



第六章：

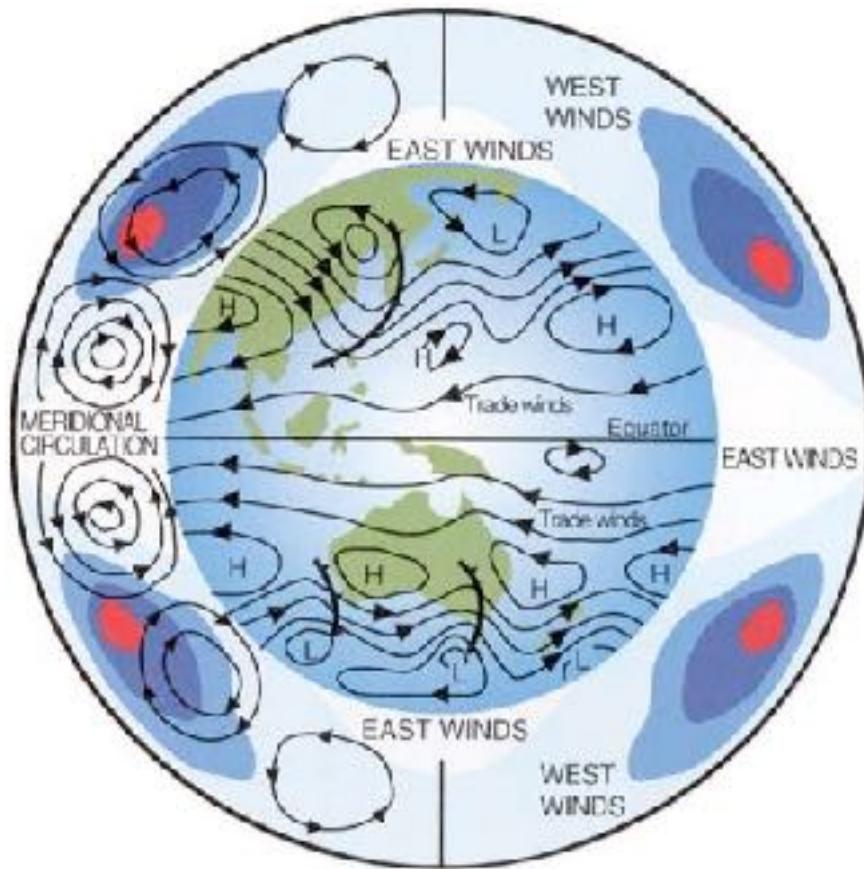
能量与水汽的 分布、平衡与输送

授课教师： 张洋

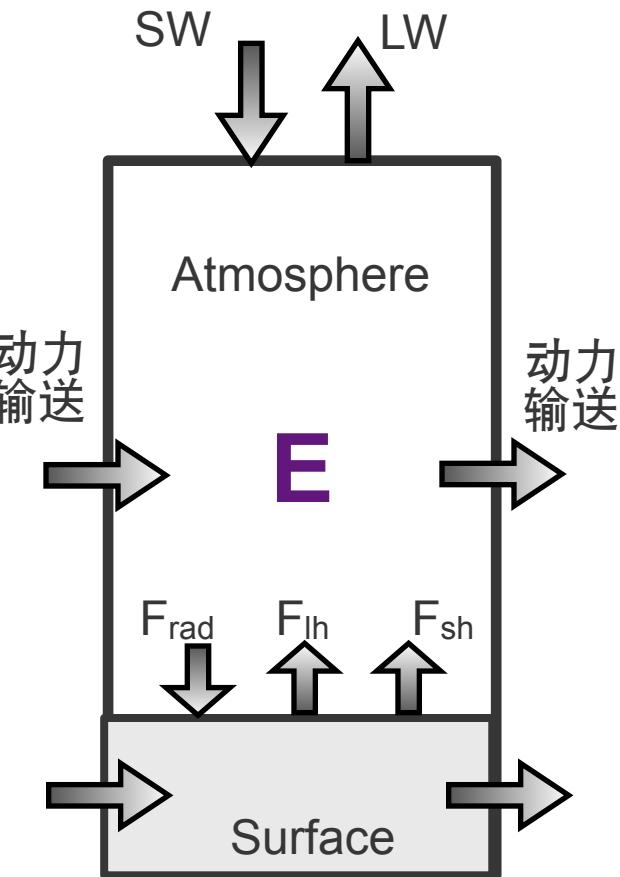
2021. 12. 14



Energy and water vapor



At any latitude:





Outline



- Total energy
 - Distribution of each component
 - Water vapor and latent heat
 - Budget equations
 - Transports



Basic forms of energy



- Kinetic energy (动能): $K = \frac{1}{2}(u^2 + v^2 + w^2) \approx \frac{1}{2}(u^2 + v^2)$
- Internal energy (内能): $I = c_v T$
- Gravitational-potential energy (位能): $\Phi = gz$
- Total potential energy: $\int_0^\infty \rho(I + \Phi) dz = \frac{1}{g} \int_0^{p_s} c_p T dp$
- Latent energy (相变潜热能): $LH = Lq$
- Total energy: $E = I + \Phi + LH + K$

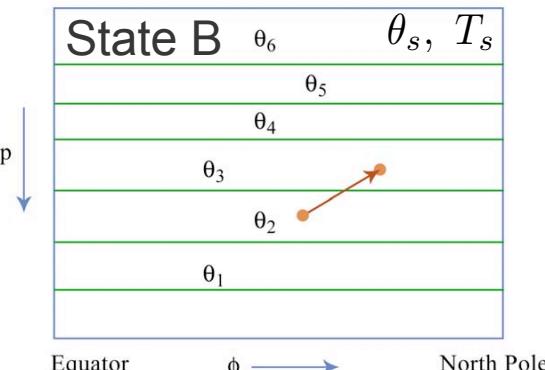
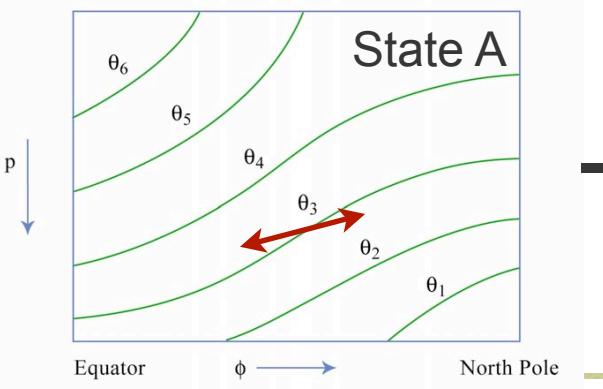


Basic forms of energy



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- Total potential energy:

$$\int_0^\infty \rho(I + \Phi) dz = \frac{1}{g} \int_0^{p_s} c_p T dp$$



= Available
potential energy



Distribution of each component



■ Total energy:

$$E = I + \Phi + LH + K$$

70.4%	27.1%	2.5%	0.05%
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However, it represents a considerable fraction of the energy *available* for the general circulation.

However, only **0.5%** are *available* to be converted for the general circulation.

Only **10%** of the available potential energy converted to KE. The atmosphere is a low efficiency heat engine.



Distribution of each component



■ Total energy:

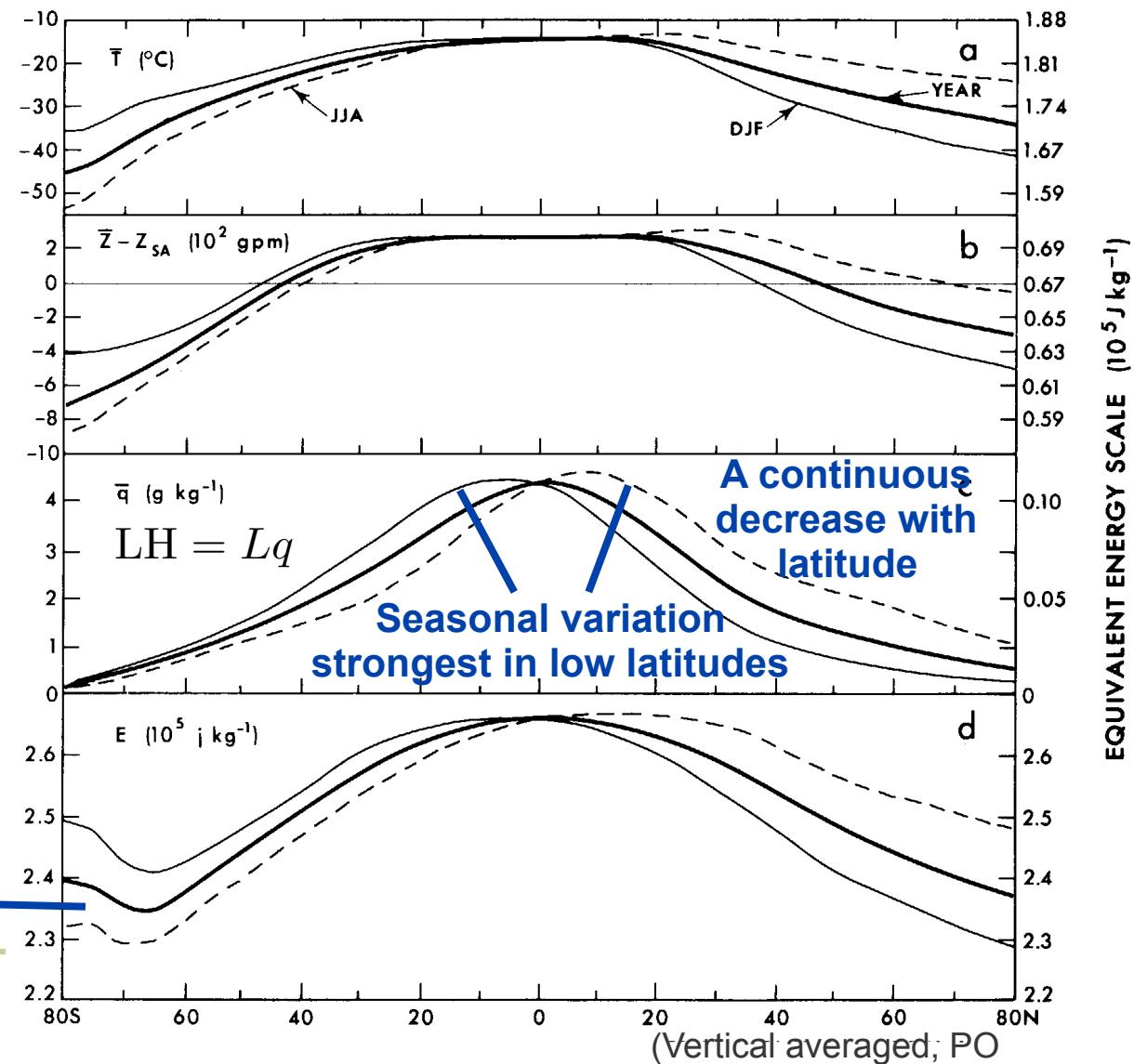
$$E = I + \Phi + LH + K$$

70.4% 27.1% 2.5% 0.05%

However, only **0.5%** are **available** to be converted for the general circulation.

■ Meridional distribution

Due to the topography





Basic forms of energy



■ Basic forms of energy:

■ **Kinetic energy (动能):** $K = \frac{1}{2}(u^2 + v^2 + w^2) \approx \frac{1}{2}(u^2 + v^2)$

$$\int_0^\infty \rho(I + \Phi)dz = \frac{1}{g} \int_0^{p_s} (c_v T + RT)dp = \frac{1}{g} \int_0^{p_s} c_p T dp$$

■ **Available potential energy (有效位能):**

The “approximate” expression of Lorenz (1955): $P \propto \frac{1}{V} \int_V \left(\frac{\theta - \theta_s}{\theta_s} \right)^2 dV$

$$P = \frac{1}{2} \int_0^{p_s} \frac{T_s}{\gamma_d - \gamma_s} \left(\frac{T - T_s}{T_s} \right)^2 dp = \frac{c_p}{2g} \int_0^{p_s} \Gamma (T - T_s)^2 dp$$

$$\Gamma = (\gamma_d/T_s) (\gamma_d - \gamma_s)^{-1} = -\frac{R}{c_p p} \left(\frac{p_s}{p} \right)^{\frac{R}{c_p}} \left(\frac{\partial \theta_s}{\partial p} \right)^{-1}$$



