Assignments 3

Question#1: Hadley cell under different forcing

Held-Hou(1980) 讨论了当外部强迫的经向分布呈二次勒让德多项式，即 $\frac{\Theta_E(\phi,z)}{\Theta_o} = 1 - \frac{2}{3}\Delta_H P_2(\sin\phi) + \Delta_v(\frac{z}{H} - \frac{1}{2})$的情况下，哈德莱环流内的风场、温度场、环流的空间范围等将怎样随纬度和外力强迫的强度而变化。如果将外力强迫的空间分布改为

$$\frac{\Theta_E(\phi,z)}{\Theta_o} = 1 - \Delta_H(\sin\phi - \frac{1}{2}) + \Delta_v(\frac{z}{H} - \frac{1}{2}),$$

1. 请推导出哈德莱环流内的高空风场和垂直平均位温场$\frac{\tilde{\Theta}}{\Theta_o}$将如何随纬度分布;
2. 同样利用小角度假设，请推导出环流的空间范围ϕ_H 的表达式。如果设 $r = \frac{gH}{\Omega^2 a^2}$, 请分别画出当 $\Delta_H = 1/3$ 和 $\Delta_H = 1/6$ 时，与Held-Hou的情况相比，ϕ_H 怎样随 r 而变化。
3. 选做题目：在此情况下，近地面风场的分布有怎样变化。

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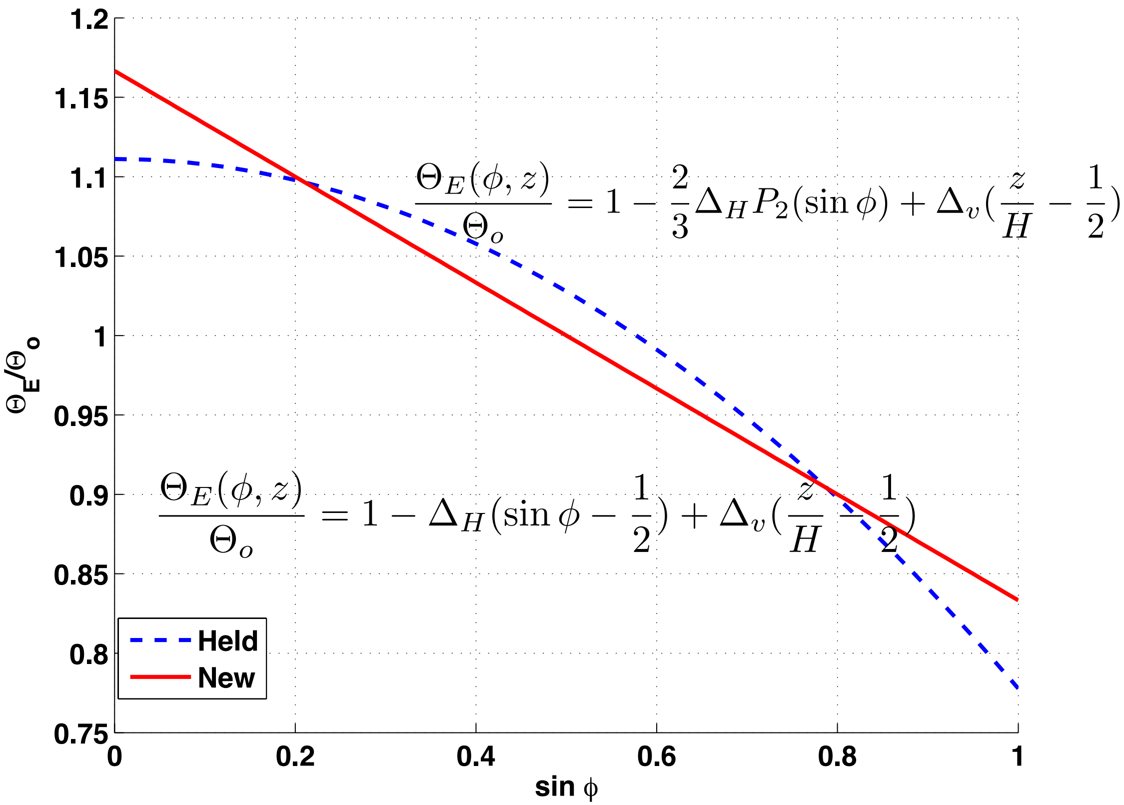

