Assignments 4

Question#1: a generalized EP flux

在第四章中，我们从准地转近似下的纬向平均风场、温度场的趋势方程出发，定义了E–P通量。但是该定义下的E–P通量并没有考虑到大气湿过程的影响。如果从第三章介绍的水汽方程出发，我们可以按照以下步骤定义出一个包含大气大尺度运动中湿过程作用的广义的E–P通量。

1）在准地转近似下，如果我们按照对热力学方程的简化方法，将比湿（specific humidity）q，分解成一个标准比湿 q_s（reference specific humidity）和变化量q'，并且同样假设$\partial q/\partial p$的水平变化很小，请证明在准地转近似下p坐标系下的纬向平均比湿$[q]$的变化方程为：

$$\frac{\partial [q']}{\partial t} + \frac{\partial q_s}{\partial p}[\omega] = -[C - S] - \frac{\partial}{\partial y}[v^* q^*],$$

其中$C - S$为水汽方程在准地砖近似下的源汇项，表征由大尺度运动所带来的净凝结率。

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水汽方程出发：

$$\frac{Dq}{Dt} = s(q)$$

$$q(x,y,p,t) = q_s(p) + q'(x,y,p,t)$$

$$q'(x,y,p,t) = [q'] + q^*$$

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设 $u=[u]+u^*$， $v=[v]+v^*$， $\omega=[\omega]+\omega^*$

$$\frac{\partial[q']}{\partial t}+\frac{\partial([q][v]+[q^*v^*])}{\partial y}+\frac{\partial([q][\omega]+[q^*\omega^*])}{\partial p}=[s(q)] \qquad (4)$$

对于中纬度大尺度流，经向水汽输送的涡旋分量相比平均分量起主导作用，即

$\frac{\partial[q][v]}{\partial y} << \frac{\partial[q^*v^*]}{\partial y}$；由于准地转关系，$\frac{\partial[q^*v^*]}{\partial y} >> \frac{\partial[q^*\omega^*]}{\partial p}$

故：$$\frac{\partial[q']}{\partial t}+\frac{\partial[q^*v^*]}{\partial y}+\frac{\partial[q][\omega]}{\partial p}=[s(q)] \qquad (5)$$

又 $\frac{\partial q}{\partial p}$ 的水平变化很小，即 $\frac{\partial[q][\omega]}{\partial p} \sim [\omega]\frac{\partial q_s}{\partial p}$；水汽的源汇项为 $[s(q)]=-[C-S]$

即得证：$$\frac{\partial[q']}{\partial t}+[\omega]\frac{\partial q_s}{\partial p}=-[C-S]-\frac{\partial[q^*v^*]}{\partial y} \qquad (6)$$

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2）如果重新定义一个非绝热加热项Q_m，使得$Q_m = Q - L[C - S](\frac{p}{p_o})^{R/c_p}$，

请推导出一个关于$[\theta + \frac{L}{c_p}q']$的变化方程。

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联系第四章中纬向平均风场和温度场的变化方程:

$$\frac{\partial [u]}{\partial t} = -\frac{\partial([u^* v^*])}{\partial y} + f[v] + [F_x]$$

$$\frac{\partial [\theta]}{\partial t} + [\omega]\frac{\partial \theta_s}{\partial p} = -\frac{\partial([\theta^* v^*])}{\partial y} + \left(\frac{p_o}{p}\right)^{R/c_p} \frac{[Q]}{c_p}$$

$$\frac{\partial [q']}{\partial t} + [\omega]\frac{\partial q_s}{\partial p} = -\frac{\partial([v^* q^*])}{\partial y} - [C - S]$$

将上述两式线性相加:

$$\frac{\partial [\theta + \frac{L}{c_p} q']}{\partial t} + [\omega]\frac{\partial}{\partial p}\left(\theta_s + \frac{L}{c_p} q_s\right) = -\frac{\partial}{\partial y}\left([\theta^* v^*] + \frac{L}{c_p}[v^* q^*]\right) + \left(\frac{p_o}{p}\right)^{R/c_p} \frac{[Q_m]}{c_p}$$

$$Q_m = Q - L[C - S]\left(\frac{p}{p_o}\right)^{R/c_p}$$

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3）根据以上推导出的新方程和准地转近似下$[u]$的变化方程，请重新定义一个广义的E–P通量$\mathcal{F}_m$，使得新的E–P通量中包含了eddy对水汽输送的作用；并且证明，在湿绝热($Q_m = 0$)和无摩擦的情况下，平衡状态下的$\mathcal{F}_m$满足$\nabla \cdot \mathcal{F}_m = 0$, 并请根据水汽输送的空间分布讨论：在实际大气中，新定义的E–P通量的$\nabla \cdot \mathcal{F}_m$应该有怎样的变化？eddy 对水汽的输送作用将对维持 Ferrel 环流起到怎样的作用?

