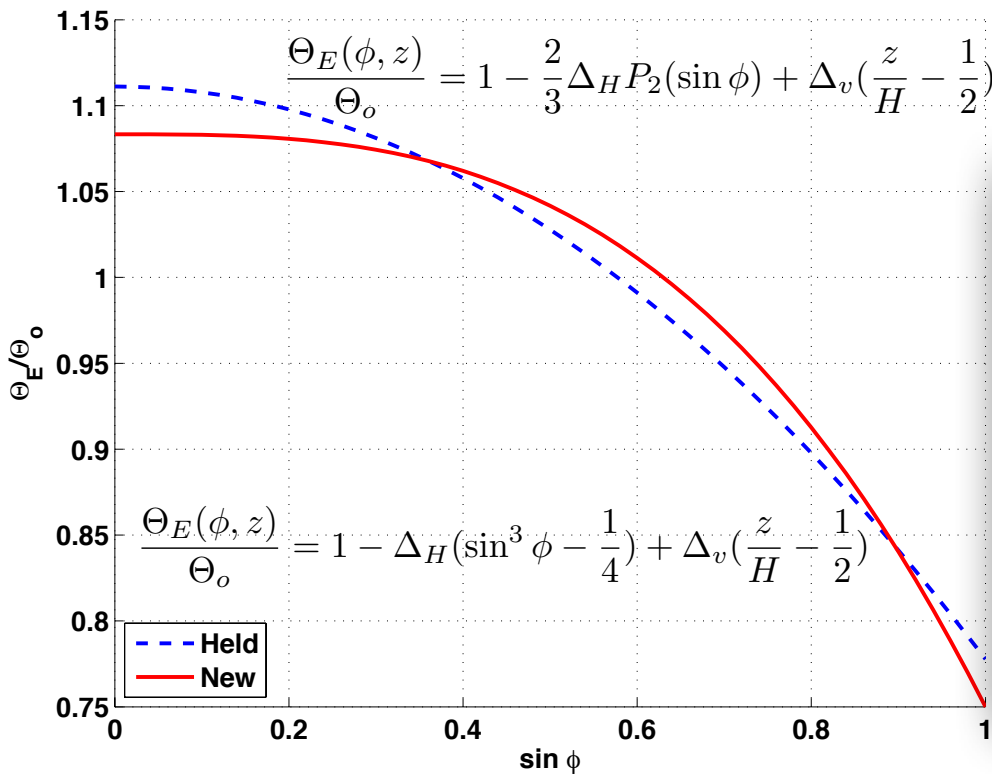




Assignments 3



Question#1: Hadley cell under different forcing



Held-Hou(1980) 讨论了当外部强迫的经向分布呈二次勒让德多项式, 即 $\frac{\Theta_E(\phi, z)}{\Theta_o} = 1 - \frac{2}{3} \Delta_H P_2(\sin \phi) + \Delta_v \left(\frac{z}{H} - \frac{1}{2} \right)$ 的情况下, 哈德莱环流内的风场、温度场、环流的空间范围等将怎样随纬度和外力强迫的强度而变化。

如果将外力强迫的空间分布改为

$$\frac{\Theta_E(\phi, z)}{\Theta_o} = 1 - \Delta_H \left(\sin^3 \phi - \frac{1}{4} \right) + \Delta_v \left(\frac{z}{H} - \frac{1}{2} \right),$$

