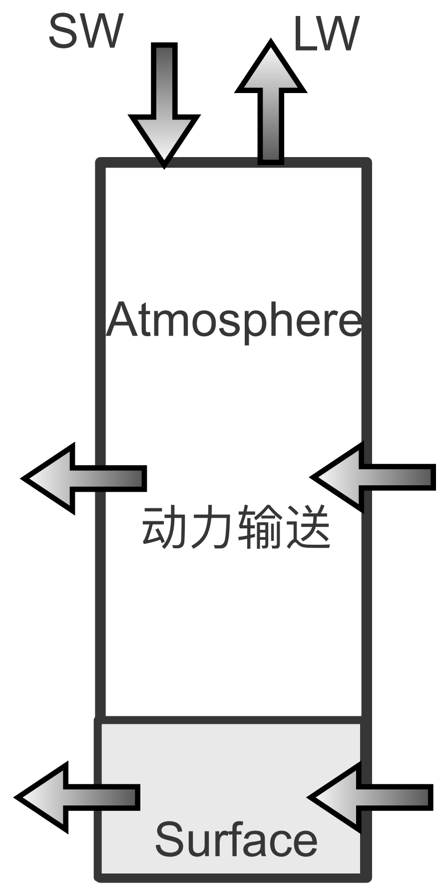
Assignments 2

Question #1

假设在大气层顶（TOA），在多年全年平均的情况下，入射的太阳辐射随纬度的分布满足 $Q = Q_o \cdot s(x)$, $s(x) = s_o \cdot P_o(x) + s_2 \cdot P_2(x)$，其中，$P_o(x) = 1$, $P_2(x) = \frac{1}{2}(3x^2 - 1)$, $s_o = 1$, $s_2 = -0.473$, $x = sin\phi$, ϕ 为纬度。

如果假设大气层顶的向外净长波辐射为I，行星反照率为α，且忽略它们随纬度和经度的变化：

1. 请写出在能量平衡的情况下，大气和海洋的总经向能量输送应满足什么条件？
2. 按（1）中的条件，大气和海洋的总经向能量输送的最大值应出现在什么纬度？
3. 请与实际情况的大气海洋的能量输送相比较，讨论（2）结果是否与实际状况相符合？



Assignments 2

Question #1

能量平衡满足：

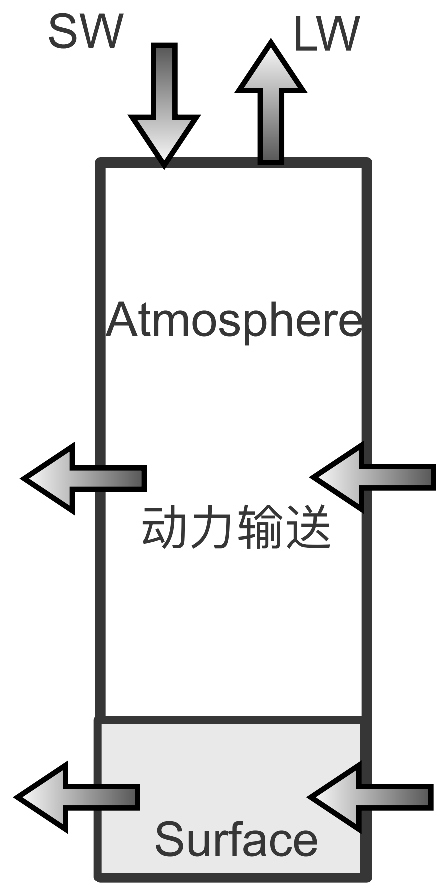
$$Q_o \cdot s(x) \cdot (1-\alpha) - I + F = 0$$

F - 经向能量输送的辐合辐散

or

$$Q_o s(x)(1-\alpha) - I = F_{rad} = \frac{1}{2\pi a^2 \cos\phi}\frac{\partial}{\partial\phi}f(\phi)$$

$$f(\phi) \quad - \quad \text{meridional energy transport by atmosphere and oceans}$$