Distribution of each component

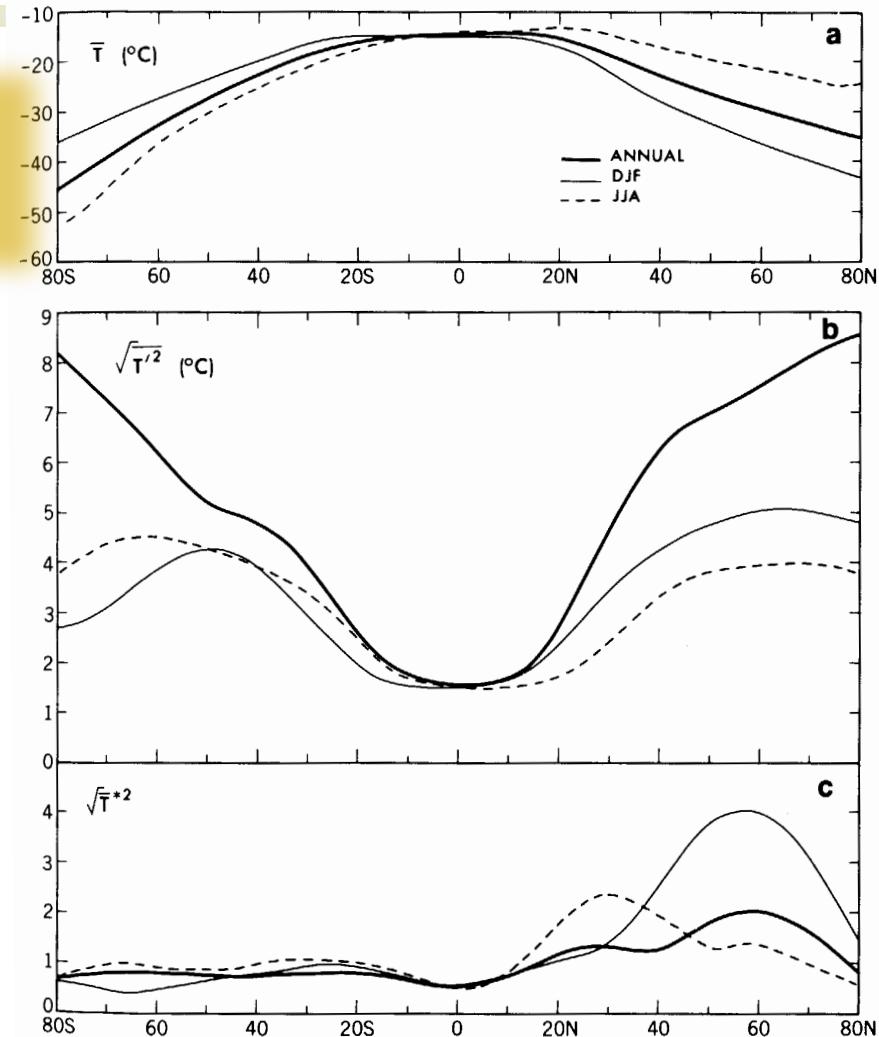
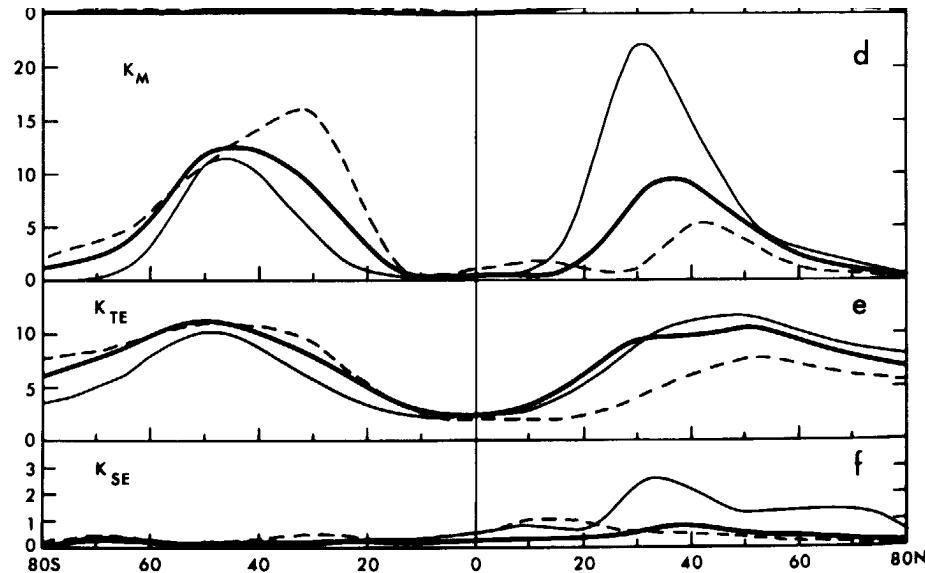


■ Total energy:

The available
energy:

$$E = I + \Phi + LH + K$$

70.4% 27.1% 2.5% 0.05%





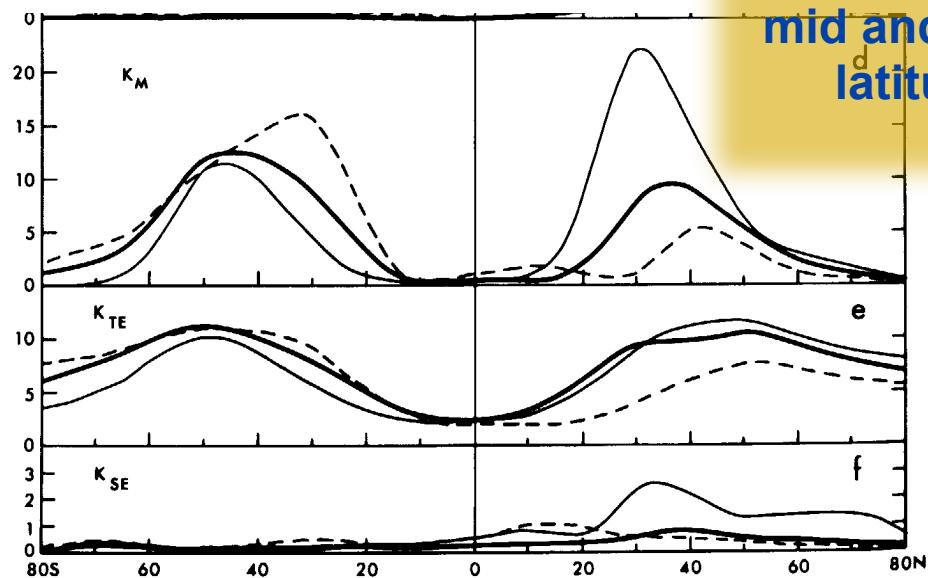
Distribution of each component



■ Total energy:

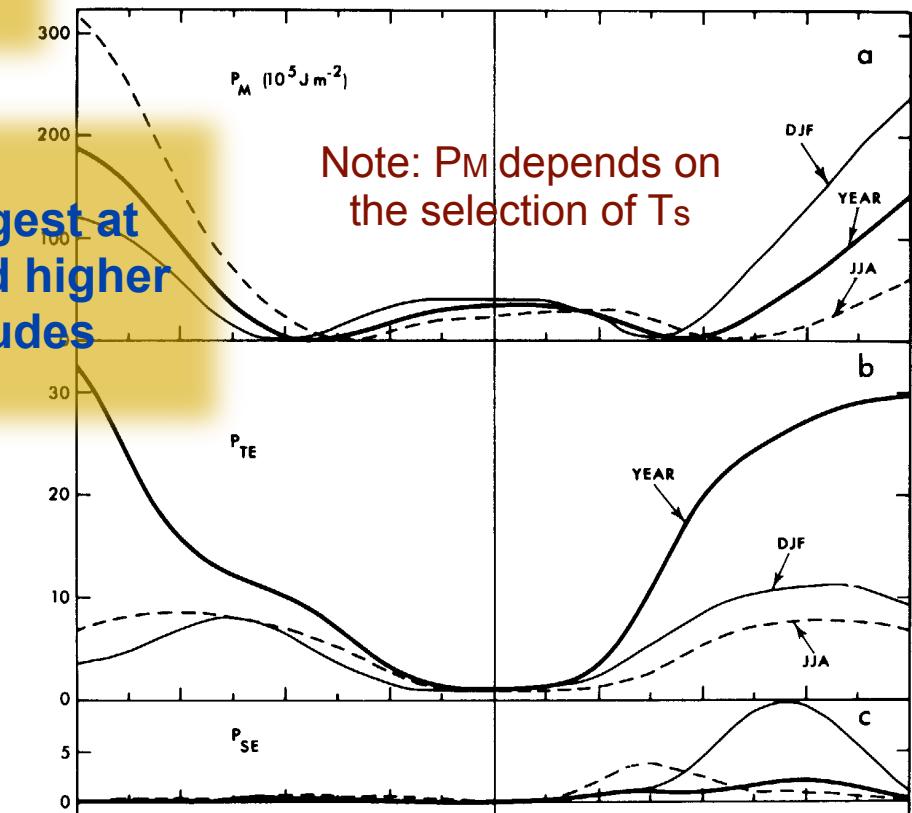
$$E = I + \Phi + LH + K$$

70.4% 27.1% 2.5% 0.05%



The available
energy:

Strongest at
mid and higher
latitudes



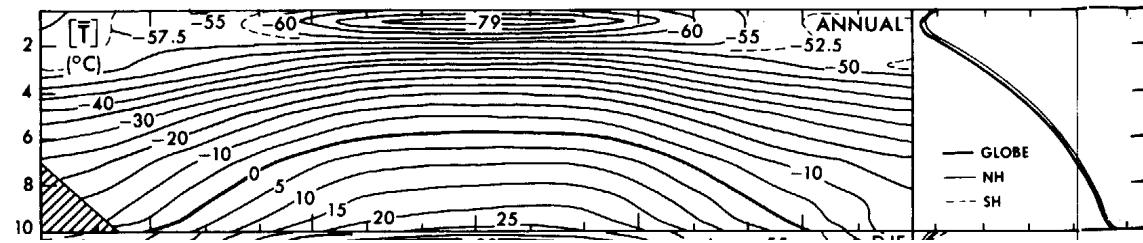


Distribution of each component



■ Total energy:

$$E = I + \Phi + LH + K$$



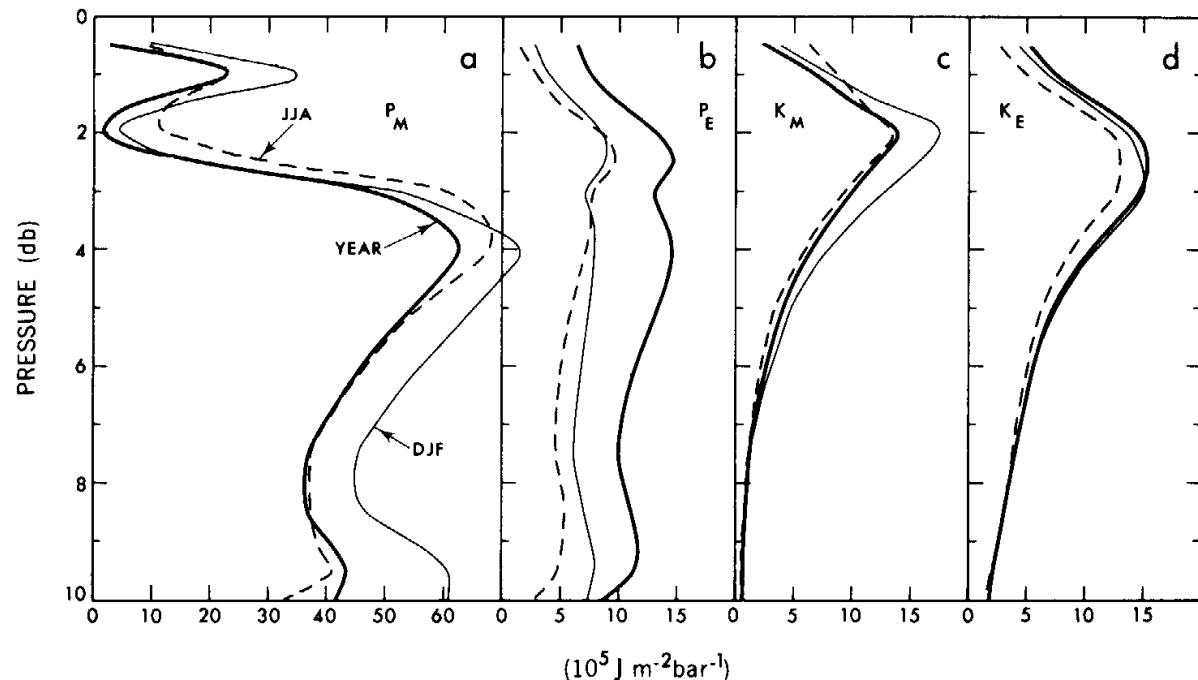
■ Vertical distribution:

Total energy:

decreases with height.

The available energy:

PE: peaks near tropopause and surface;
KE: strongest at tropopause.