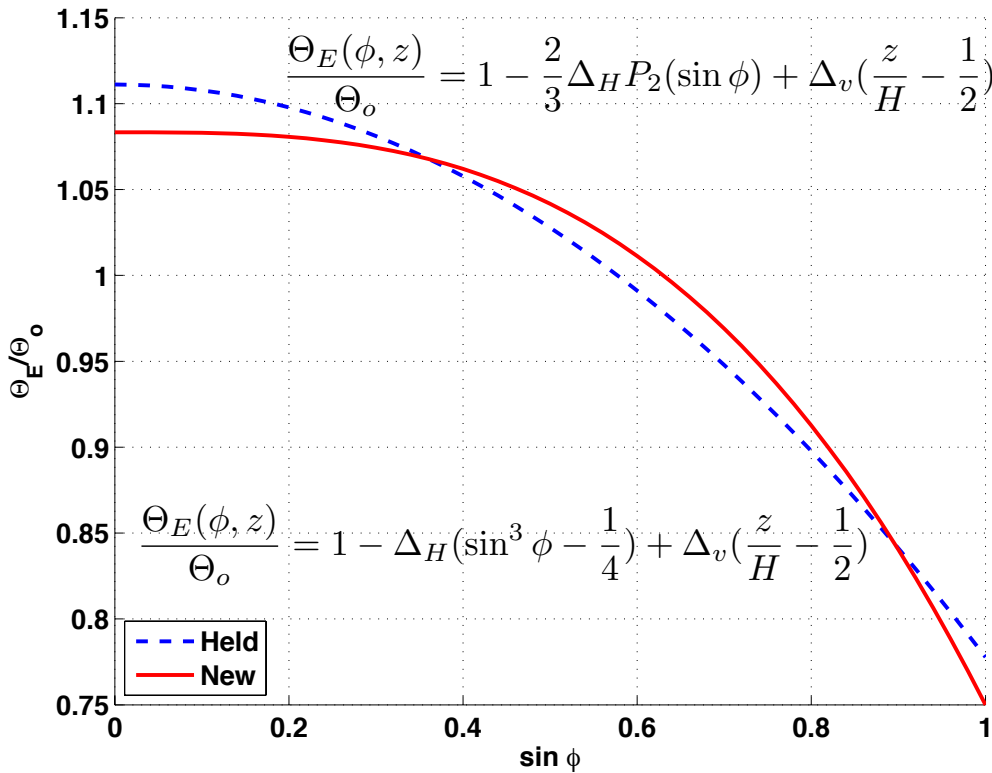
1. 请推导出哈德莱环流内的高空风场和垂直平均位温 $\frac{\tilde{\Theta}}{\Theta_o}$ 将如何随纬度分布;
2. 同样利用小角度假设, 请推导出环流的空间范围 ϕ_H 的表达式。如果设 $r \equiv \frac{gH}{\Omega^2 a^2}$, 请分别画出当 $\Delta_H = 1/3$ 和 $\Delta_H = 1/6$ 时, 与Held-Hou的情况相比, ϕ_H 怎样随 r 而变化。
3. 选做题目: 在此情况下, 近地面风场的分布有怎样变化。



Assignments 3



Question#1: Hadley cell under different forcing



- Angular momentum:

$$[u] = \Omega a \frac{\sin^2 \phi}{\cos \phi} \equiv U_M$$

- Thermal wind relation:

$$f[u(H) - u(0)] + \frac{\tan \phi}{a} [u^2(H) - u^2(0)] = -\frac{gH}{a\Theta_o} \frac{\partial \tilde{\Theta}}{\partial \phi}$$

$$\frac{\tilde{\Theta}(0) - \tilde{\Theta}(\phi)}{\Theta_o} = \frac{\Omega^2 a^2 \sin^4 \phi}{2gH \cos^2 \phi}$$

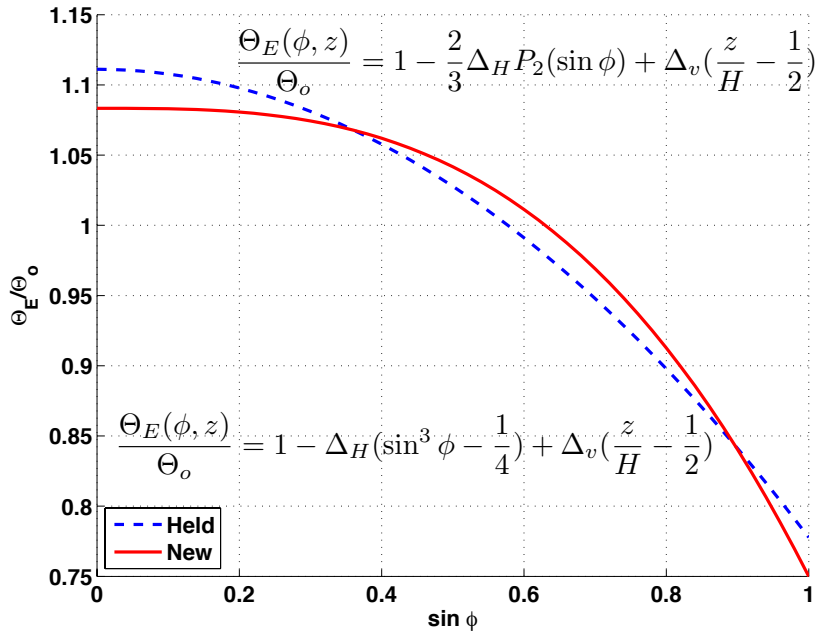
Need to know $\frac{\tilde{\Theta}(0)}{\Theta_o}$



Assignments 3



Question#1: Hadley cell under different forcing



- Temperature should be continuous at the edge:

$$\tilde{\Theta}(\phi_H) = \tilde{\Theta}_E(\phi_H)$$

- Hadley cell does not produce net heating but just carry heat poleward over the

$$\int_0^{\phi_H} \tilde{\Theta} \cos \phi d\phi = \int_0^{\phi_H} \tilde{\Theta}_E \cos \phi d\phi$$