4）请根据新定义出的E–P通量，定义出新的剩余环流(residual circulation, $[\tilde{v}_m]$, $[\tilde{\omega}_m]$)，并讨论此时剩余环流的含义是什么？相对于新的剩余环流，新的TEM方程(Transformed Eulerian Mean Equations)应该是什么？同时，也请写出，如果用剩余环流来表述，(1)问中推导出的水汽方程将如何改写，eddy强迫项应变为什么？

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得到重新定义的纬向平均动量和热力学方程:

$$\frac{\partial [u]}{\partial t} = -\frac{\partial([u^* v^*])}{\partial y} + f[v] + [F_x]$$

$$\frac{\partial[\theta + \frac{L}{c_p} q']}{\partial t} + [\omega]\frac{\partial}{\partial p}(\theta_s + \frac{L}{c_p} q_s) = -\frac{\partial}{\partial y}([\theta^* v^*] + \frac{L}{c_p}[v^* q^*]) + \left(\frac{p_o}{p}\right)^{R/c_p} \frac{[Q_m]}{c_p}$$

由上述两式可定义一个新的EP通量:

$$\mathcal{F}_m \equiv -[u^* v^*]\,\mathbf{j} + f\frac{[v^*\theta^*] + \frac{L}{c_p}[v^* q^*]}{\partial/\partial p(\theta_s + \frac{L}{c_p} q_s)}\,\mathbf{k}$$

在湿绝热、无摩擦的情况下，平衡状态的方程满足:

$$-\frac{\partial([u^* v^*])}{\partial y} + f[v] = 0$$

$$[\omega]\frac{\partial}{\partial p}(\theta_s + \frac{L}{c_p} q_s) + \frac{\partial}{\partial y}([\theta^* v^*] + \frac{L}{c_p}[v^* q^*]) = 0$$

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则此时纬向平均环流：

$$[v] = \frac{1}{f}\frac{\partial([u^*v^*])}{\partial y} \qquad [\omega] = -\frac{\partial}{\partial y}\frac{([\theta^*v^*] + \frac{L}{c_p}[v^*q^*])}{\frac{\partial}{\partial p}(\theta_s + \frac{L}{c_p}q_s)}$$

将上述两式代入连续方程可得：

$$\frac{\partial}{\partial y}(\nabla \cdot \mathcal{F}_m) = 0$$

利用边界条件，如极地处的 $\nabla \cdot \mathcal{F}_m = 0$ 可得：

$$\nabla \cdot \mathcal{F}_m = 0$$

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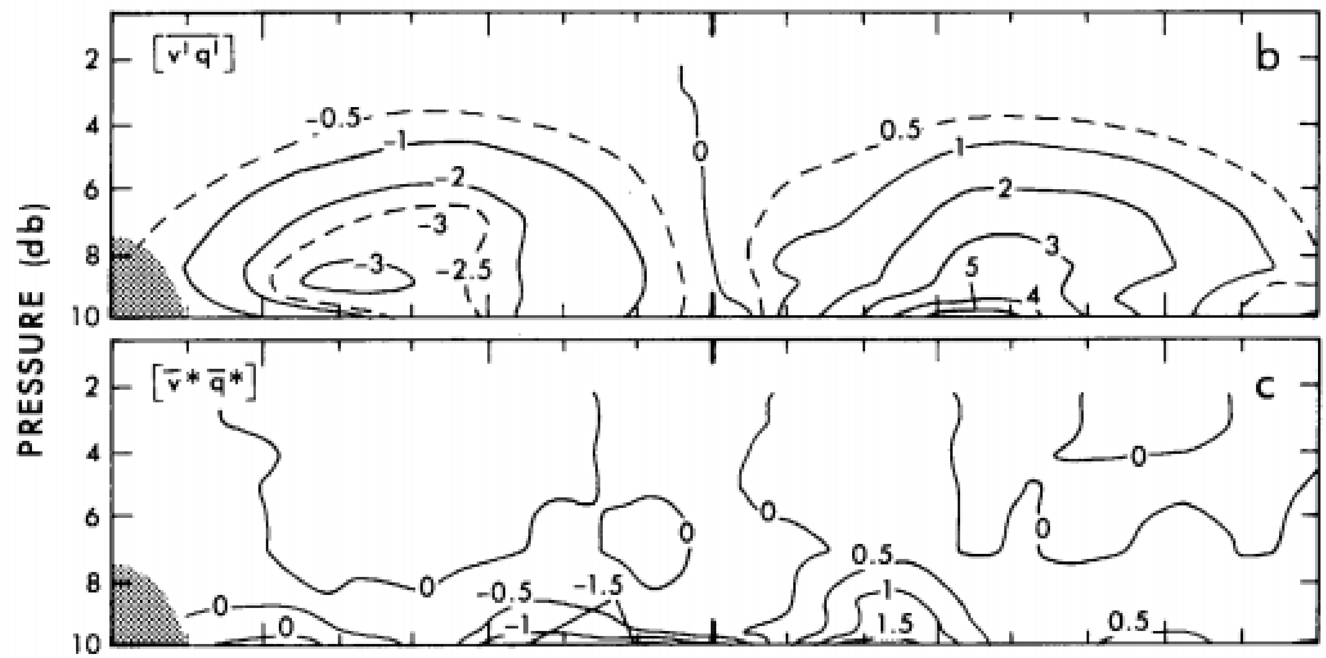
对Ferrel环流的影响：

