Assignments 2

In equilibrium,

$$Qs(x)\mathcal{A}(T) - I(T) + F(T) = 0$$

The snow line case: infrared cooling $I = A + BT$

$$F(T) = C(\bar{T} - T)$$

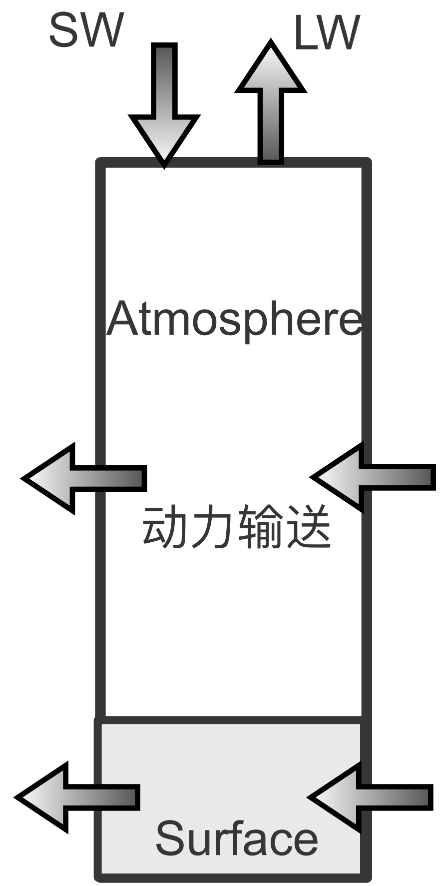
Assume:

$$\begin{aligned} \mathcal{A}(T) &= \quad \alpha = 0.4, \quad \text{for } T < T_{snow} \\ &= \quad \beta = 0.7, \quad \text{for } T > T_{snow} \\ &= \quad \tfrac{\alpha+\beta}{2}, \quad \text{for } T = T_{snow} \end{aligned}$$

$$T_{snow} = -10^oC$$

$$s(x) = 1 - 0.241(3x^2 - 1)$$

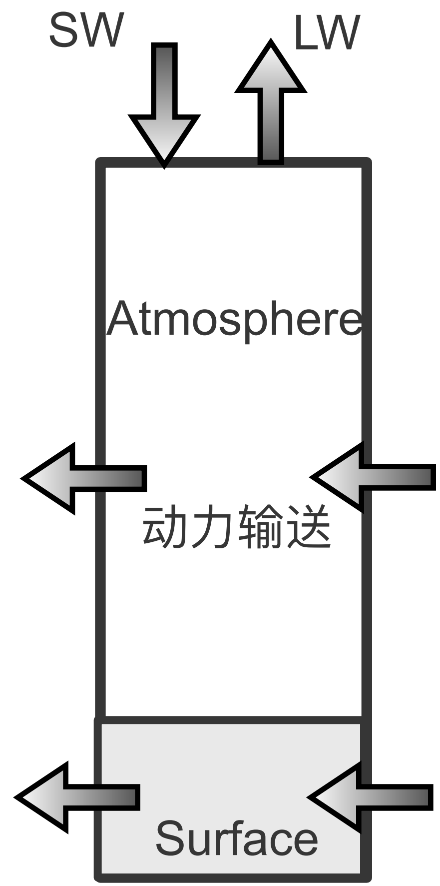
$$A = 211.1\,Wm^{-2}, \text{ and } B = 1.55\,Wm^{-2}(^oC)^{-1}$$



Simple energy balance climate models

In equilibrium,

$$Qs(x)\mathcal{A}(T) - I(T) + F(T) = 0$$



The snow line case - HOMEWORK#2:

- Let $\alpha = 0.45$. Keep A and B the same.
  - Determine β such that $\bar{T}$ remains unchanged for current climate (x_s and Q_o);
  - Determine C for the above choice of α and β ;
  - Compute Q(xs);
  - Discuss any difference between these results and those obtained for $\alpha = 0.4$, $\beta = 0.7$. In particular, how has the global stability changed and why?