Assume small ϕ , $\sin \phi \sim \phi$

$$\phi_H = \frac{15}{8} \frac{gH\Delta_H}{\Omega^2 a^2}$$

$$\frac{\tilde{\Theta}(0) - \tilde{\Theta}(\phi)}{\Theta_o} = \frac{\Omega^2 a^2 \sin^4 \phi}{2gH \cos^2 \phi}$$

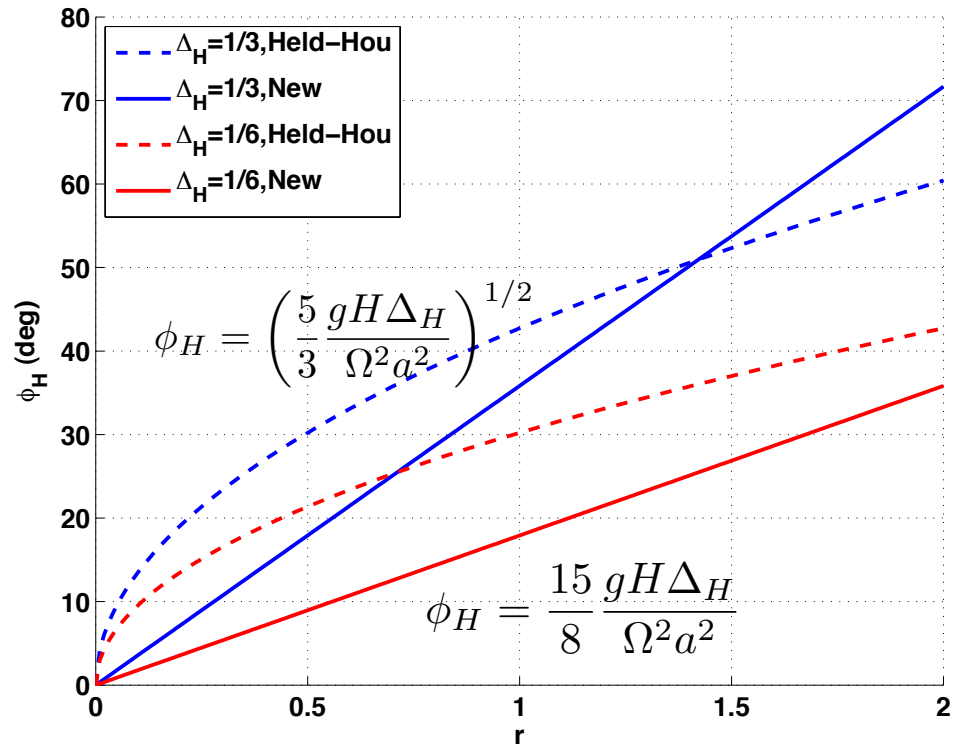
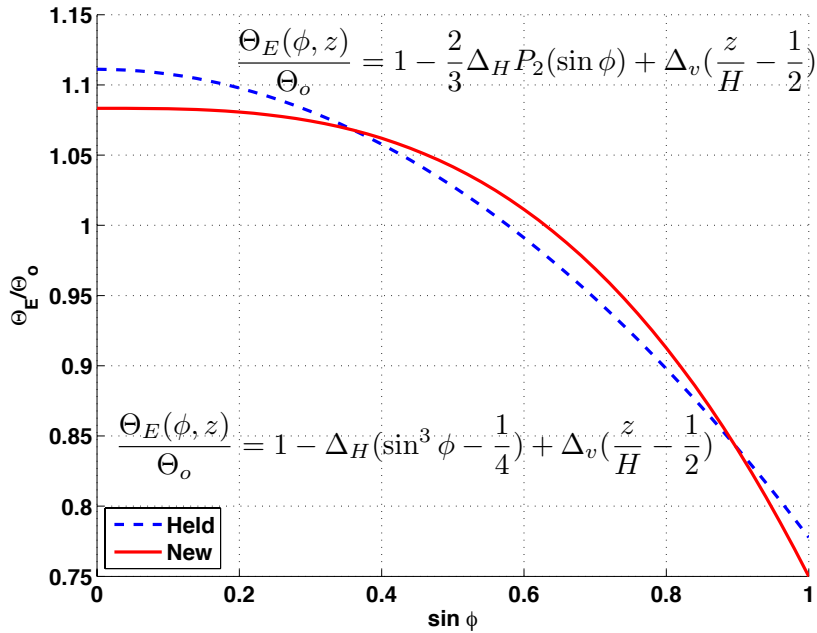
$$\frac{\tilde{\Theta}(0)}{\Theta_o} \approx \frac{\tilde{\Theta}_E(0)}{\Theta_o} - \Delta_H \phi_H^3 + \frac{\Omega^2 a^2}{2gH} \phi_H^4 = \frac{\tilde{\Theta}_E(0)}{\Theta_o} - \frac{1}{16} \Delta_H \phi_H^3$$