Outline



- Total energy
 - Distribution of each component
 - Water vapor and latent heat
 - Budget equations
 - Transport



Latent heat and water vapor



■ Total energy:

$$E = I + \Phi + \text{LH} + K$$

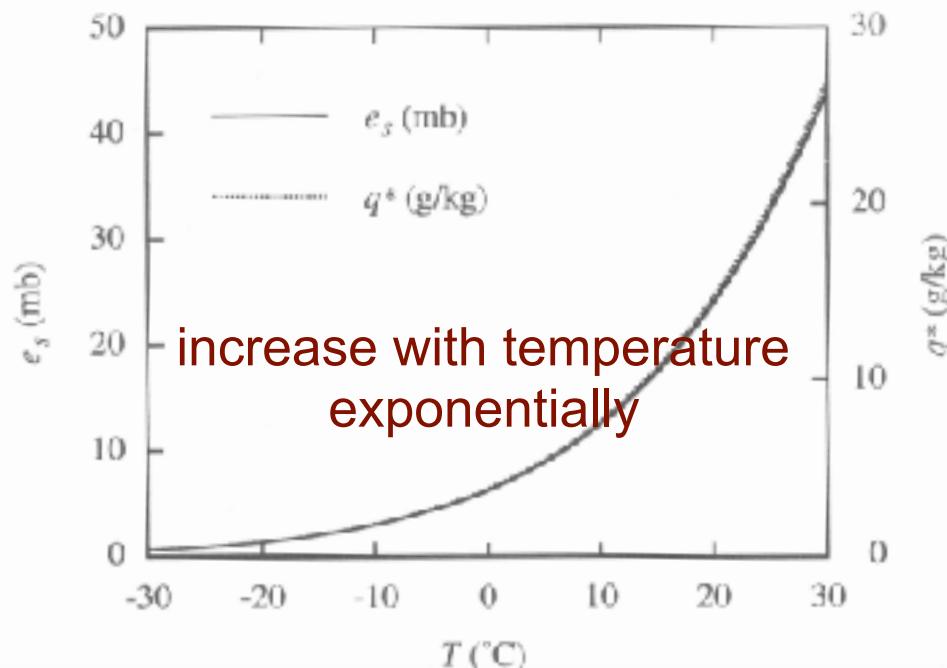
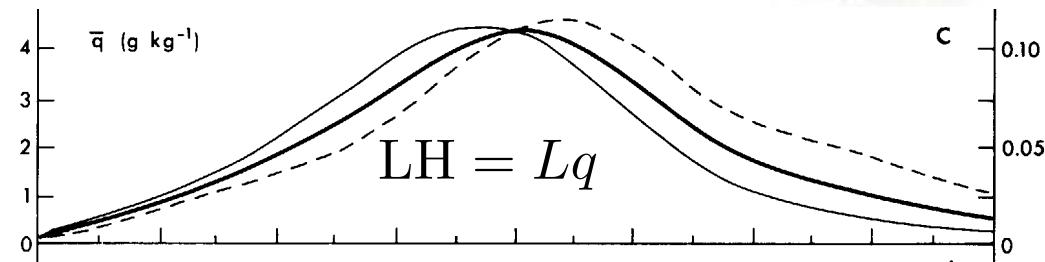
■ Clausis-Clapeyron relation:

$$\frac{de_s}{dT} = \frac{L}{T(\alpha_v - \alpha_l)} = \frac{Le_s}{R_v T^2}$$

$$e_s \cong 6.11 \times e^{\frac{L}{R_v} \left(\frac{1}{273} - \frac{1}{T} \right)}$$

$$q^* \approx 0.62 (e_s/p)$$

$$q = \text{RH} \times q^*$$





Latent heat and water vapor



■ Total energy:

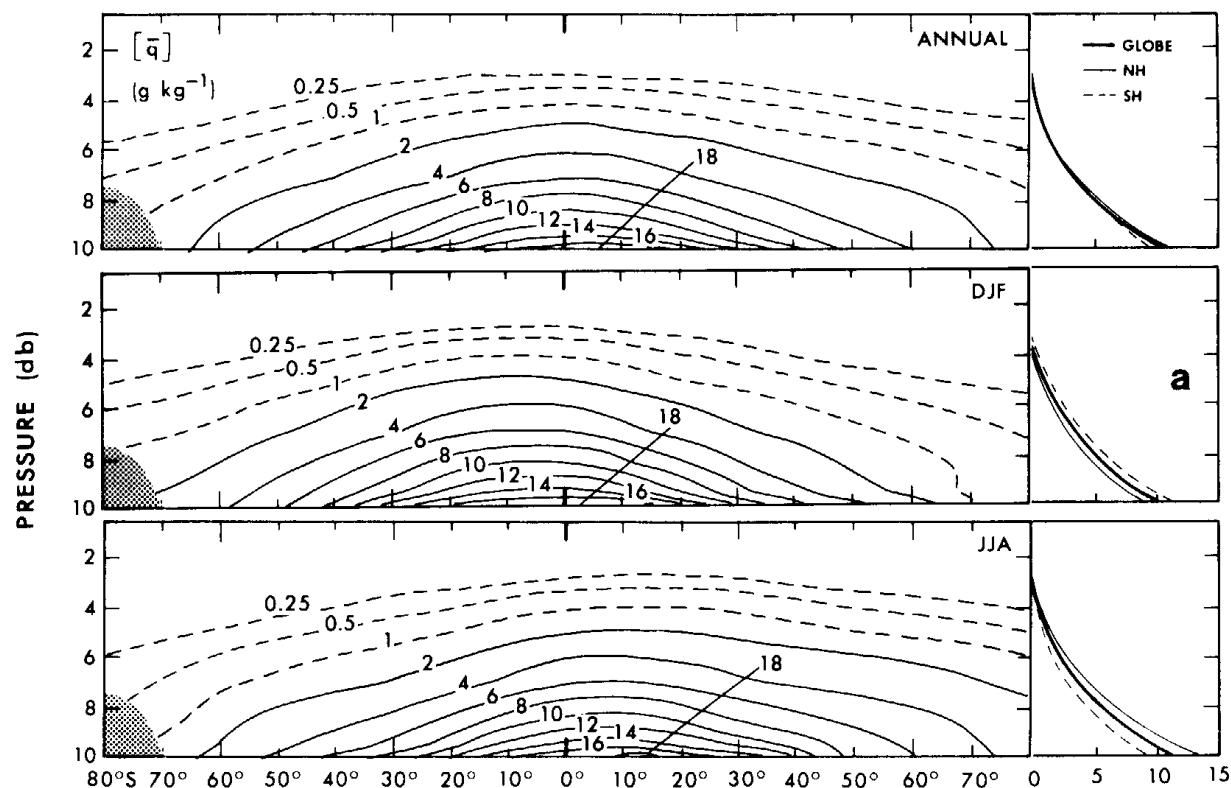
$$E = I + \Phi + \text{LH} + K$$

■ Vertical distribution:

- Decrease rapidly with height (almost exponentially);
- More than 50% water vapor is concentrated below 850 hPa(2km scale height);
- More than 90% water vapor is concentrated below 500 hPa;

$$\text{LH} = Lq \quad q = \text{RH} \times q^*$$

$$e_s \cong 6.11 \times e^{\frac{L}{R_v} \left(\frac{1}{273} - \frac{1}{T} \right)}$$





Latent heat and water vapor



■ Total energy:

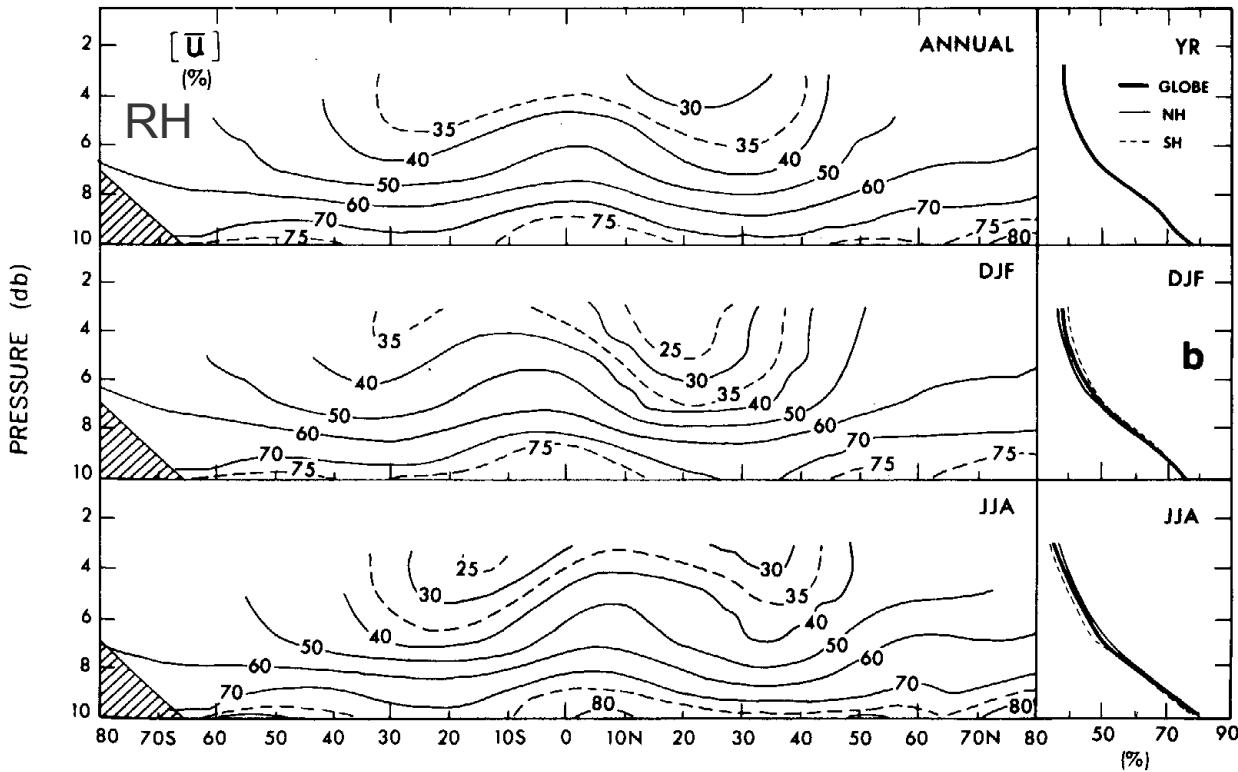
$$E = I + \Phi + \text{LH} + K$$

■ Distribution of RH:

- Vertically: the decrease with height is weaker than specific humidity(80%-30%)
- Meridional variations tend to increase with height, which shows close relation to the sinking and rising branches of the mean meridional variation.
- Obvious seasonal variation

$$\text{LH} = Lq \quad q = \text{RH} \times q^*$$

$$e_s \cong 6.11 \times e^{\frac{L}{R_v} \left(\frac{1}{273} - \frac{1}{T} \right)}$$





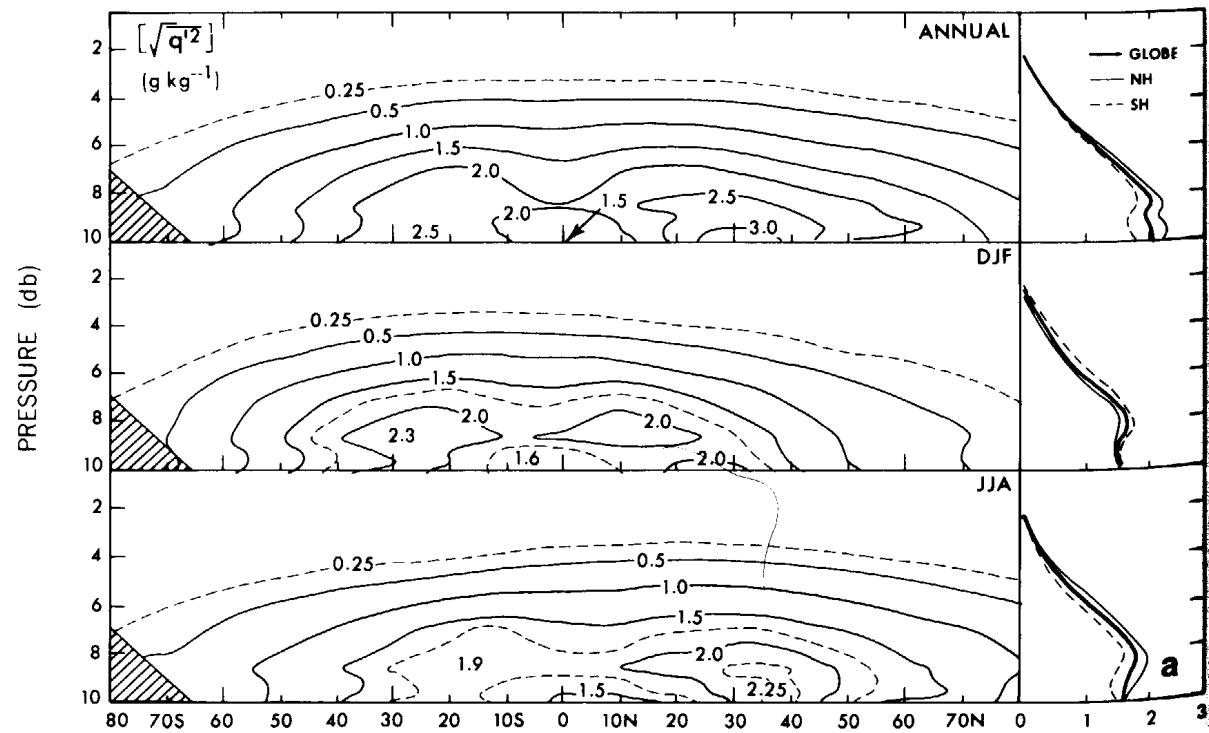
Latent heat and water vapor



- Variability of specific humidity in time (transient standard deviation):

$$LH = Lq \quad q = RH \times q^*$$

$$e_s \cong 6.11 \times e^{\frac{L}{R_v} \left(\frac{1}{273} - \frac{1}{T} \right)}$$



- Vertical: peak near the surface
- Meridional: peak around 20-30 degree
- Obvious seasonal variation