Determine the value of β

In equilibrium,

$$Qs(x)\mathcal{A}(T) - I(T) + F(T) = 0$$

The snow line case: infrared cooling $I = A + BT$

$$F(T) = C(\bar{T} - T)$$

Hemisphere average:

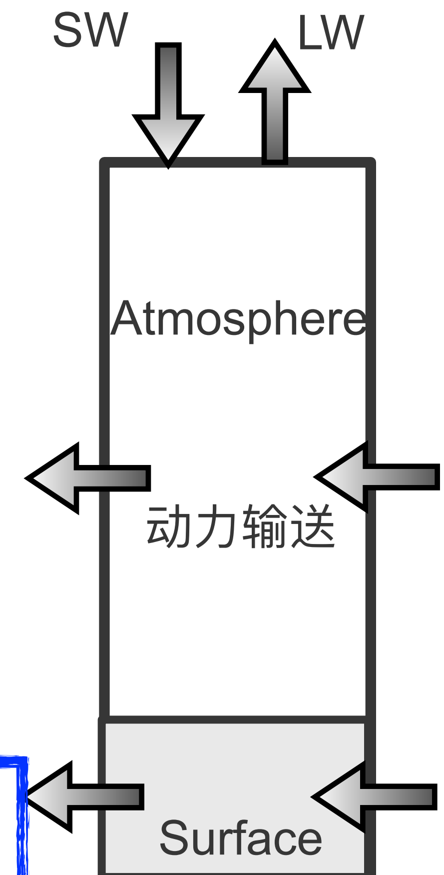
$$\bar{T} = \int_0^1 T dx \quad \bar{I} = \int_0^1 I dx \quad F(I) = (C/B)(\bar{I} - I)$$

Radiation balance

Unchanged

$\beta = 0.698$

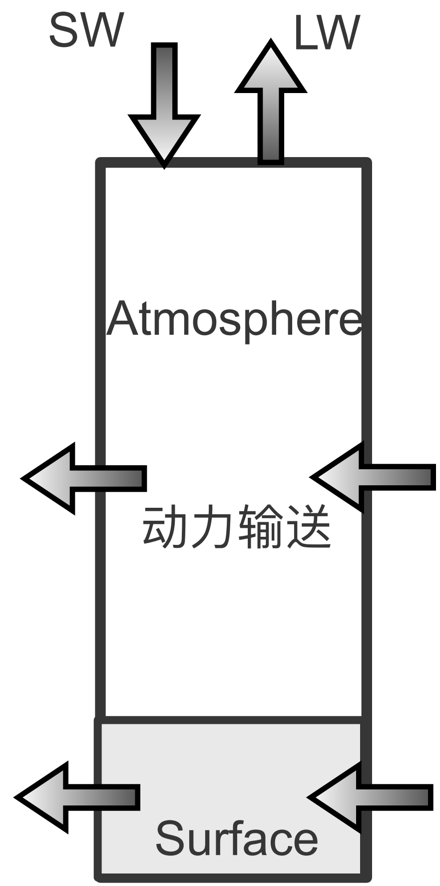
$$\begin{aligned} \bar{I}/Q &= \int_0^1 s(x)\mathcal{A}(x)dx \\ &= (\beta - \alpha)(1.241x_s - 0.241x_s^3) + \alpha \end{aligned}$$



Determine the value of C

In equilibrium,

$$Qs(x)\mathcal{A}(T) - I(T) + F(T) = 0$$



The snow line case:

$$I/Q = \frac{\frac{C}{B}\bar{I}/Q + s(x)\mathcal{A}(x, x_s)}{1 + \frac{C}{B}}$$

Determine C using current climate:

$$I(x_s) = I(0.95) = I(T_{snow}), \text{ get the value of } \frac{C}{B}$$

$$\frac{C}{B} = \frac{\frac{I_{snow}}{Q_o} - \frac{\alpha+\beta}{2}s(x_s)}{\frac{\bar{I}}{Q_o} - \frac{I_{snow}}{Q_o}}$$