Assignments 3



Question#1: Hadley cell under different forcing





- From the zonal momentum equation

$$\frac{1}{a \cos^2 \phi} \frac{\partial}{\partial \phi} \left(\cos^2 \phi \int_0^H uvdz \right) = -Cu(0)$$

$$\frac{1}{\Theta_0} \int_0^H v\Theta dz \approx V\Delta_V$$

$$\frac{1}{H} \int_0^H \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} (v\Theta \cos \phi) dz = \frac{\tilde{\Theta}_E - \tilde{\Theta}}{\tau}$$

Then, mass flux V can be solved. Similarly, we have the momentum flux,

$$\int_0^H uvdz \approx VU_m$$

$$0 = -\nabla \cdot (\mathbf{v}u) + fv + \frac{uv \tan \theta}{a} + \frac{\partial}{\partial z} \left(\nu \frac{\partial u}{\partial z} \right)$$

$$0 = -\nabla \cdot (\mathbf{v}v) - fu - \frac{u^2 \tan \theta}{a} - \frac{1}{a} \frac{\partial \Phi}{\partial \theta} + \frac{\partial}{\partial z} \left(\nu \frac{\partial v}{\partial z} \right), \quad (1)$$

$$0 = -\nabla \cdot (\mathbf{v}\Theta) - (\Theta - \Theta_E)\tau^{-1} + \frac{\partial}{\partial z} \left(\nu \frac{\partial \Theta}{\partial z} \right)$$

$$0 = -\nabla \cdot \mathbf{v}$$

$$\frac{\partial \Phi}{\partial z} = g\Theta/\Theta_0$$

with boundary conditions

$$\text{at } z = H: \quad w = 0; \quad \frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = \frac{\partial \Theta}{\partial z} = 0$$

$$\text{at } z = 0: \quad w = 0; \quad \frac{\partial \Theta}{\partial z} = 0;$$

(1a)

$$\nu \frac{\partial u}{\partial z} = Cu; \quad \nu \frac{\partial v}{\partial z} = Cv$$



- From thermodynamic equation:

$$\frac{1}{H\Theta_0} \int_0^H \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} (v\Theta \cos \phi) dz = \frac{\bar{\Theta}_e - \bar{\Theta}}{\tau \Theta_0} = \frac{1}{\tau} \left(\frac{1}{16} \left(\frac{15}{8} \right)^3 \Delta_H^4 r^3 - \Delta_H \phi^3 + \frac{1}{2} \frac{\Omega^2 a^2}{gH} \phi^4 \right)$$

$$\frac{1}{\Theta_0} \int_0^H (v\Theta) dz = \int_0^\phi \frac{Ha}{\tau} \left(\frac{1}{16} \left(\frac{15}{8} \right)^3 \Delta_H^4 r^3 - \Delta_H \phi^3 + \frac{1}{2} \frac{\Omega^2 a^2}{gH} \phi^4 \right) d\phi$$

$$\frac{1}{\Theta_0} \int_0^H (v\Theta) dz = \frac{Har^4 \Delta_H^5}{\tau} \frac{1}{16} \left(\frac{15}{8} \right)^4 \left[\frac{\phi}{\phi_H} - 4 \left(\frac{\phi}{\phi_H} \right)^4 + 3 \left(\frac{\phi}{\phi_H} \right)^5 \right]$$

Then, mass flux V can be solved.

$$\int_0^H uvdz \approx VU_m$$

$$Cu(0) = -\frac{\Omega}{\Delta v} \frac{Har^5 \Delta_H^6}{\tau} \frac{1}{16} \left(\frac{15}{8} \right)^5 \left[3 \left(\frac{\phi}{\phi_H} \right)^2 - 24 \left(\frac{\phi}{\phi_H} \right)^5 + 21 \left(\frac{\phi}{\phi_H} \right)^6 \right]$$



$$Cu(0) = -\frac{\Omega}{\Delta v} \frac{Har^5 \Delta_H^6}{\tau} \frac{1}{16} \left(\frac{15}{8}\right)^5 \left[3\left(\frac{\phi}{\phi_H}\right)^2 - 24\left(\frac{\phi}{\phi_H}\right)^5 + 21\left(\frac{\phi}{\phi_H}\right)^6\right]$$