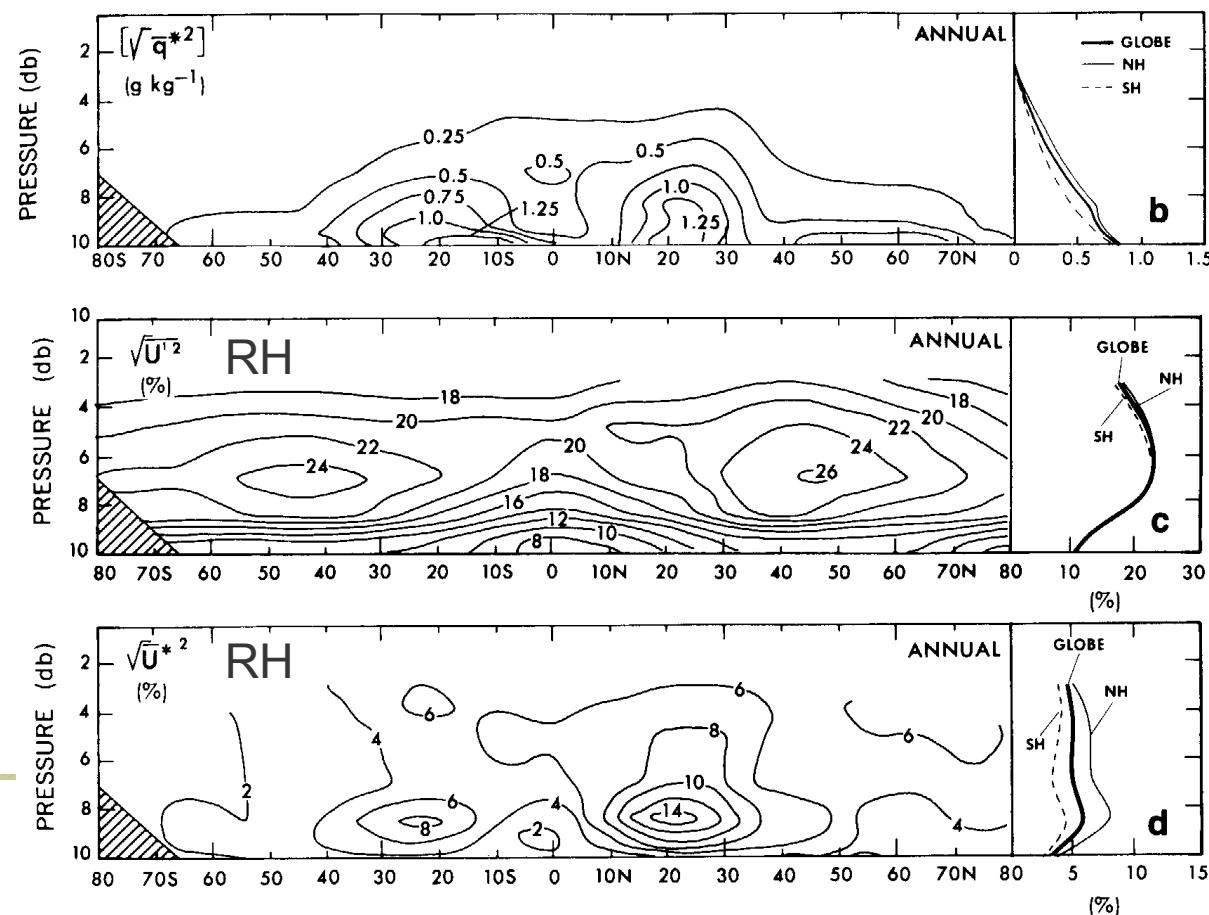
Latent heat and water vapor



- Variability of specific humidity and relative humidity in space and time:
- Stationary variation of q : peak near the surface, 20-30 degree
- Transient variation of RH: peak around 700 hPa, midlatitudes
- Stationary variation of RH: peak near surface, 20-30 degree

$$LH = Lq \quad q = RH \times q^*$$

$$e_s \cong 6.11 \times e^{\frac{L}{R_v} \left(\frac{1}{273} - \frac{1}{T} \right)}$$





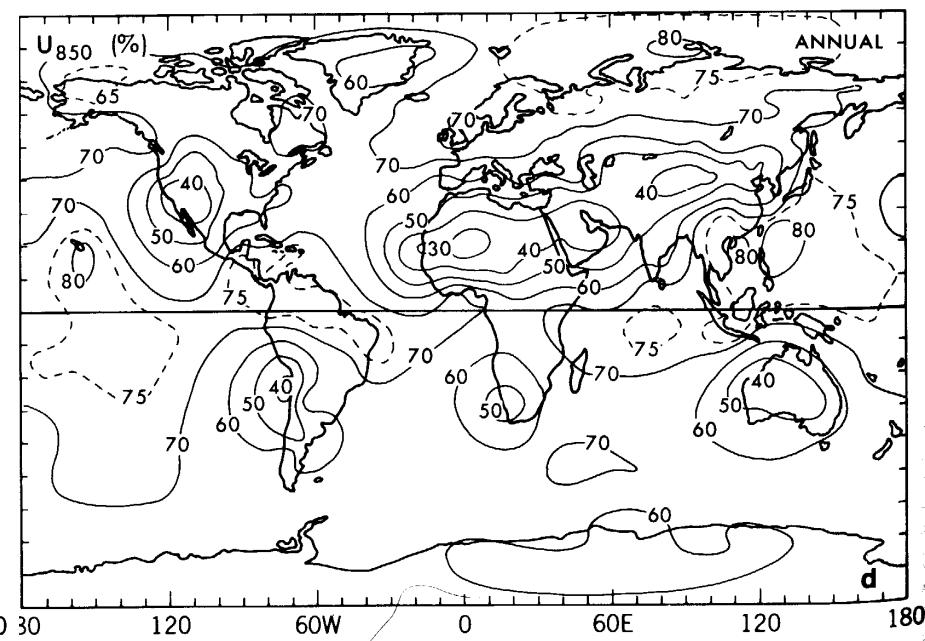
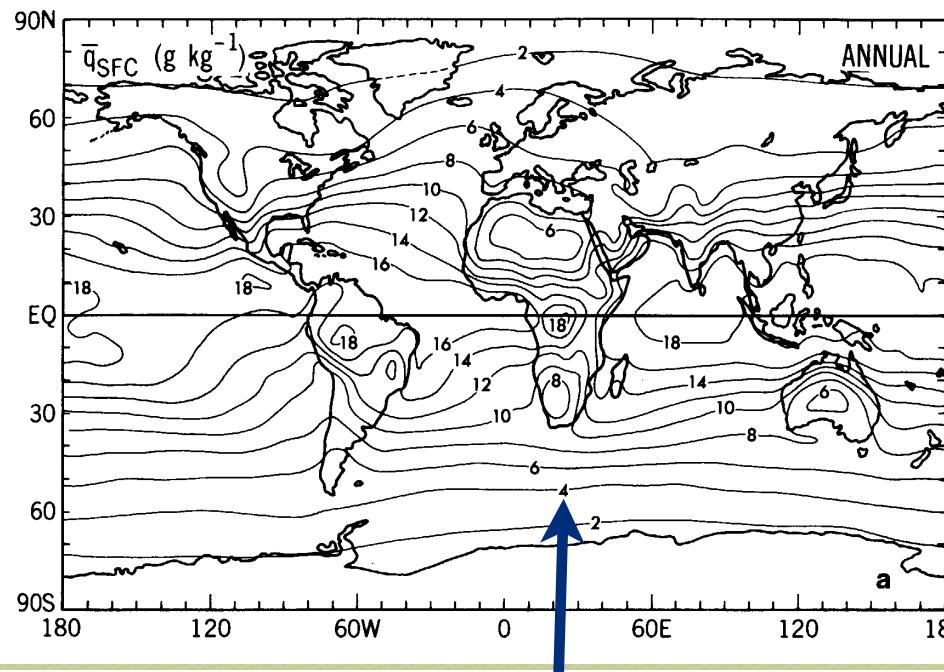
Latent heat and water vapor



Strong land-sea contrast in both specific humidity and relative humidity;
Over the ocean, RH-70% with weak spatial variation; Land, RH-30% at the mid and west coast of continents.

$$\text{LH} = Lq \quad q = \text{RH} \times q^*$$

$$e_s \cong 6.11 \times e^{\frac{L}{R_v} \left(\frac{1}{273} - \frac{1}{T} \right)}$$



Ocean: varies consistent with temperature



Outline



- Total energy
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 - Budget equations
 - Transport



The budget equations



■ Momentum equation:

$$\left(\frac{du}{dt} \right)_p - fv = - \left(\frac{\partial \Phi}{\partial x} \right)_p + F_x$$

$$\left(\frac{dv}{dt} \right)_p + fu = - \left(\frac{\partial \Phi}{\partial y} \right)_p + F_y$$

■ Continuity equation:

$$\nabla_p \cdot \mathbf{v} + \frac{\partial \omega}{\partial p} = 0$$

■ Thermodynamic equation:

$$\left(\frac{d \ln \theta}{dt} \right)_p = \frac{Q}{c_p T}$$

■ Water vapor (budget) equation:
q - specific humidity

$$\left(\frac{dq}{dt} \right)_p = s(q) + D$$

$$\left(\frac{d}{dt} \right)_p = \left(\frac{\partial}{\partial t} \right)_p + u \left(\frac{\partial}{\partial x} \right)_p + v \left(\frac{\partial}{\partial y} \right)_p + \omega \frac{\partial}{\partial p}$$



The budget equations



$$\left(\frac{dq}{dt} \right)_p = s(q) + D \quad \left(\frac{d \ln \theta}{dt} \right)_p = \frac{Q_{\text{RAD}} + Q_{\text{LH}} + Q_{\text{B}}}{c_p T}$$

D - molecular and turbulent eddy diffusion through the boundaries

$s(q)$ - source-sink term $s(q) = e - c$

e - the rate of evaporation per unit mass

c - the rate of condensation per unit mass

Diabatic heating $Q = Q_{\text{RAD}} + Q_{\text{LH}} + Q_{\text{B}}$

$$Q_{\text{LH}} = -L \left(\frac{dq}{dt} \right)$$

Q_{B} - diabatic heating due to boundary layer processes, i.e. sensible heat



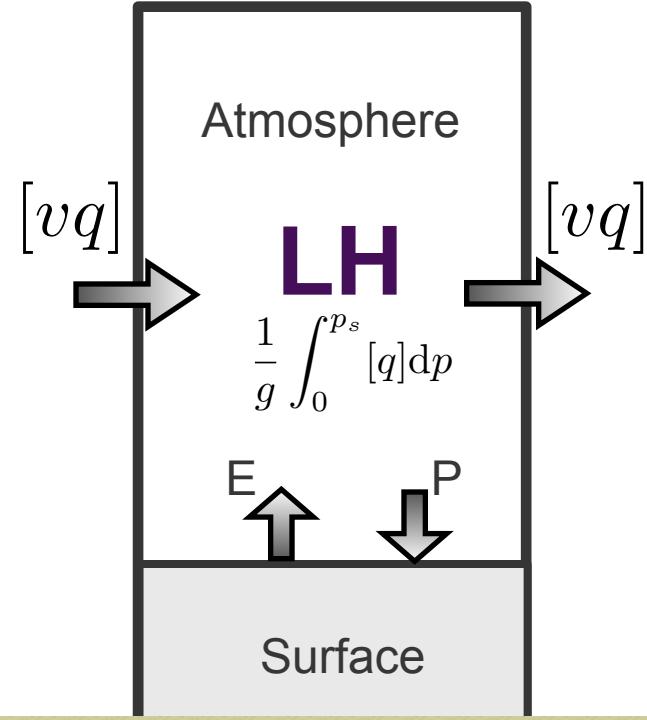
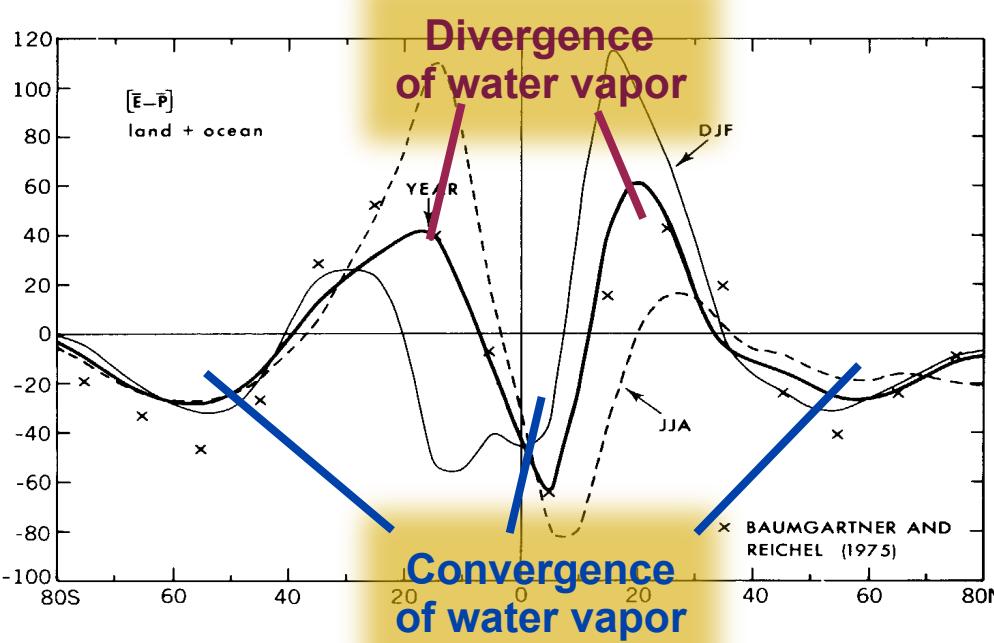
The budget equation of water vapor



$$\left(\frac{dq}{dt} \right)_p = s(q) + D \quad s(q) = e - c$$

Integrate above equation vertically and over a latitudinal belt:

$$\frac{\partial}{\partial t} \int_0^{p_s} [q] \frac{dp}{g} = - \frac{\partial}{\partial y} \int_0^{p_s} [vq] \frac{dp}{g} + [E - P]$$