$Q_o = 340\,Wm^{-2},\ C = 3.16$ 相较于原来3.34，减弱

Determine the value of C

In equilibrium,

$$Qs(x)\mathcal{A}(T) - I(T) + F(T) = 0$$

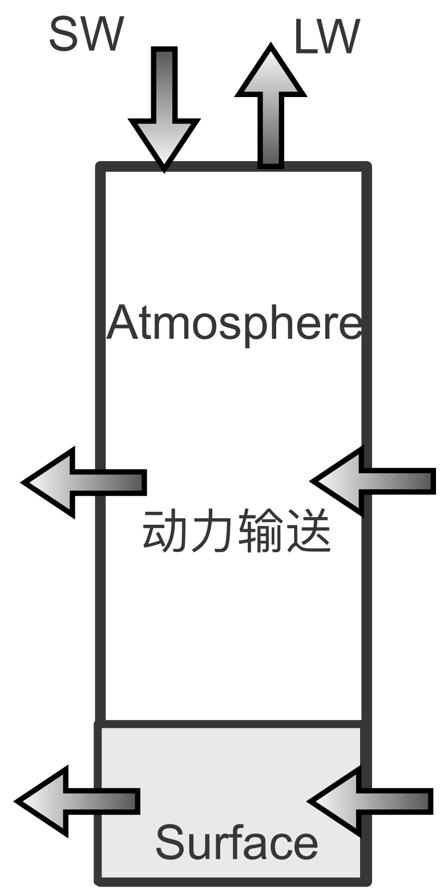
The snow line case:

$$I/Q = \frac{\frac{C}{B}\bar{I}/Q + s(x)\mathcal{A}(x, x_s)}{1 + \frac{C}{B}}$$

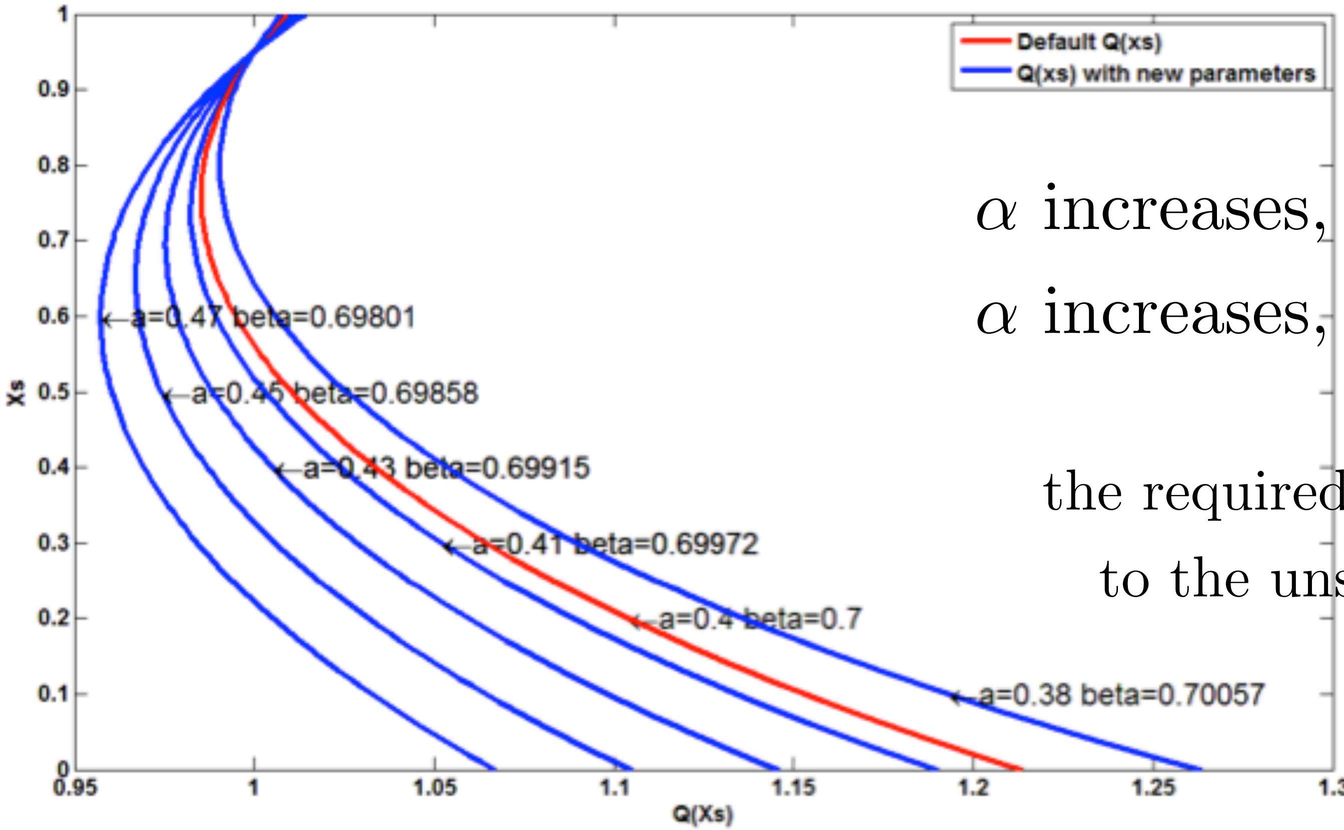
Then

$$Q(x_s) = \frac{(1 + \frac{C}{B})(A + BT_{snow})}{\frac{C}{B}\bar{I}/Q + s(x_s)\frac{\alpha+\beta}{2}}$$

$$\begin{aligned}\bar{I}/Q &= \int_0^1 s(x)\mathcal{A}(x)dx \\ &= (\beta - \alpha)(1.241x_s - 0.241x_s^3) + \alpha\end{aligned}$$



Plot $Q(x_s)$, note the choice of Q_o



α increases, β decreases

α increases, C decreases

the required variation of Q to the unstable increases

System becomes more stable !

Assignments 2

Question #2

假设在大气层顶（TOA），在多年全年平均的情况下，入射的太阳辐射随纬度的分布满足

$$
\begin{aligned}
Q &= Q_o * s(x) \\
s(x) &= s_o P_o(x) + s_2 P_2(x)
\end{aligned}
$$

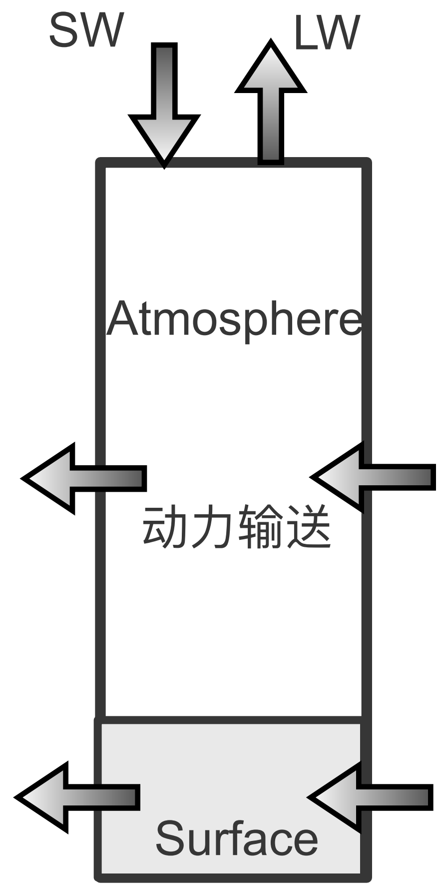
其中

$$P_o(x) = 1, P_2(x) = \frac{1}{2}(3x^2 - 1), s_o = 1, s_2 = -0.473, x = sin\phi,$$

大气层顶的向外净长波辐射为I，其随纬度的分布满足

$$
\begin{aligned}
I &= Q_o * i(x) \\
i(x) &= i_o P_o(x) + i_2 P_2(x) \qquad i_o = 0.687, i_2 = -0.165
\end{aligned}
$$