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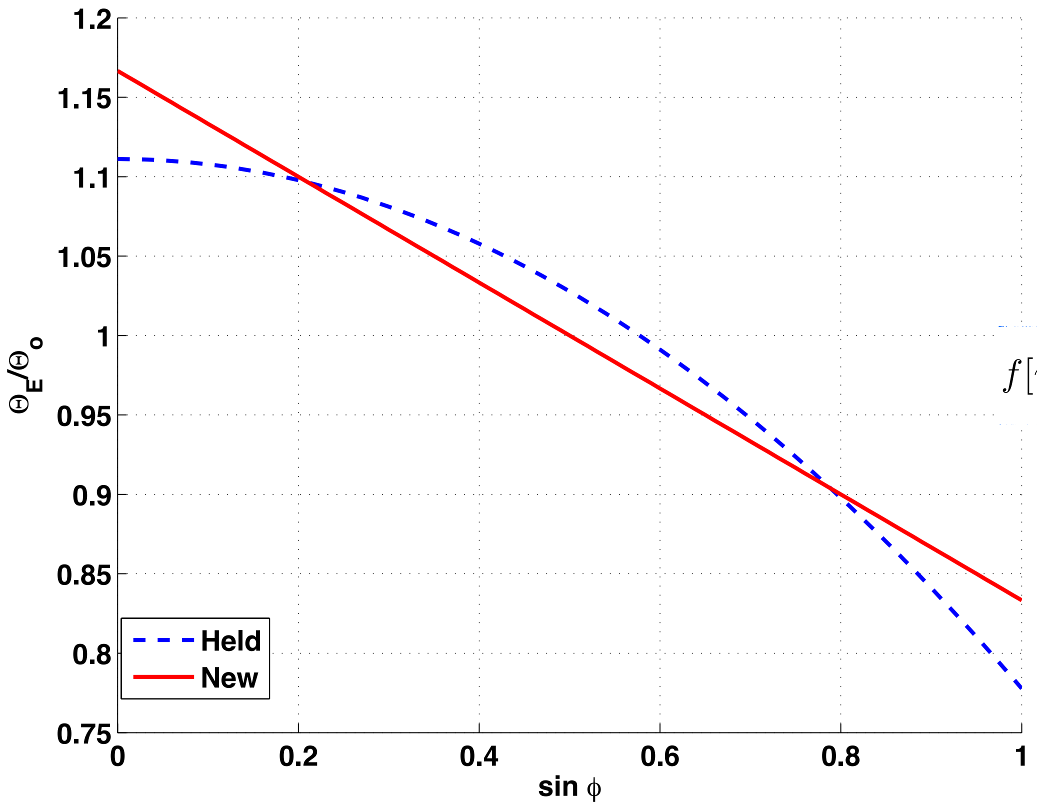

- Angular momentum:

$$[u] = \Omega a \frac{\sin^2\phi}{\cos\phi} \equiv U_M$$

- Thermal wind relation:

$$f[u(H) - u(0)] + \frac{\tan\phi}{a}[u^2(H) - u^2(0)] = -\frac{gH}{a\Theta_o}\frac{\partial\tilde{\Theta}}{\partial\phi}$$

$$\frac{\tilde{\Theta}(0) - \tilde{\Theta}(\phi)}{\Theta_o} = \frac{\Omega^2 a^2}{2gH}\frac{\sin^4\phi}{\cos^2\phi}$$

Need to know $\dfrac{\tilde{\Theta}(0)}{\Theta_o}$

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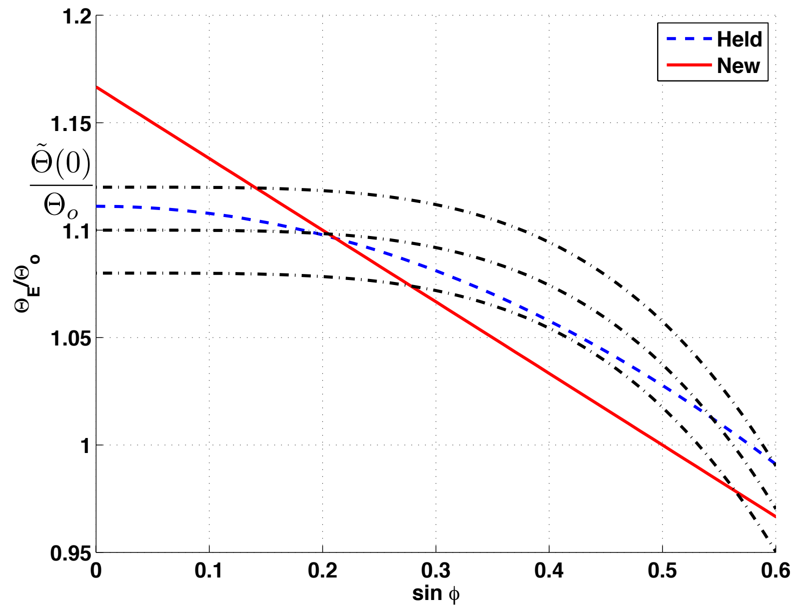