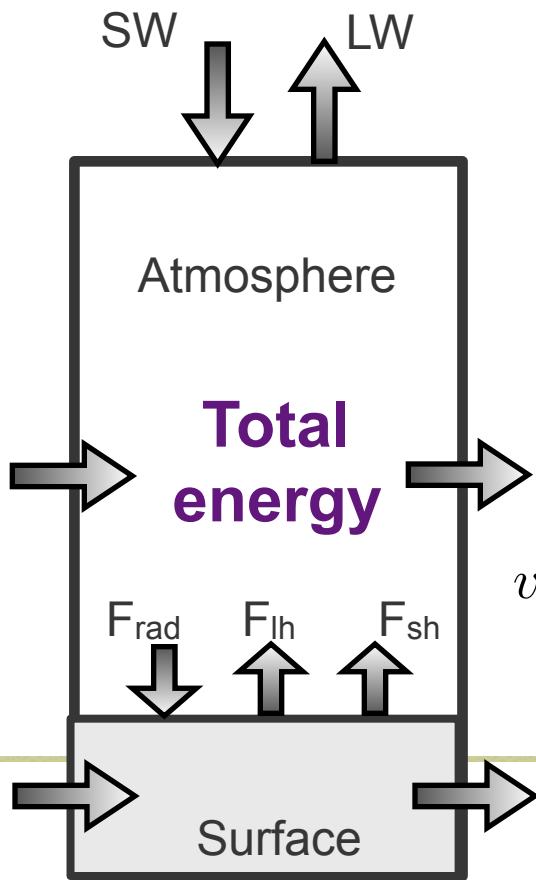
The energy budget



From the momentum, thermodynamic equation and the water vapor budget:

$$Q_{\text{RAD}} + Q_B \approx -g \frac{\partial}{\partial p} (F_{\text{rad}} + F_{\text{sh}})$$

$$\begin{aligned} \frac{\partial}{\partial t} \int_0^{p_s} (c_v T + gz + Lq + K) \frac{dp}{g} &= - \int_0^{p_s} \nabla \cdot \mathbf{v} (c_p T + gz + Lq + K) \frac{dp}{g} \\ &\quad + \int_0^{p_s} (Q + uF_x + vF_y) \frac{dp}{g} \end{aligned}$$



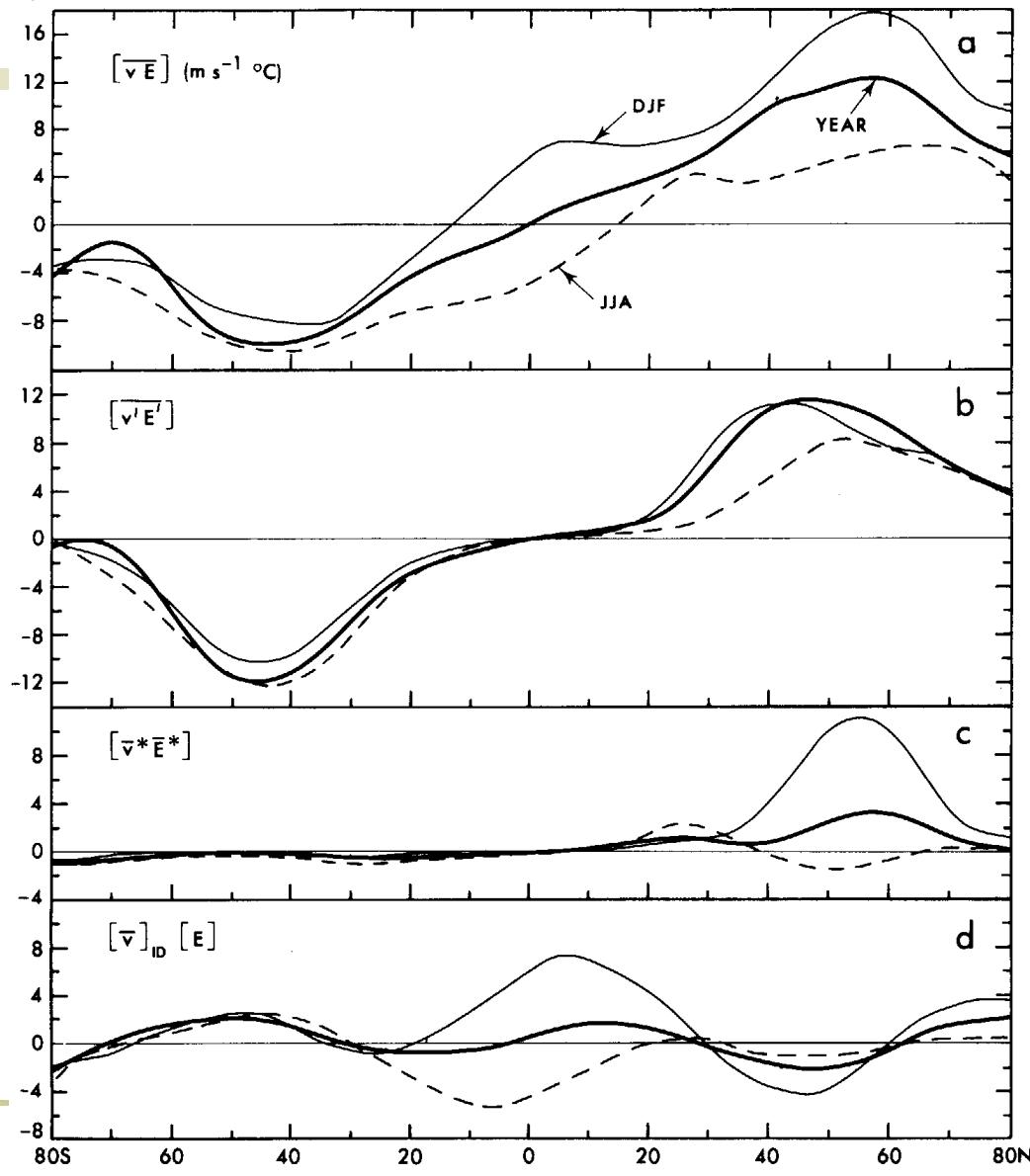
After simplification and zonal average:

$$\begin{aligned} \frac{\partial}{\partial t} \int_0^{p_s} [c_p T + Lq + K] \frac{dp}{g} &= - \frac{\partial}{\partial y} \int_0^{p_s} [v(c_p T + gz + Lq + K)] \\ &\quad + [F_{\text{rad}}]^{\text{top}} - [F_{\text{rad}} + F_{\text{sh}} + F_{\text{lh}}]^{\text{surf}} \end{aligned}$$



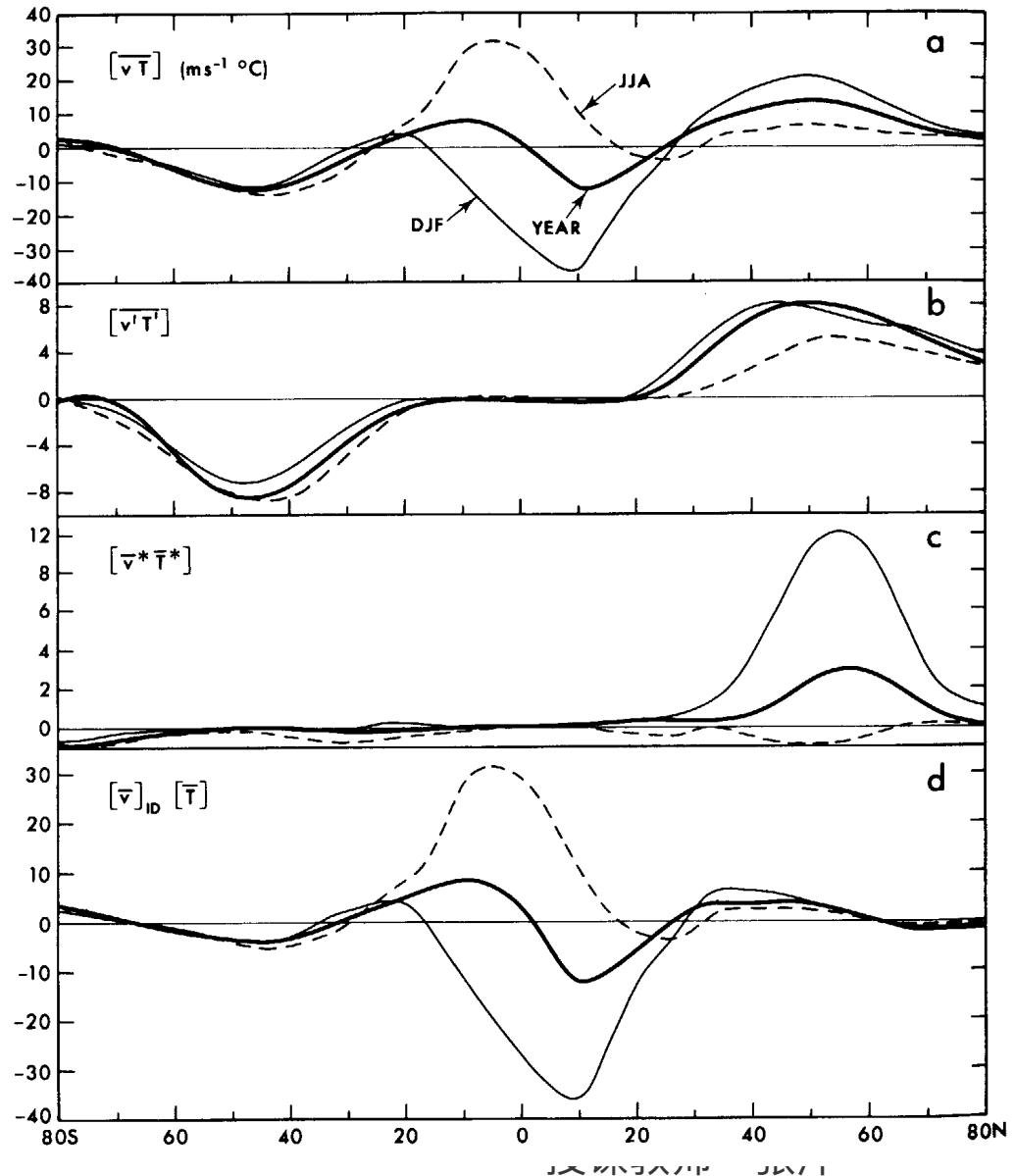
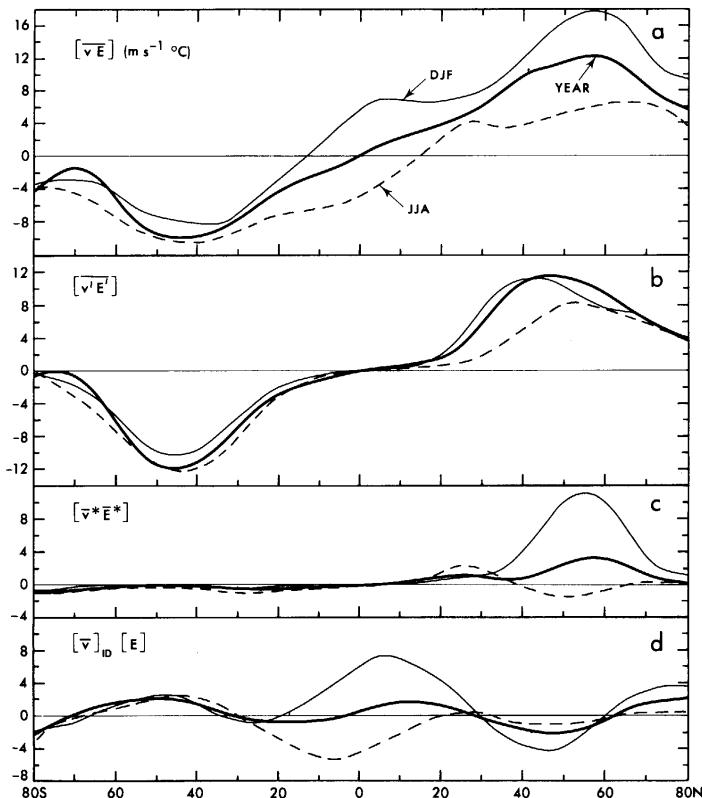
Total energy transport

- Strongest in midlatitudes;
- Contribution from transient eddies is dominant;
- Stationary eddies are only obvious in the N.H.
- Mean flow contribution is dominant in equatorial region, with strong seasonal variation