经向能量输送最大时，应满足 $\frac{1}{2\pi a^2 \cos\phi}\frac{\partial}{\partial\phi}f(\phi) = 0$

Assignments 2

Question #1

经向能量输送最大时，应满足 $$\frac{1}{2\pi a^2\cos\phi}\frac{\partial}{\partial\phi}f(\phi)=0$$

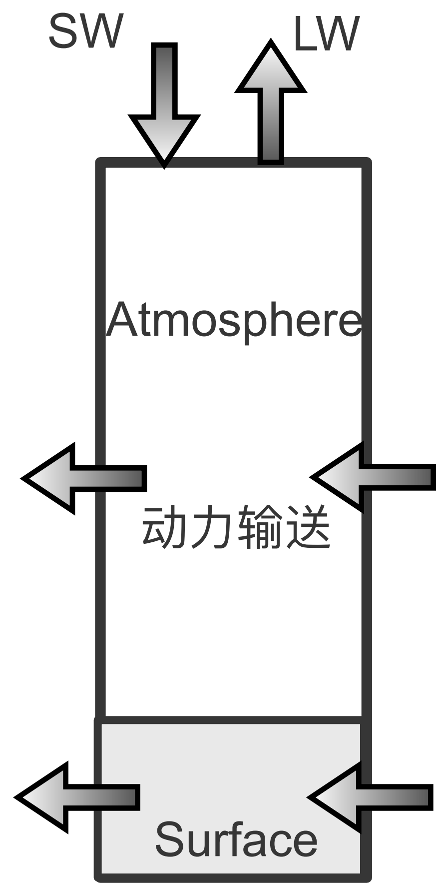
即在该纬度

$$F_{rad}=Q_o s(x)(1-\alpha)-I=0$$

全球积分的长波和短波能量应该相等：

$$2\pi a^2\int_{-1}^{1}[Q_o s(x)(1-\alpha)-I]dx=0$$

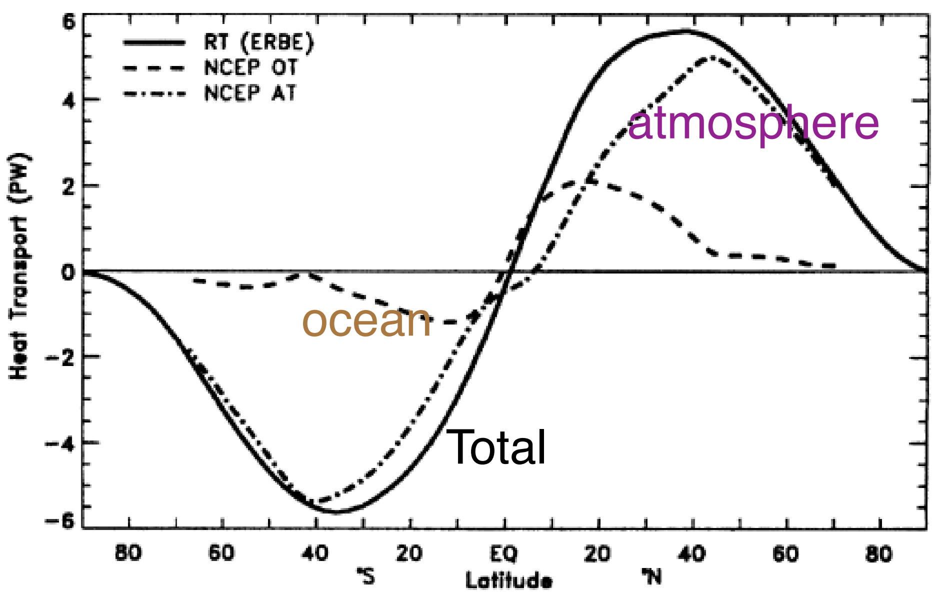
$\phi\approx 35^o$，以上两式同时满足，经向能量输送最大。



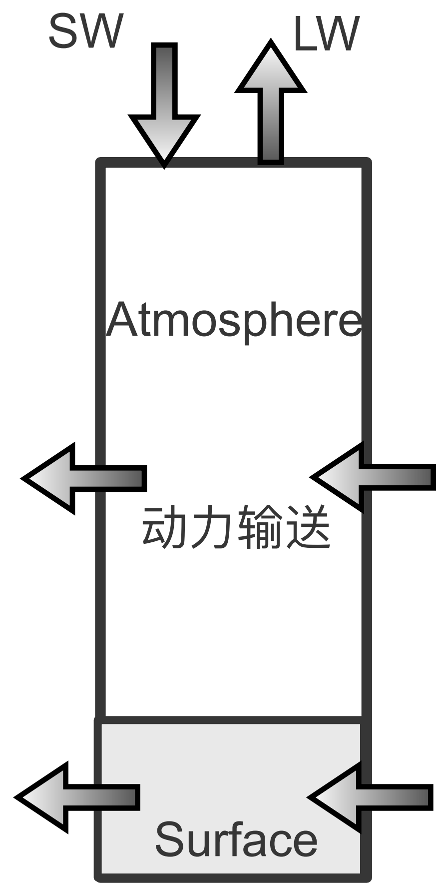
Assignments 2

Question #1

