Chapter 2

Simple energy balance climate models

Supplemental reading:[1]

Budyko (1969)

Held and Suarez (1974)

Lindzen and Farrell (1977)

North (1975)

Sellers (1969)

We will consider energy balance climate models because they are the simplest models wherein the interactions of radiation (including the effect of snow on albedo) and dynamic heat transport can be considered. In the above references some attempt is made to justify the realism of the models. Although this is probably worth thinking about, we are here only concerned with the illustrative aspects rather than detailed realism.

[1] A complete list of references is given at the end of this book. Those references that are particularly useful to a given chapter are listed at the beginning of that chapter. Sometimes specific pages and/or chapters will be noted.

These models are typically characterized as follows:

1. Only latitude dependences are considered; that is, the models are spatially one-dimensional (though time dependence is also sometimes considered).

2. Global energy budgets are assumed to be expressible in terms of surface temperatures.

3. Planetary albedo is taken to depend primarily on ice and/or snow cover or the lack thereof.

4. The convergence of dynamic heat fluxes is generally represented by either a simple diffusion law or by a linear heating law wherein local heating is proportional to deviations of the global mean temperature from the local surface temperature. *The primary feature of the heat transport is that it carries heat from warmer to colder regions.* Both of the above representations do this.

5. Generally, only annual mean conditions are considered.

The starting point for such models is an equation of the form

$$\begin{aligned} C\frac{\partial T(x,t)}{\partial t} &= \text{incoming solar radiation} \\ &\quad -\text{infrared cooling} \\ &\quad -\text{divergence of heat flux,} \end{aligned} \tag{2.1}$$

where C is some heat capacity of the atmosphere-ocean system, T the surface temperature (°C), t is time, and $x = \sin\theta$, where θ is the latitude. It is somewhat more convenient to deal with x rather than θ.

Under the assumption that the *total* global energy budget can be expressed in terms of the surface temperature, the first term on the right-hand side of Equation 2.1 is generally taken to be the total insolation as might be determined by a satellite above the atmosphere. It is typically written as

$$\text{incoming solar radiation} = Qs(x)A(T), \tag{2.2}$$

where Q is one quarter of the solar constant (Why?) and $s(x)$ is a function whose integral from the equator to the pole is unity and which

represents the annually averaged latitude distribution of incoming radiation. This function is discussed in Held and Suarez (1974). Finally, $\mathcal{A}(T)$ is 1 minus the planetary albedo; $\mathcal{A}$ is allowed to depend on temperature. In most simple climate models, a temperature T_s is identified with the onset of ice (snow) cover such that for $T > T_s$ there is no ice (snow), and for $T < T_s$ there is. The most important change in $\mathcal{A}$ is due to T passing through T_s. We will specify $\mathcal{A}$ more explicitly later.

Again under the assumption that global energy budgets can be expressed in terms of surface temperature, one writes

$$\text{infrared cooling} = I(T). \tag{2.3}$$

The justification for Equation 2.3 is that temperature profiles have more or less the same shape at all latitudes. Hence cooling, which depends on the temperature at all levels, ought to be expressible in terms of surface temperature, since the temperature at all levels is related to the surface temperature. In fact, temperature profiles at different latitudes are somewhat different (*viz.* Figure 2.1). Moreover, Held and Suarez (1974) have shown that 500mb temperatures correlate better with infrared emission than do surface temperatures. Nevertheless, it is the surface temperature which relates to the formation of ice, and which therefore must be used in simple climate models. The fact that total infrared emission is not perfectly related to surface temperature is merely an indication that a significant portion of the emitted radiation originates in the atmosphere. Similarly, not all of the incoming radiation is absorbed at the surface; in practice, some of the incoming radiation is *not* directly involved in the surface energy budget.

As a rule, models based on Equation 2.1 take little account of clouds and cloud feedbacks. In truth we hardly know how to include such feedbacks. It is probably impossible in such a simple model. However, to the extent that clouds can be specified in terms of latitude and surface temperature, their effects on incoming radiation and on infrared emission can be included in Equations 2.2 and 2.3.