- Temperature should be continuous at the edge:

$$\tilde{\Theta}(\phi_H) = \tilde{\Theta}_E(\phi_H)$$

- Hadley cell does not produce net heating but just carry heat poleward over the

$$\int_0^{\phi_H} \tilde{\Theta}\cos\phi d\phi = \int_0^{\phi_H} \tilde{\Theta}_E \cos\phi d\phi$$

Assume small ϕ, $\sin\phi \sim \phi$

$$\phi_H = \left(\frac{5}{4}\frac{gH\Delta_H}{\Omega^2 a^2}\right)^{1/3}$$

$$\frac{\tilde{\Theta}(0) - \tilde{\Theta}(\phi)}{\Theta_o} = \frac{\Omega^2 a^2}{2gH}\frac{\sin^4\phi}{\cos^2\phi}$$

$$\frac{\tilde{\Theta}(0)}{\Theta_o} \approx \frac{\tilde{\Theta}_E(0)}{\Theta_o} - \frac{3}{8}\Delta_H\phi_H$$

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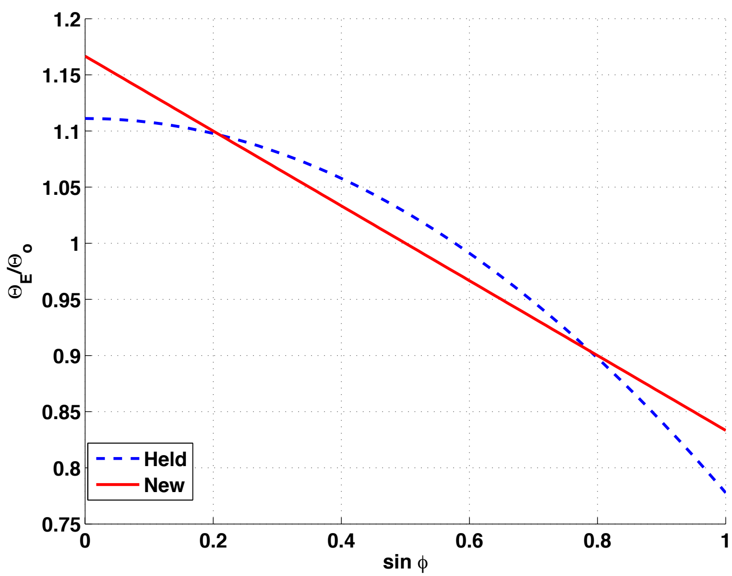