假设行星反照率为α，且不随经向和纬向变化。请写出在能量平衡的情况下，大气和海洋的总经向能量传输应满足什么条件？按此条件，请利用合适的边界条件，推导出大气和海洋的总经向能量输送将如何随纬度φ和太阳辐射强度Q_o变化？



Assignments 2

Question #2

能量平衡满足：

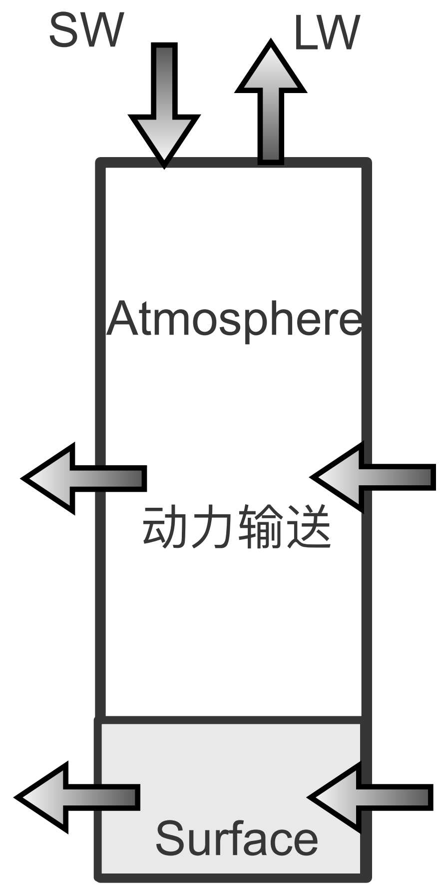
$$Q_o \cdot s(x) \cdot (1-\alpha) - I + F = 0$$

F - 经向能量输送的辐合辐散

or

$$Q_o s(x)(1-\alpha) - I = F_{rad} = \frac{1}{2\pi a^2 \cos\phi} \frac{\partial}{\partial \phi} f(\phi)$$

$f(\phi)$ — meridional energy transport by atmosphere and oceans



Assignments 2

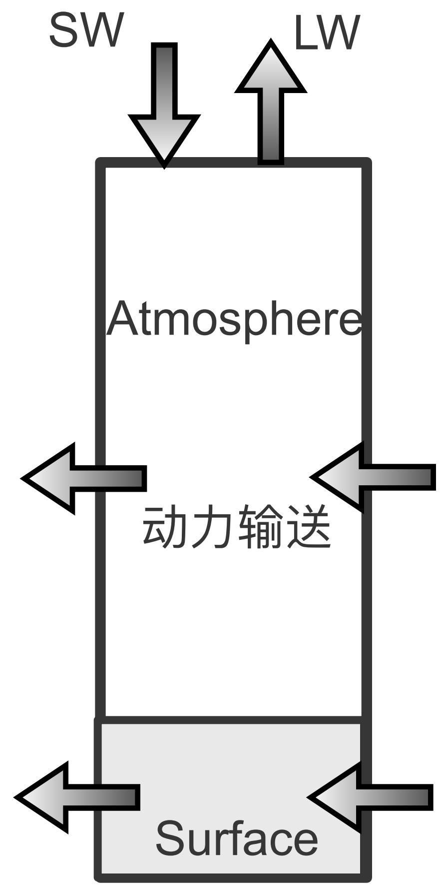
Question #2

全球积分的长波和短波能量应该相等：

$$2\pi a^2 \int_{-1}^{1} [Q_o s(x)(1-\alpha) - I]dx = 0$$

由上式可得，行星返照率

$$\alpha = 0.313$$



Assignments 2

Question #2

总经向能量传输

$$f = 2\pi a^2 \int_x^1 [Q_o\, s(x)(1-\alpha) - I]dx$$

代入可得：

$$f = 0.16\pi a^2 Q_o(sin\phi - sin^3\phi) + C$$

利用边界条件，在极地，经向传输为0， 可得 C=0。