$\phi \approx 35^o$，以上两式同时满足，经向能量输送最大。



Wunsch (2005)，J. Climate



与实际情况大致相符

Assignments 2

Question #2:

$$Qs(x)\mathcal{A}(T) - I(T) + F(T) = 0$$

The snow line case: infrared cooling $I = A + BT$

$$F(T) = C(\bar{T} - T)$$

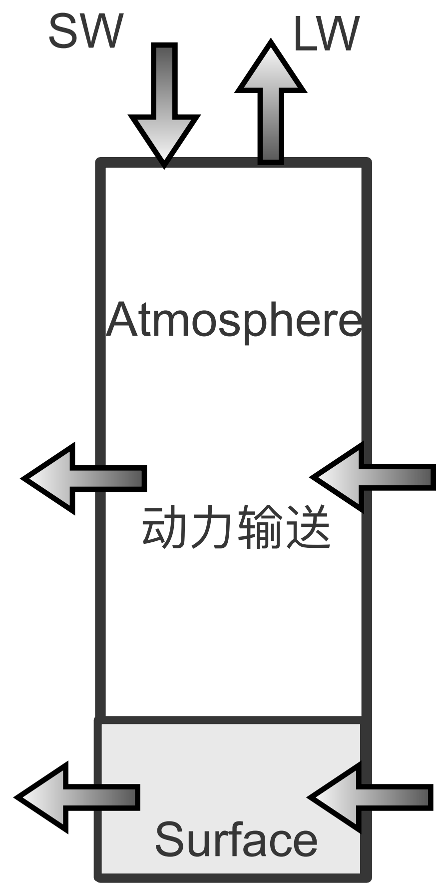
Assume:

$$\begin{aligned} \mathcal{A}(T) = &\quad \alpha = 0.4, && \text{for } T < T_{snow} \\ = &\quad \beta = 0.7, && \text{for } T > T_{snow} \\ = &\quad \tfrac{\alpha+\beta}{2}, && \text{for } T = T_{snow} \end{aligned}$$