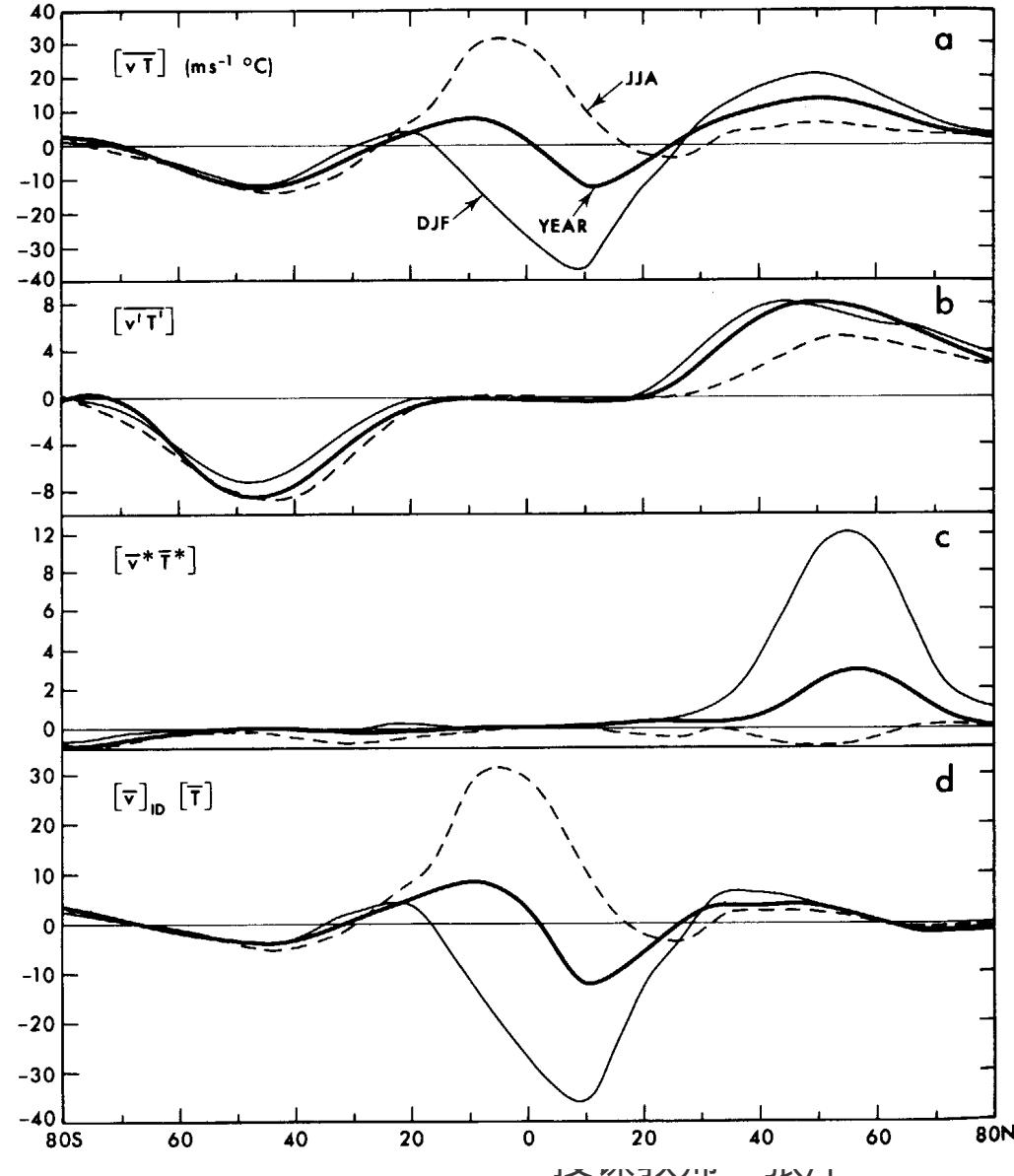
Sensible heat transport





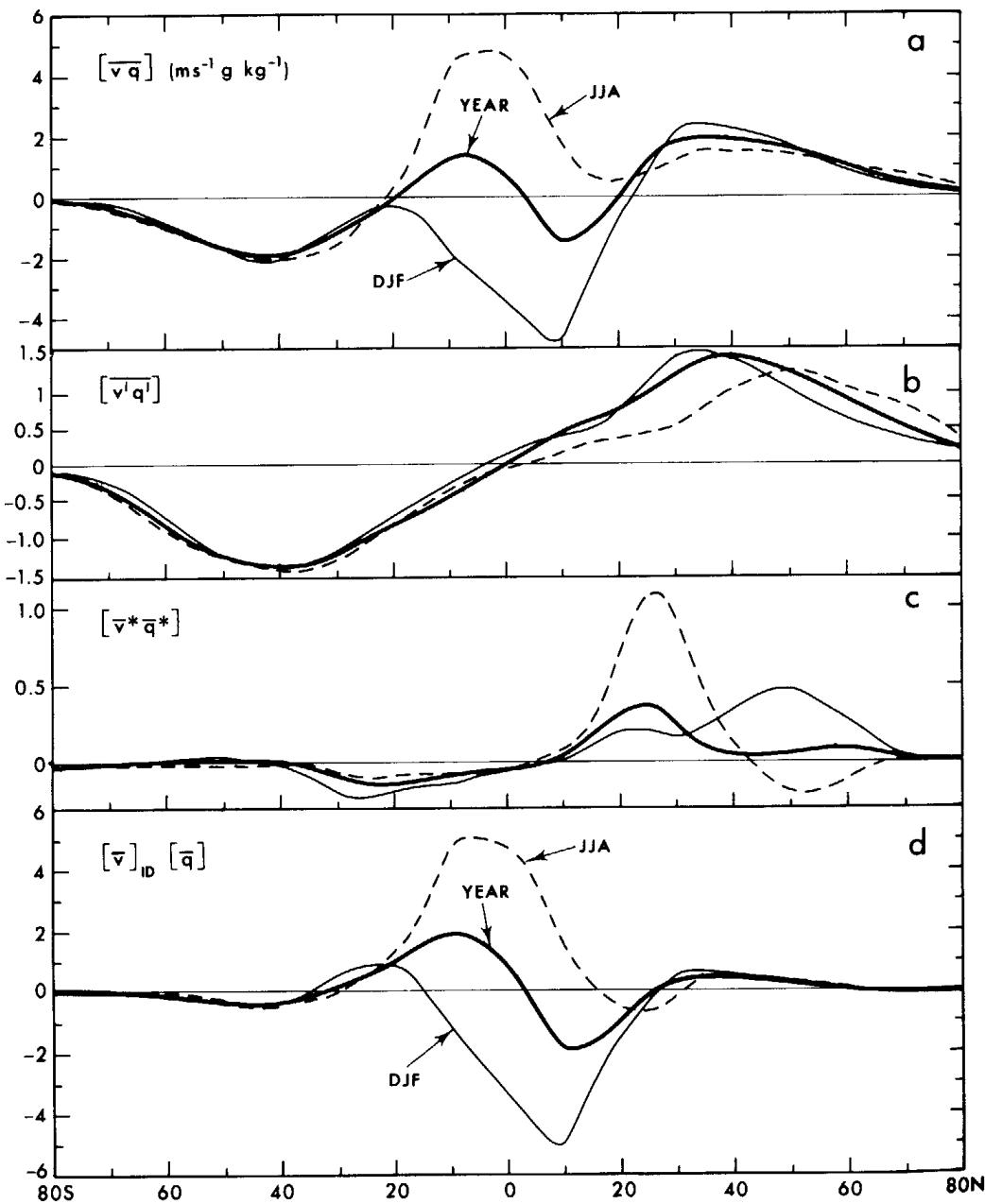
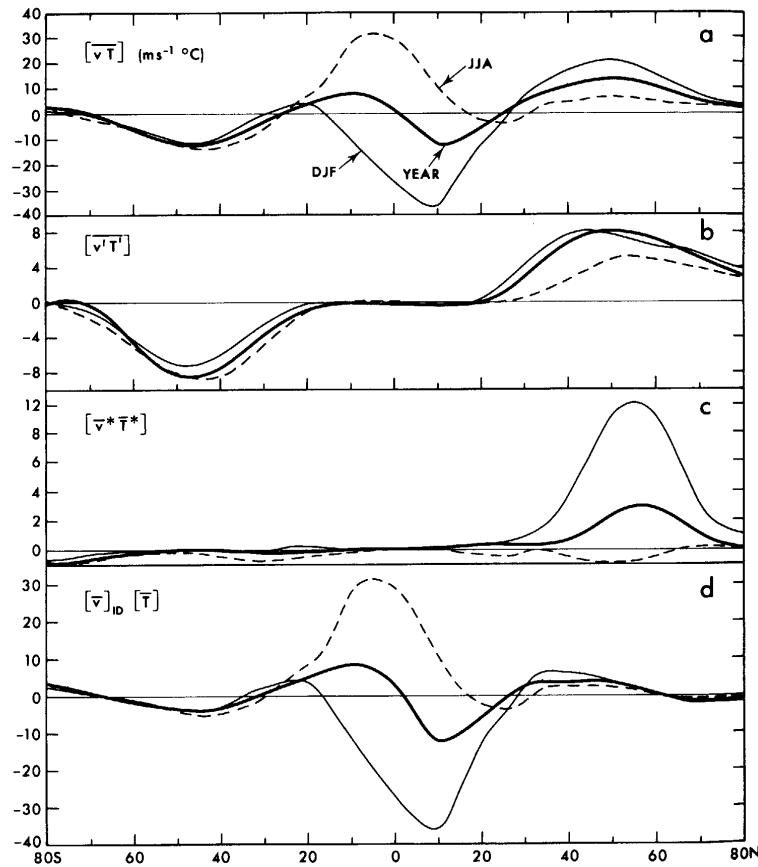
Sensible heat transport

- Strongest in both mid and low latitudes;
- Midlatitude: contribution from transient eddies is dominant;
- Stationary eddies are only obvious in the N.H.
- Mean flow contribution is dominant in equatorial region, with strong seasonal variation. But its direction is opposite to the total energy transport.





Latent heat transport



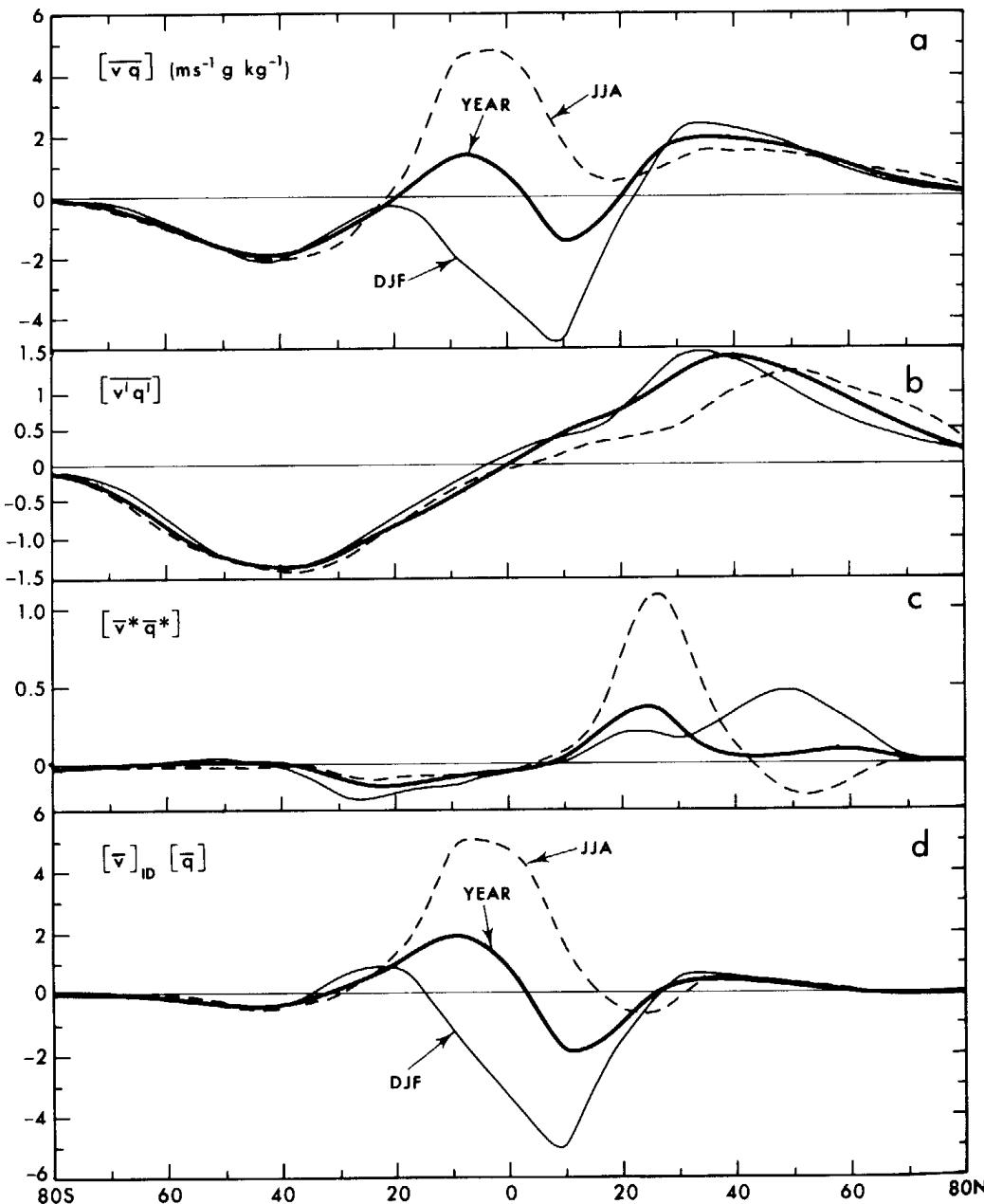


Latent heat transport

The Latitudinal distribution and seasonal variation of the latent heat transport are generally similar to the sensible heat transport.