$$[\omega] = -\frac{\partial}{\partial y}\frac{([\theta^* v^*] + \frac{L}{c_p}[v^* q^*])}{\frac{\partial}{\partial p}(\theta_s + \frac{L}{c_p} q_s)}$$

水汽输送的分布：



可知其对Ferrel环流的影响主要在中低层，其在南北方向上的分布与感热输送大体相似，只是大值区较感热分布更加偏向低纬地区。因此加强了eddy对平均环流的强迫。主要表现为：

1）增强Ferrel环流的强度

2）增加了Ferrel环流向南延伸的范围。

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$$\frac{\partial [u]}{\partial t} = -\frac{\partial([u^* v^*])}{\partial y} + f[v] + [F_x]$$

$$\frac{\partial[\theta + \frac{L}{c_p} q']}{\partial t} + [\omega]\frac{\partial}{\partial p}(\theta_s + \frac{L}{c_p} q_s) = -\frac{\partial}{\partial y}([\theta^* v^*] + \frac{L}{c_p}[v^* q^*]) + \left(\frac{p_o}{p}\right)^{R/c_p} \frac{[Q_m]}{c_p}$$

由上述两式可定义一个新的EP通量：

$$\mathcal{F}_m \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*] + \frac{L}{c_p}[v^* q^*]}{\partial/\partial p(\theta_s + \frac{L}{c_p} q_s)}\,\mathbf{k}$$

可重新定义剩余环流：

$$\tilde{v}_m = [v] - \frac{\partial}{\partial p}\frac{[v^*\theta^*] + \frac{L}{c_p}[v^* q^*]}{\partial/\partial p(\theta_s + \frac{L}{c_p} q_s)}$$

$$\tilde{\omega}_m = [\omega] + \frac{\partial}{\partial y}\frac{[v^*\theta^*] + \frac{L}{c_p}[v^* q^*]}{\partial/\partial p(\theta_s + \frac{L}{c_p} q_s)}$$

剩余环流为平衡掉eddy强迫（包括大尺度的eddy潜热强迫)之后的经向环流，可理解为由非湿绝热加热、摩擦力等外力强迫而引发的环流。

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$$\frac{\partial [u]}{\partial t} = -\frac{\partial([u^*v^*])}{\partial y} + f[v] + [F_x]$$

$$\frac{\partial[\theta + \frac{L}{c_p}q']}{\partial t} + [\omega]\frac{\partial}{\partial p}(\theta_s + \frac{L}{c_p}q_s) = -\frac{\partial}{\partial y}([\theta^* v^*] + \frac{L}{c_p}[v^* q^*]) + \left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q_m]}{c_p}$$

由上述两式可定义一个新的EP通量:

$$\mathcal{F}_m \equiv -[u^*v^*]\,\mathbf{j} + f\frac{[v^*\theta^*] + \frac{L}{c_p}[v^*q^*]}{\partial/\partial p(\theta_s + \frac{L}{c_p}q_s)}\,\mathbf{k}$$

可得到TEM方程:

$$\frac{\partial [u]}{\partial t} = f\tilde{v}_m + \nabla\cdot\mathcal{F}_m + [F_x]$$

$$\frac{\partial \tilde{v}_m}{\partial y} + \frac{\partial \tilde{\omega}_m}{\partial p} = 0$$

$$\frac{\partial[\theta + \frac{L}{c_p}q']}{\partial t} = -\tilde{\omega}_m\frac{\partial}{\partial p}(\theta_s + \frac{L}{c_p}q_s) + \left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q_m]}{c_p}$$

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$$\frac{\partial [q']}{\partial t} + [\omega]\frac{\partial q_s}{\partial p} = -\frac{\partial([v^* q^*])}{\partial y} - [C-S]$$