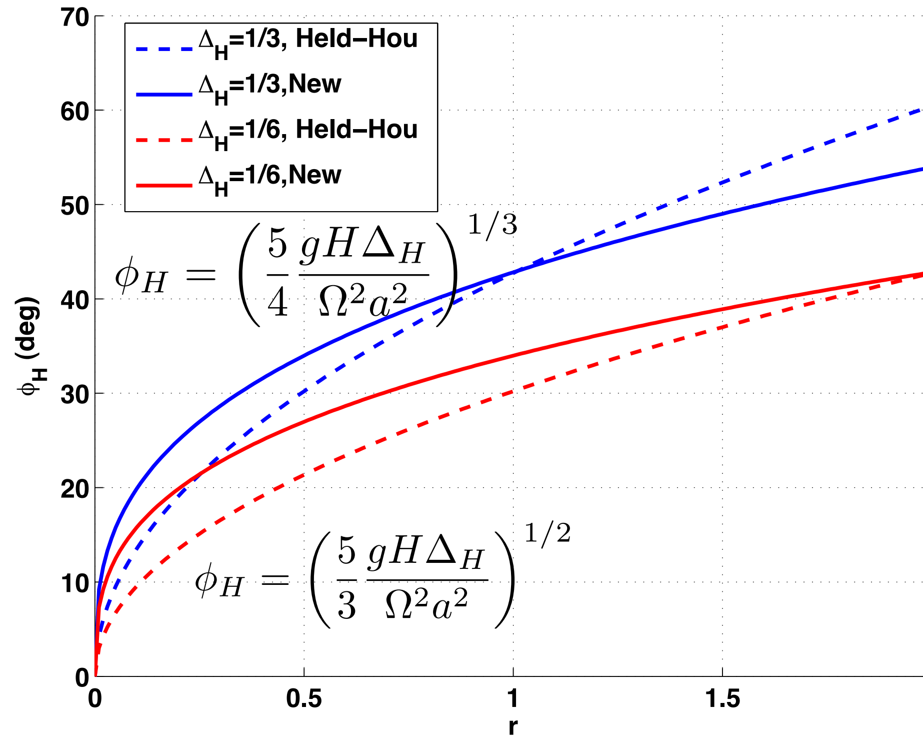

Held-Hou model

-Surface winds

- From the zonal momentum equation

$$\frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}\left(\cos^2\phi\int_0^H uvdz\right) = -Cu(0)$$

$$\frac{1}{\Theta_o}\int_0^H v\Theta dz \approx V\Delta_V$$

$$\frac{1}{H}\int_0^H \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(v\Theta\cos\phi)dz = \frac{\tilde{\Theta}_E - \tilde{\Theta}}{\tau}$$

Then, mass flux V can be solved. Similarly, we have the momentum flux,

$$\int_0^H uvdz \approx VU_m$$

$$\left.\begin{aligned} 0 &= -\nabla\cdot(\mathbf{v}u) + fv + \frac{uv\tan\theta}{a} + \frac{\partial}{\partial z}\left(\nu\frac{\partial u}{\partial z}\right) \\ 0 &= -\nabla\cdot(\mathbf{v}v) - fu - \frac{u^2\tan\theta}{a} - \frac{1}{a}\frac{\partial\Phi}{\partial\theta} + \frac{\partial}{\partial z}\left(\nu\frac{\partial v}{\partial z}\right) \\ 0 &= -\nabla\cdot(\mathbf{v}\Theta) - (\Theta - \Theta_E)\tau^{-1} + \frac{\partial}{\partial z}\left(\nu\frac{\partial\Theta}{\partial z}\right) \\ 0 &= -\nabla\cdot\mathbf{v} \\ \frac{\partial\Phi}{\partial z} &= g\Theta/\Theta_0 \end{aligned}\right\}, \quad (1)$$

with boundary conditions

$$\left.\begin{aligned} &\text{at } z = H: \quad w = 0; \quad \frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = \frac{\partial\Theta}{\partial z} = 0 \\ &\text{at } z = 0: \quad w = 0; \quad \frac{\partial\Theta}{\partial z} = 0; \\ &\nu\frac{\partial u}{\partial z} = Cu; \quad \nu\frac{\partial v}{\partial z} = Cv \end{aligned}\right\}. \quad (1a)$$

-Surface Winds

- From thermodynamic equation:

$$\frac{1}{H}\int_0^H \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(v\theta\cos\phi)dz = \frac{\widetilde{\Theta}_E - \widetilde{\Theta}}{\tau}$$

由上式再结合小角度假设可得 $\frac{1}{\Theta_0}\int_0^H v\theta dz = \frac{Ha}{\tau}\int\frac{\widetilde{\Theta}_E - \widetilde{\Theta}}{\Theta_0}d\phi$

$\widetilde{\Theta}_E$ 和 $\widetilde{\Theta}$ 的表达式代入可得

$$\frac{1}{\Theta_0}\int_0^H v\theta dz = \frac{Ha}{\tau}[\frac{3}{8}\phi_H\Delta_H\phi - \frac{\Delta_H}{2}\phi^2 + \frac{\Omega^2 a^2}{10gH}\phi^5]$$

$\frac{1}{\Theta_o}\int_0^H v\Theta dz \approx V\Delta_V$ then, mass flux V can be solved.

$$\int_0^H uvdz = \frac{\Omega a\phi^2}{\Delta_V}\frac{1}{\Theta_0}\int_0^H v\theta dz = \frac{H\Omega a^2}{\Delta_V\tau}[\frac{3}{8}\phi_H\Delta_H\phi^3 - \frac{\Delta_H}{2}\phi^4 + \frac{\Omega^2 a^2}{10gH}\phi^7]$$

$$u(0) = -\frac{1}{Ca}\frac{\partial}{\partial\phi}(\int_0^H uvdz) = -\frac{H\Omega a\Delta_H\phi_H^3}{8C\Delta_V\tau}[9(\frac{\phi}{\phi_H})^2 - 16(\frac{\phi}{\phi_H})^3 + 7(\frac{\phi}{\phi_H})^6]$$