Meridional moisture transport by stationary eddy: **strongest in summer in subtropics.**

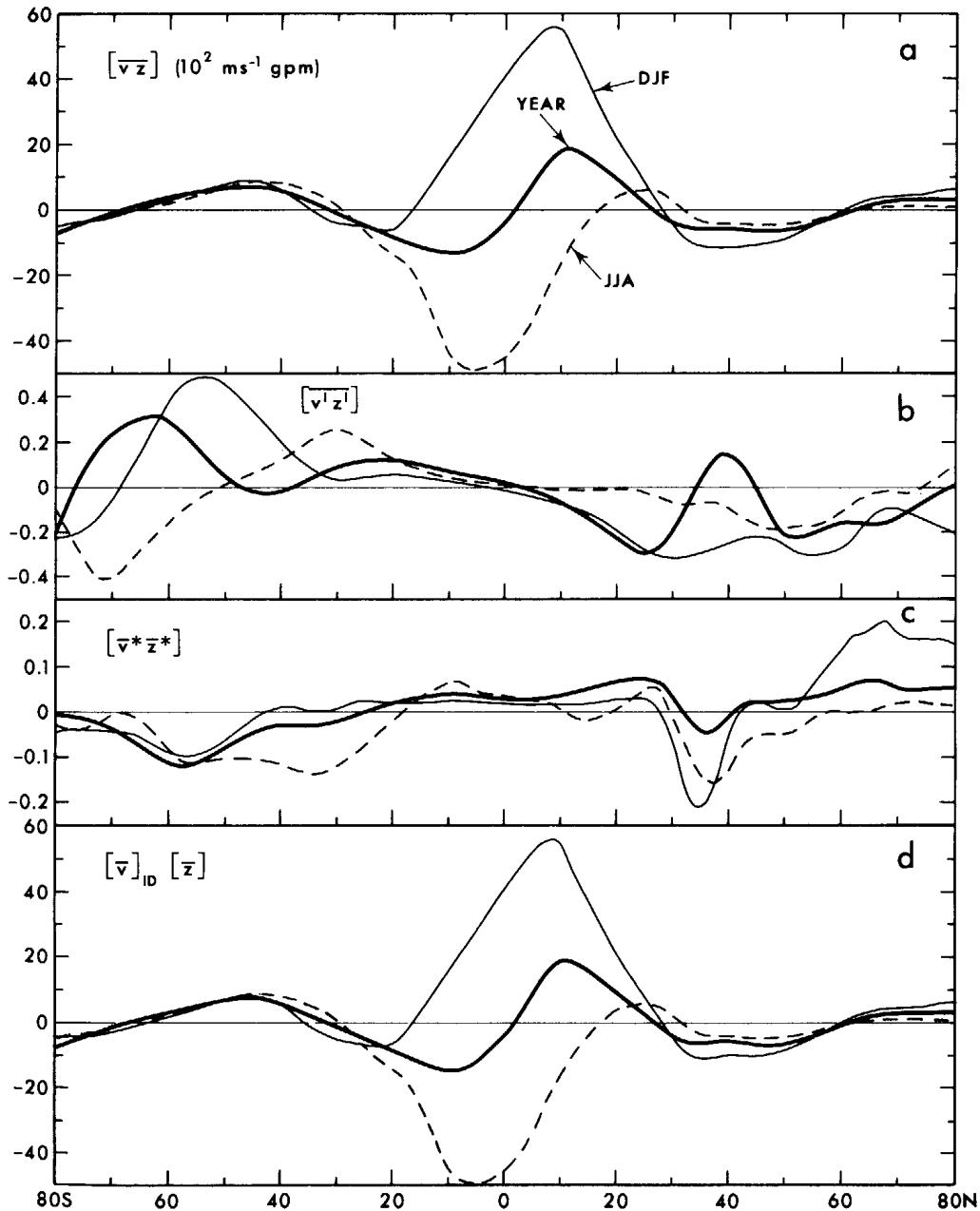
Transport by Hadley cell: **lower branch effective in transfer moisture into ITCZ.**





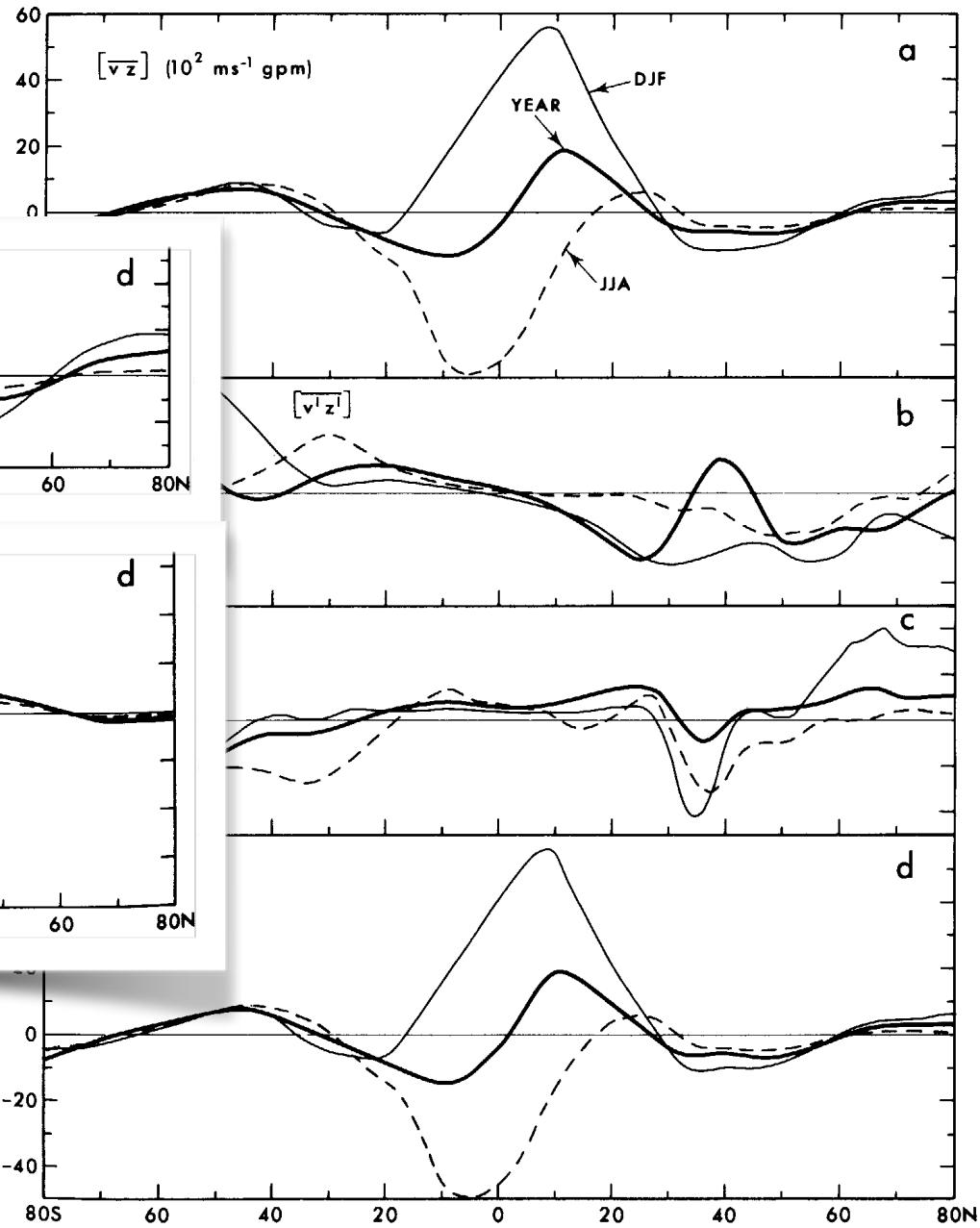
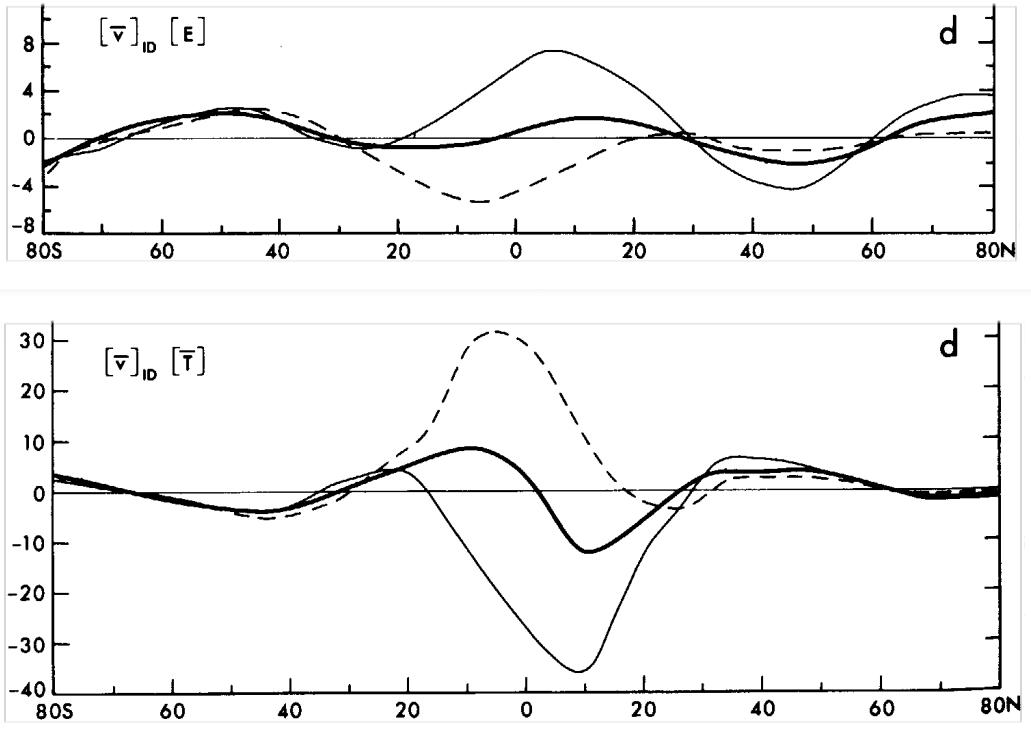
Gravitational-potential energy transport

- Strongest in low latitudes;
- The directions of energy transport in low latitudes are opposite to the sensible and latent heat transport;
- Zonal mean circulation transport dominant;





Gravitational-potential energy transport



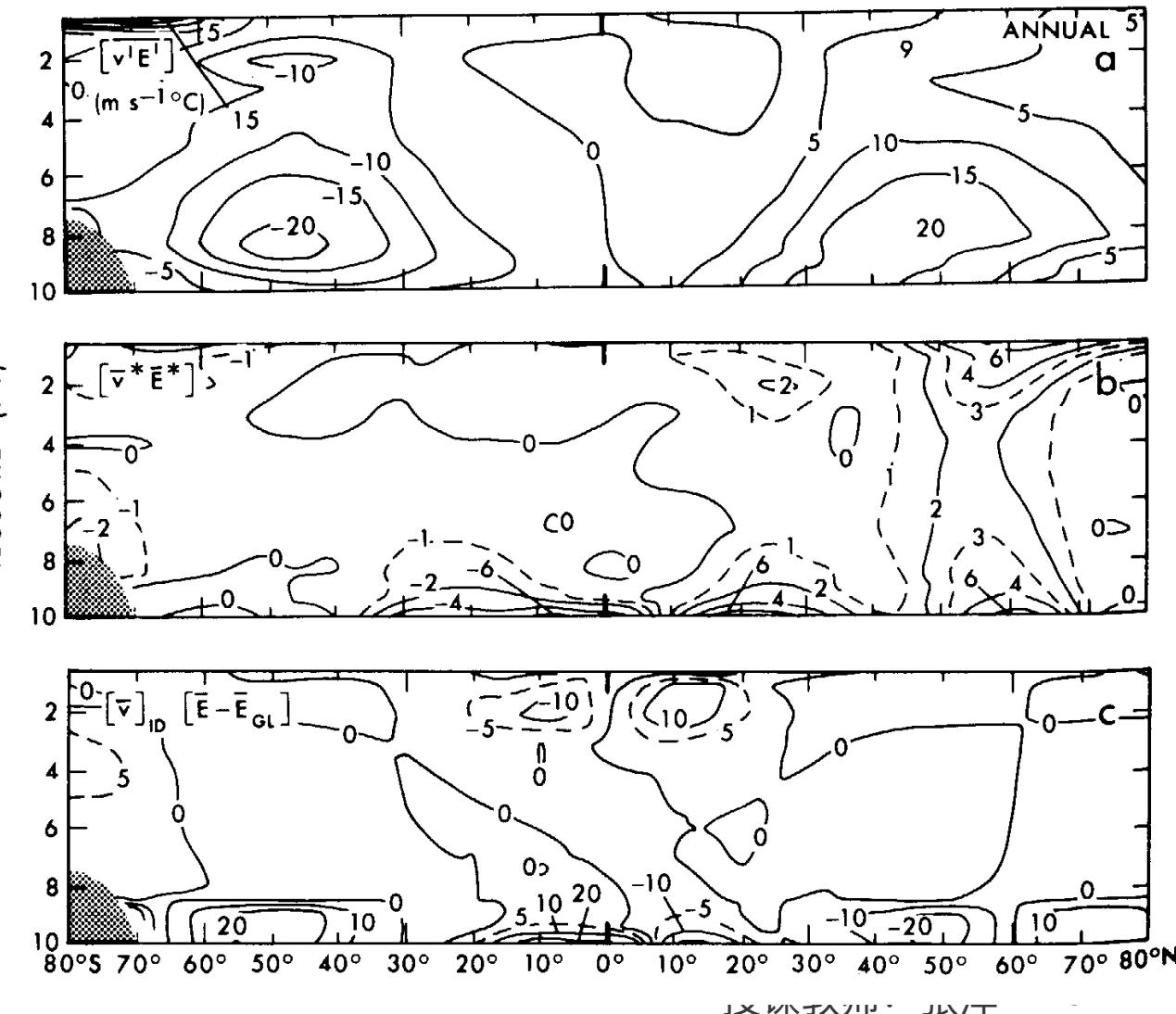


Total energy transport: vertical distribution



Transient components:
two peaks in vertical
direction (around 800 and
200 hPa).

Zonal-mean flow: two
peaks in vertical direction
(around 200 hPa and
near surface).