可重新定义剩余环流:

$$\tilde{v}_m = [v] - \frac{\partial}{\partial p}\frac{[v^*\theta^*] + \frac{L}{c_p}[v^* q^*]}{\partial/\partial p(\theta_s + \frac{L}{c_p} q_s)}$$

$$\tilde{\omega}_m = [\omega] + \frac{\partial}{\partial y}\frac{[v^*\theta^*] + \frac{L}{c_p}[v^* q^*]}{\partial/\partial p(\theta_s + \frac{L}{c_p} q_s)}$$

可得到新的水汽方程:

$$\frac{\partial [q']}{\partial t} + [\tilde{\omega}_m]\frac{\partial q_s}{\partial p} = -[C-S] - \frac{\partial}{\partial y}\left([q^* v^*] - \frac{\left[\left(\theta^* + \frac{L}{C_p} q^*\right) v^*\right]}{\frac{\partial \theta_s}{\partial p} / \frac{\partial q_s}{\partial p} + \frac{L}{C_p}}\right)$$

eddy 强迫项变为:

$$[q^* v^*] - \frac{\left[\left(\theta^* + \frac{L}{C_p} q^*\right) v^*\right]}{\frac{\partial \theta_s}{\partial p} / \frac{\partial q_s}{\partial p} + \frac{L}{C_p}}$$

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$\nabla \cdot \mathcal{F}_m, \nabla \cdot \mathcal{F}$

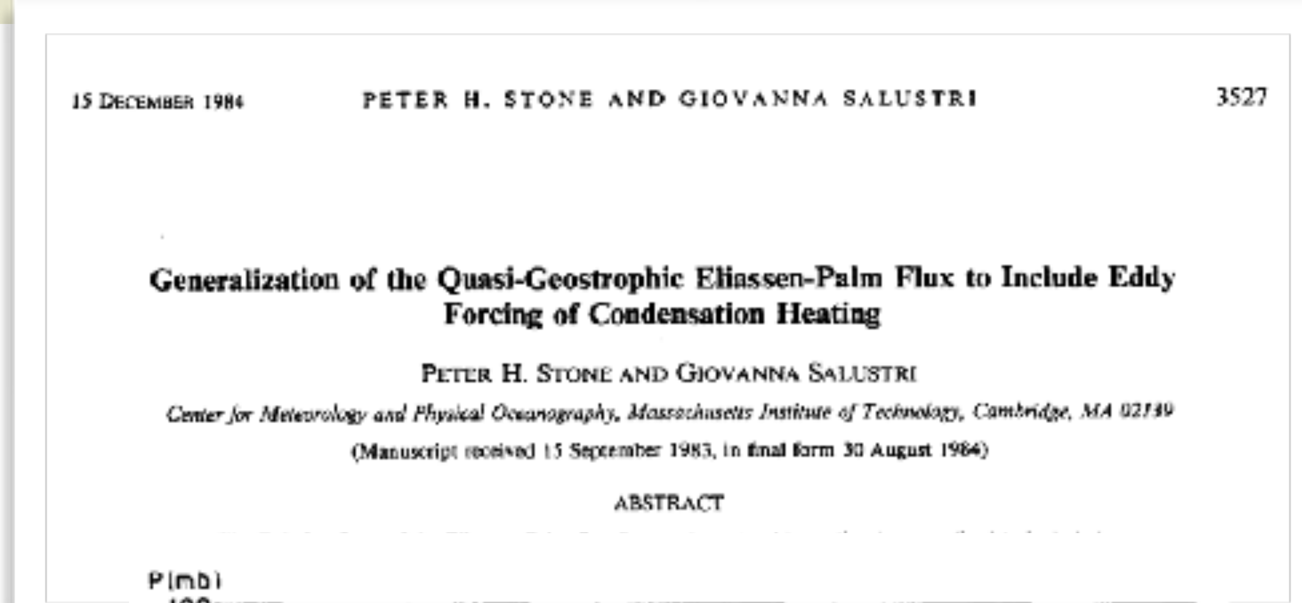
的比较如图所示，
具体可参看该参考文献

$$\mathcal{F}_m \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*] + \frac{L}{c_p}[v^* q^*]}{\partial/\partial p(\theta_s + \frac{L}{c_p} q_s)}\,\mathbf{k}$$

$$\frac{\partial \tilde{v}_m}{\partial y} + \frac{\partial \tilde{\omega}_m}{\partial p} = 0$$

$$\frac{\partial [u]}{\partial t} = f\tilde{v}_m + \nabla \cdot \mathcal{F}_m + [F_x]$$

$$\frac{\partial[\theta + \frac{L}{c_p} q']}{\partial t} = -\tilde{\omega}_m \frac{\partial}{\partial p}(\theta_s + \frac{L}{c_p} q_s) + \left(\frac{p_o}{p}\right)^{R/c_p} \frac{[Q_m]}{c_p}$$



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Generalization of the Quasi-Geostrophic Eliassen-Palm Flux to Include Eddy Forcing of Condensation Heating

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ABSTRACT