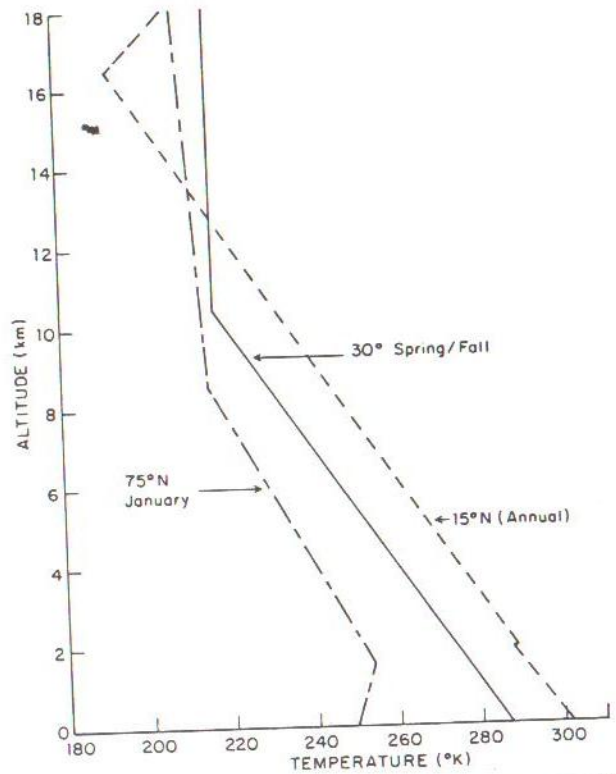

Figure 2.1: Vertical temperature profiles for various latitudes. (From U.S. Standard Atmosphere Supplements, 1966.)

The divergence of atmospheric and oceanic heat flux must also be expressed in terms of an operator on surface temperature; that is,

$$\text{div flux} - F[T], \tag{2.4}$$

where F is some operator. Usually F is a linear operator, although Held and Suarez (1974) and North (1975) have also considered nonlinear operators as suggested by Green (1970) and Stone (1973). The common choices for $F[T]$ are a linear relation first suggested by Budyko (1969):

$$F[T] = C\,(\bar{T} - T), \tag{2.5}$$

where $\bar{T}$ is the average of T over all latitudes[2], and a diffusion law (first used in this context by Sellers (1969)):

$$F[T] = \frac{\partial}{\partial x}\left[(1-x^2)D\frac{\partial T}{\partial x}\right], \tag{2.6}$$

where C and D in (2.5) and (2.6) are constants; they are generally chosen to simulate some features of the existing climate.

The most common application of (2.1) involves assuming a steady state and seeking a relation between the equilibrium position of the ice line and the solar constant. Usually, one linearizes (2.3) to obtain

$$I = A + BT, \tag{2.7}$$

and replaces T by I as defined in (2.7). Equation 2.1 becomes[3]

$$Qs(x)\mathcal{A}(I) - I + F^*[I] = 0. \tag{2.8}$$

One identifies the ice line with a temperature T_s, or equivalently $I_s = A + BT_s$. We shall use x_s to identify the value of x at $I = I_s$. Moreover, variations in $\mathcal{A}$ are taken to be due solely to whether or not there is an ice surface, that is,

$$\mathcal{A} = \mathcal{A}(x, x_s). \tag{2.9}$$

'C' or 'D' in (2.4) will be chosen so that for the present climate $T = T_s$ at the present annually averaged value of x_s (i.e., $x_s \simeq 0.95$). Obtaining the dependence of x_s on Q (or more conveniently, the dependence of Q on x_s) is straightforward. If we write

$$I = Q\tilde{I}(x), \tag{2.10}$$

and assume F to be a linear operator, then we may divide (2.8) by Q, yielding

$$-F^*[\tilde{I}] + \tilde{I} = s(x)\mathcal{A}(x, x_s). \tag{2.11}$$

[2] A general constraint on $F[T]$ is that its integral over the globe be zero (Why?).
[3] $F^* \equiv \frac{1}{B}F$

For any choice of x_s we may solve (2.11) for $\tilde{I} = \tilde{I}(x, x_s)$. It is now a trivial matter to obtain the solar constant (or, equivalently, Q) as a function of x_s. We already have

$$I(x_s) = I_s. \tag{2.12}$$

But since

$$I(x_s) = \tilde{I}(x_s, x_s)Q, \tag{2.13}$$

by combining (2.12) and (2.13) we have

$$\frac{Q}{I_s} = \frac{1}{\tilde{I}(x_s, x_s)}, \tag{2.14}$$

which is the desired relation.

Normally, we expect advancing ice (decreasing x_s) to be associated with decreasing Q. Such a situation is generally stable in the sense that the time-dependent version (2.1) indicates that perturbations away from the equilibria defined by (2.14) decay in time. This stability is easy to understand intuitively. If, for example, one decreased x_s while holding Q constant, then Q would be larger than needed for that value of x_s and the resulting warming would cause x_s to increase. This is, in fact, the situation when we do *not* have transport. Surprisingly, the introduction of transport always leads to some values of x_s where decreasing x_s is associated with increasing Q – an unstable situation leading to an ice covered earth (at least in the context of the simple model).

