Prize for basic sciences.

Dr. Lorenz is best known for the notion of the “butterfly effect,” the idea that a small disturbance like the flapping of a butterfly’s wings can induce enormous consequences.

As recounted in the book “Chaos” by James Gleick, Dr. Lorenz’s accidental discovery of chaos came in the winter of 1961. Dr. Lorenz was running simulations of weather using a simple computer model. One day, he wanted to repeat one of the simulations for a longer time, but instead of repeating the whole simulation, he started the second run in the middle, typing in numbers from the first run for the initial conditions.
Summary & Discussion

- Observations
- The Ferrel Cell
- Baroclinic eddies
  - Review: baroclinic instability and baroclinic eddy life cycle
  - Eddy-mean flow interaction, E-P flux
  - Transformed Eulerian Mean equations
- Eddy-driven jet
- The energy cycle
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1. The role of moisture;
Summary & Discussion

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1. The role of moisture;
2. Quantify (parameterize) the relation between eddies and mean flow;
3. Zonal variations.
Assignment 4, Fall 2022

在第四章中，我们从准地转近似下的纬向平均风场、温度场的趋势方程出发，定义了E-P通量。但是该定义下的E-P通量并没有考虑到大气湿过程的影响。如果从第三章介绍的水汽方程出发，我们可以按照以下步骤定义出一个包含大气大尺度运动中湿过程作用的广义的E-P通量。

1）在准地转近似下，如果我们按照对热力学方程的简化方法，将比湿（specific humidity）$q'$ 分解成一个标准比湿 $q_s$（reference specific humidity）和变化量$q''$，并且同样假设$\partial q/\partial p$的水平变化很小，请证明在准地转近似下$p$坐标系下的纬向平均比湿$q$的变化方程为:

$$
\frac{\partial [q']}{\partial t} + \frac{\partial q_s}{\partial p} [\omega] = -[C - S] - \frac{\partial}{\partial y} [v' q''],
$$

其中$C - S$为水汽方程在准地转近似下的源汇项，表征由大尺度运动所带来的凝结速率。

2）如果重新定义一个非绝热加热项$Q_m$，使得$Q_m = Q - L |C - S| (\frac{p}{\rho_o}) R/c_p$, 请推导出一个关于$\theta + \frac{L}{c_p} q'$的变化方程。

3）根据以上推导出的新方程和准地转近似下$[u]$的变化方程，请重新定义一个广义的E-P通量$\mathcal{F}_m$，使得新的E-P通量中包含了eddy对水汽输送的作用，并且证明，在湿绝热$(Q_m = 0)$和无摩擦的情况下，平衡状态下的$\mathcal{F}_m$满足$\nabla \cdot \mathcal{F}_m = 0$，并请根据水汽输送的空间分布讨论：在实际大气中，新定义的E-P通量的$\nabla \cdot \mathcal{F}_m$应该有怎样的变化？eddy对水汽的输送作用将对维持Farrell 环流起到怎样的作用？

4）请根据新定义出的E-P通量，定义出新的剩余环流(residual circulation, $[\bar{v}_m]$, $[\bar{u}_m]$), 并讨论此时剩余环流的含义是什么？相对于新的剩余环流，新的TEM方程(Transformed Eulerian Mean Equations)应该是什么？同时，也请写出，如果用剩余环流来表述，(1)问中推导出的水汽方程将如何改写，eddy强迫项应变为什么？