$$T_{snow} = -10^{o}C$$

$$s(x) = 1 - 0.241(3x^2 - 1)$$

$$A = 211.1\,Wm^{-2}, \text{ and } B = 1.55\,Wm^{-2}(^{o}C)^{-1}$$



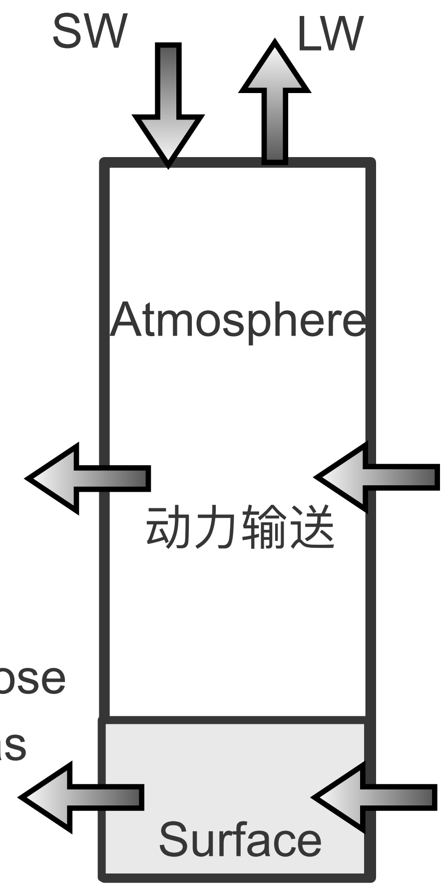
Simple energy balance climate models

In equilibrium,

$$Qs(x)\mathcal{A}(T) - I(T) + F(T) = 0$$

The snow line case - **Question #2:**

- Let $\alpha = 0.43$. Keep A and B the same.
  - Determine β such that $\bar{T}$ remains unchanged;
  - Determine C for the above choice of α and β ;
  - Compute Q(xs);
  - Discuss any difference between these results and those obtained for $\alpha = 0.4$, $\beta = 0.7$. In particular, how has the global stability changed and why?



Determine the value of β

In equilibrium,

$$Qs(x)\mathcal{A}(T) - I(T) + F(T) = 0$$

The snow line case: infrared cooling $I = A + BT$

$$F(T) = C(\bar{T} - T)$$

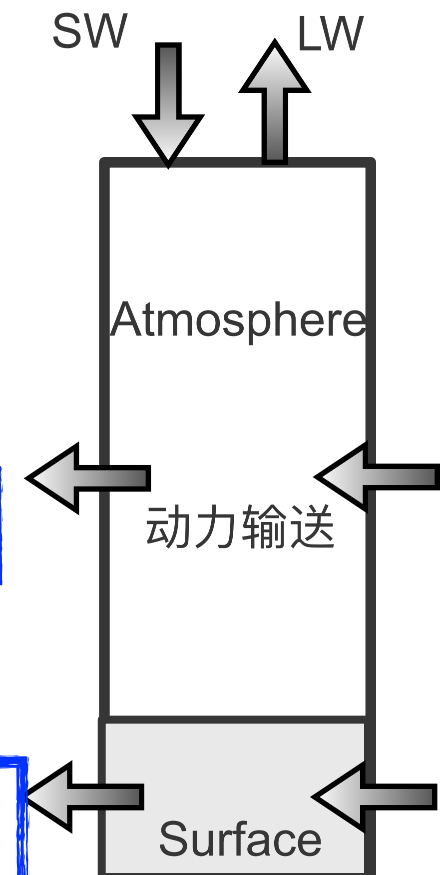
Hemisphere average:

$$\bar{T} = \int_0^1 T dx \qquad \bar{I} = \int_0^1 I dx \qquad F(I) = (C/B)(\bar{I} - I)$$

Radiation balance

$$\begin{aligned}\bar{I}/Q &= \int_0^1 s(x)\mathcal{A}(x)dx \\ &= (\beta - \alpha)(1.241x_s - 0.241x_s^3) + \alpha\end{aligned}$$