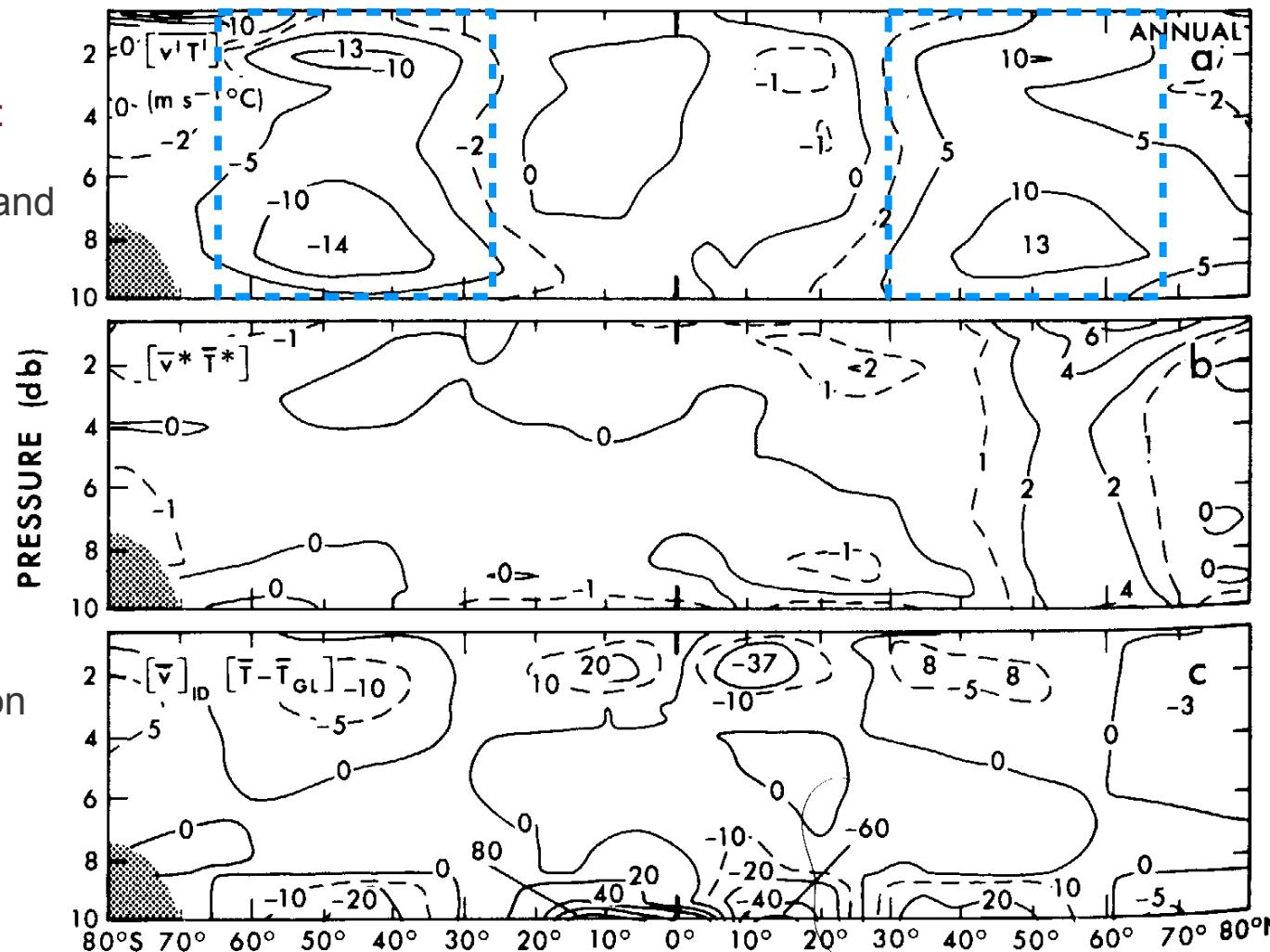




Sensible heat transport: vertical distribution



Transient components:
two peaks in vertical
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200 hpa).



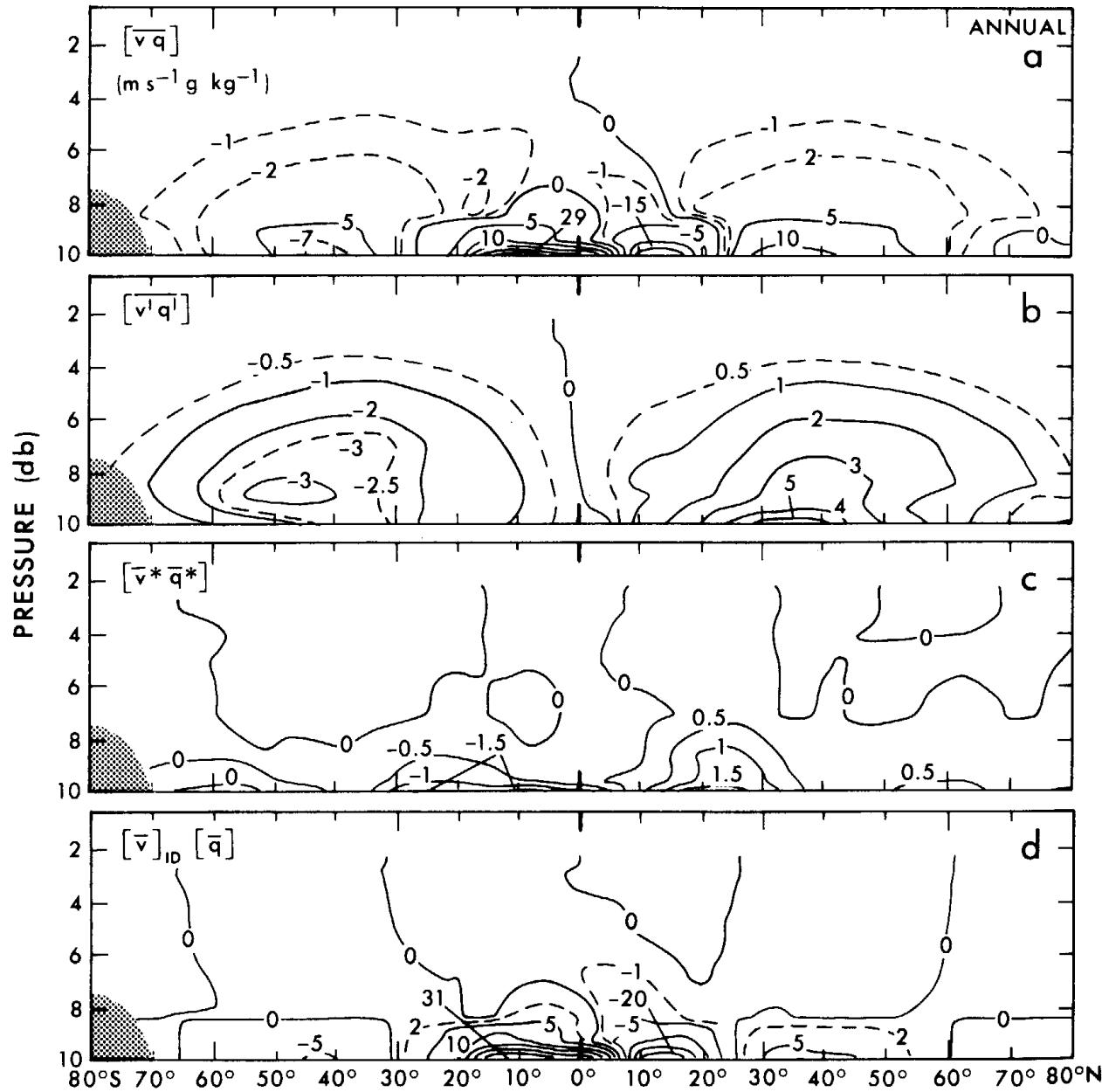
Zonal-mean flow: two
peaks in vertical direction
(around 200 hpa and
near surface).



■ Vertical distribution of latent heat transport:

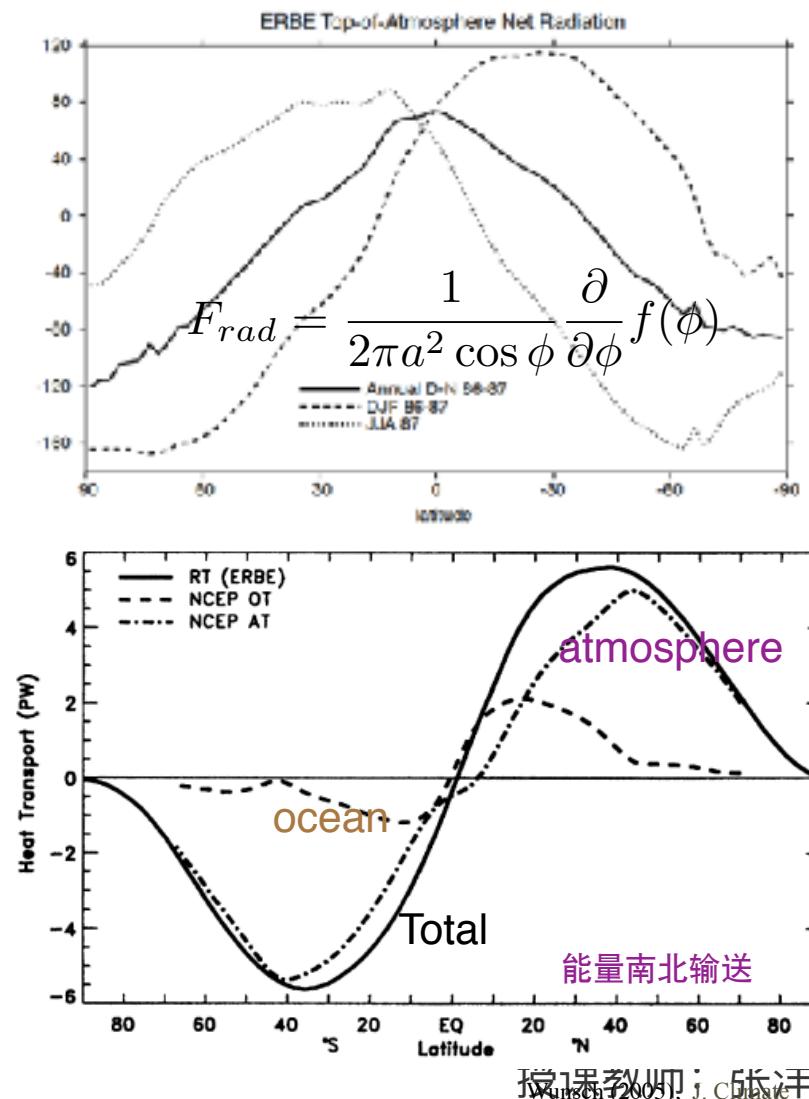
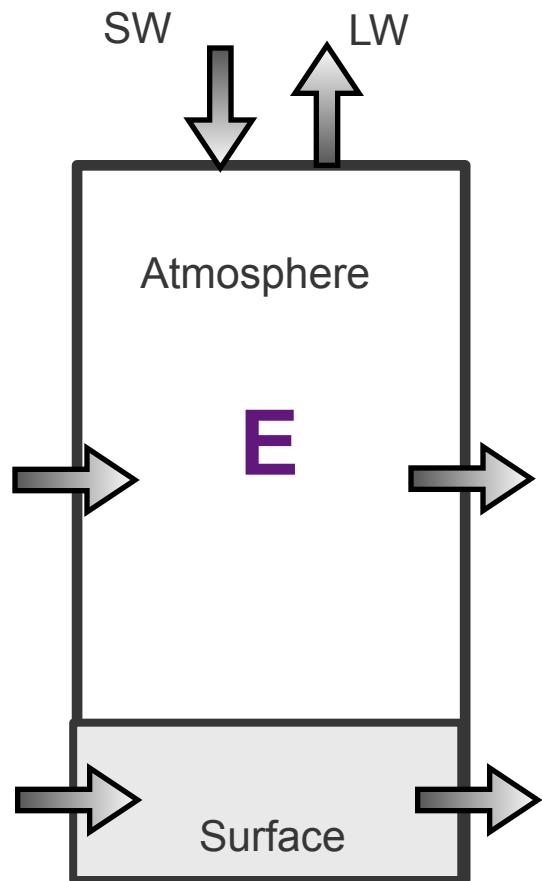
All the components are concentrated near surface

Only considerable in the boundary layer



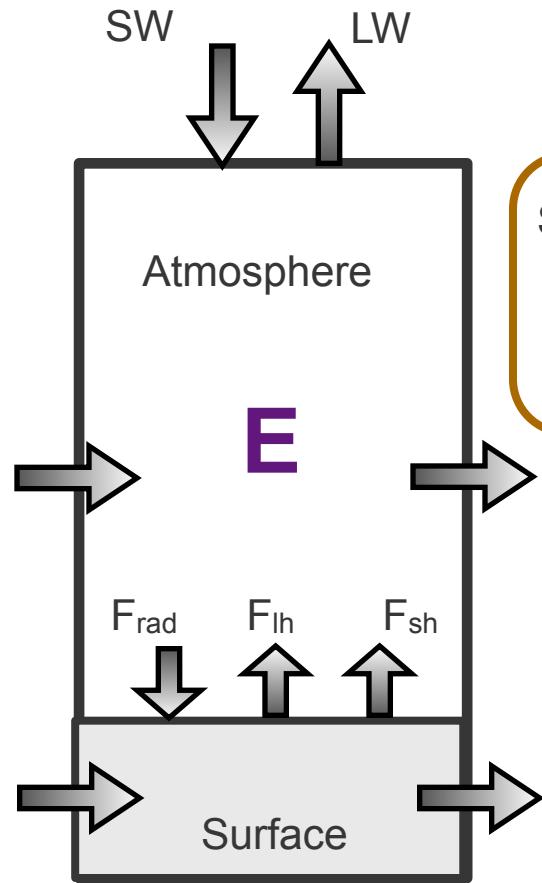


Summary: distribution, budget and transport





Summary: distribution, budget and transport



Energy transport:

$$v(c_p T + gz + Lq + K)$$

Similar meridional distribution;
different vertical distribution

Dominant component

Transient eddy dominant