Unchanged
$\beta = 0.699$



Determine the value of C

In equilibrium,

$$Qs(x)\mathcal{A}(T) - I(T) + F(T) = 0$$

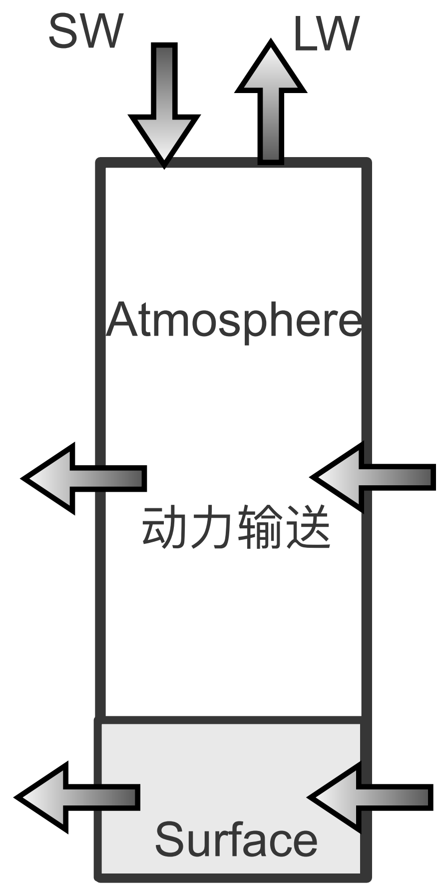
The snow line case:

$$I/Q = \frac{\frac{C}{B}\bar{I}/Q + s(x)\mathcal{A}(x, x_s)}{1 + \frac{C}{B}}$$

Determine C using current climate:

$$I(x_s) = I(0.95) = I(T_{snow}), \text{ get the value of } \frac{C}{B}$$

$$\frac{C}{B} = \frac{\frac{I_{snow}}{Q_o} - \frac{\alpha+\beta}{2}s(x_s)}{\frac{\bar{I}}{Q_o} - \frac{I_{snow}}{Q_o}}$$



$Q_o = 340\,Wm^{-2}$, $C = 3.23$, 相较于原来3.34，C减弱

Determine the value of C

In equilibrium,

$$Qs(x)\mathcal{A}(T) - I(T) + F(T) = 0$$

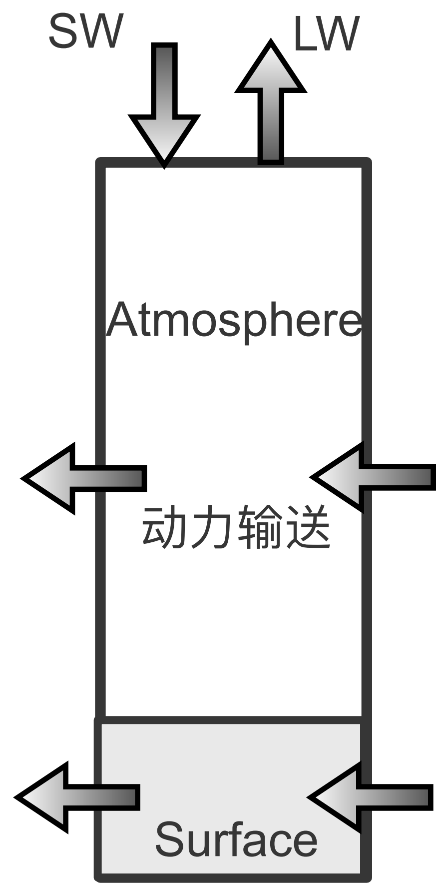
The snow line case:

$$I/Q = \frac{\frac{C}{B}\bar{I}/Q + s(x)\mathcal{A}(x, x_s)}{1 + \frac{C}{B}}$$

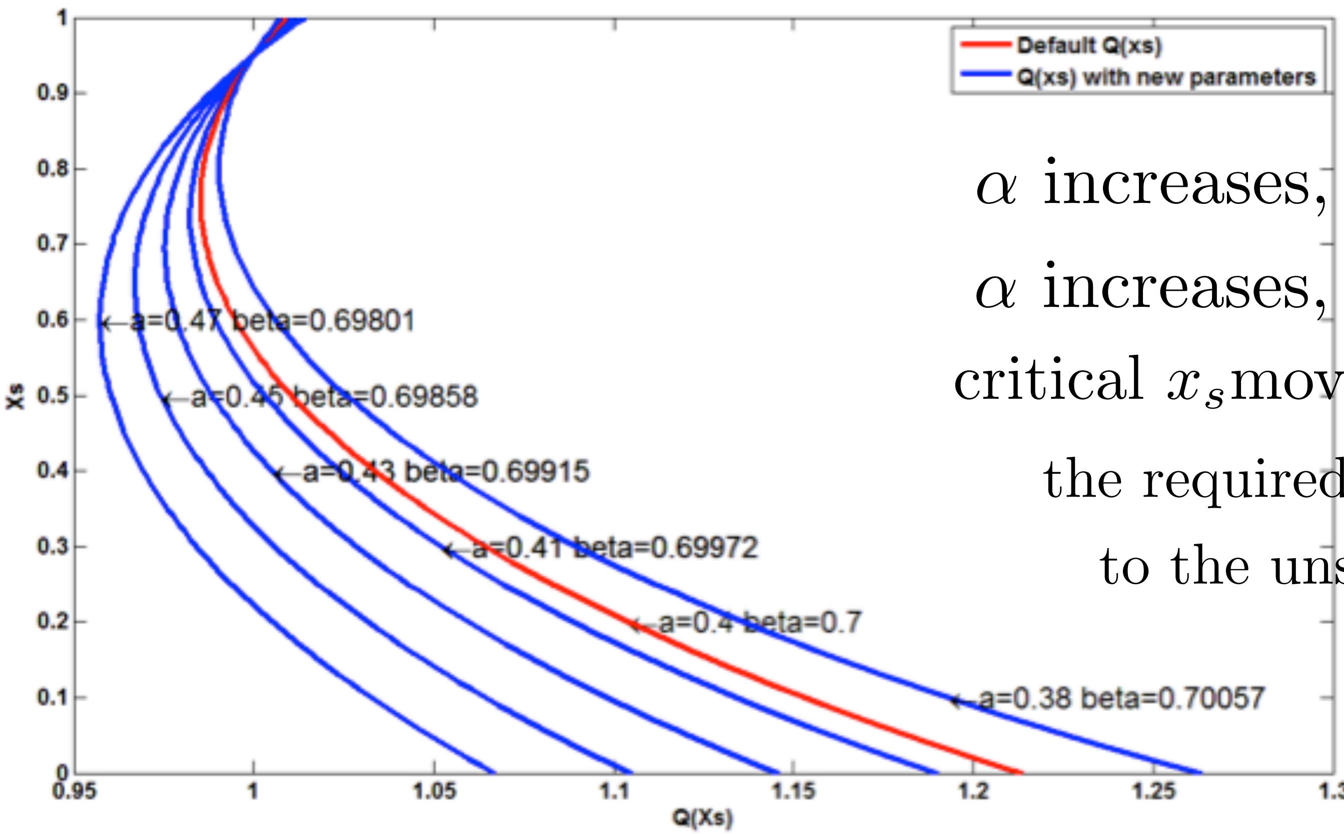
Then

$$Q(x_s) = \frac{(1 + \frac{C}{B})(A + BT_{snow})}{\frac{C}{B}\bar{I}/Q + s(x_s)\frac{\alpha+\beta}{2}}$$

$$\begin{aligned}\bar{I}/Q &= \int_0^1 s(x)\mathcal{A}(x)dx \\ &= (\beta - \alpha)(1.241x_s - 0.241x_s^3) + \alpha\end{aligned}$$



Plot $Q(x_s)$, note the choice of Q_o



α increases, β decreases

α increases, C decreases

critical x_s moves equatorwards

the required variation of Q

to the unstable increases

Simple energy balance climate models

The snow line case:

$$Q(x_s) = \frac{(1+\frac{C}{B})(A+BT_{snow})}{\frac{C}{B}\bar{I}(x_s)+s(x_s)\frac{\alpha+\beta}{2}}$$

If C is nonzero,

The destabilizing effect of heat transport

There is a minimum value of Q, below which the climate will unstably proceed to a snow/ice covered earth.