We shall examine how this occurs under particularly simple conditions. First we shall use (2.5) for $F[T]$. Next we shall take the following expression for $\mathcal{A}$:

$$\begin{aligned} \mathcal{A} &= \alpha \text{ for } T < T_s \\ \mathcal{A} &= \beta \text{ for } T > T_s. \end{aligned} \tag{2.15}$$

Common choices for α and β are:

$$\begin{aligned} \alpha &= 0.4 \\ \beta &= 0.7. \end{aligned} \tag{2.16}$$

For T_s we, for the moment, take $T_s = -10°\mathrm{C}$, and for the constants A and B in (2.7) we use $A = 211.1\,\mathrm{Wm}^{-2}$, and $B = 1.55\,\mathrm{Wm}^{-2}(°\mathrm{C})^{-1}$. Hence, $I_s = 195.6\,\mathrm{Wm}^{-2}$. For $s(x)$ we use the annual average function as approximated by North (1975):

$$s(x) \approx 1 - 0.241(3x^2 - 1). \tag{2.17}$$

For the present solar constant,

$$Q = \overset{344.4}{\cancel{334.4}}\,\mathrm{Wm}^{-2}.$$

For the above choices, straightforward analytic solutions exist. Let us begin by neglecting all transport. Equation 2.8 becomes

$$Qs(x)\mathcal{A}(x, x_s) - A - BT = 0,$$

or

$$T = \frac{Qs(x)\mathcal{A}(x, x_s) - A}{B}. \tag{2.18}$$

This radiative equilibrium value of T depends only on the local radiative budget. Its distribution is shown in Figure 2.2 as is the observed distribution. Note that the observed gradients are much smaller than in the equilibrium distribution. The value of Q associated with x_s is not unique. Any value of Q less than the value needed for $T(x_s) = T_s$ with $\mathcal{A} = \alpha$, and greater than the value needed for $T(x_s) = T_s$ with $\mathcal{A} = \beta$, is consistent with x_s. This leads to the two curves for $Q(x_s)$ shown in Figure 2.3:

$$Q_+(x_s) = \frac{A + BT_s}{s(x_s)\alpha}, \tag{2.19}$$

$$Q_-(x_s) = \frac{A + BT_s}{s(x_s)\beta}. \tag{2.20}$$

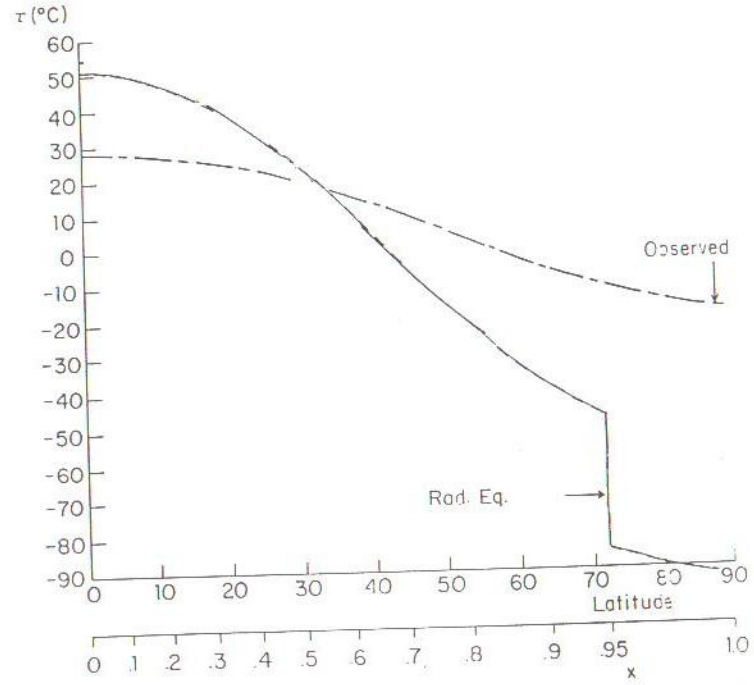

Figure 2.2: $T(\phi)$ for radiative equilibrium. Also shown is the observed $T(\phi)$. For reference purposes, x as well as ϕ is shown. Taken from Lindzen and Farrell (1977).

It will be left to the student to figure out how x_s will vary as Q is changed, but it is evident that for both (2.19) and (2.20) that decreasing Q leads to decreasing x_s and *vice versa*. (Note that $s(x_s)$ decreases monotonically with x_s.)

Introducing transport via Equation 2.5 does not eliminate the ambiguity in Q. A device for eliminating the ambiguity is to choose

$$\mathcal{A}(x_s) = \frac{\alpha + \beta}{2}. \tag{2.21}$$

This device turns out to be almost equivalent to introducing a very small conductivity which, in turn, renders T continuous at x_s (*viz.* Figure 2.3).

Now

$$F^*[\tilde{I}] = (C/B)(\bar{\tilde{I}} - \tilde{I}) \tag{2.22}$$

and

$$\bar{\tilde{I}} = \int_0^1 s(x)\,\mathcal{A}(x, x_s)\,dx \tag{2.23}$$

(Why?).

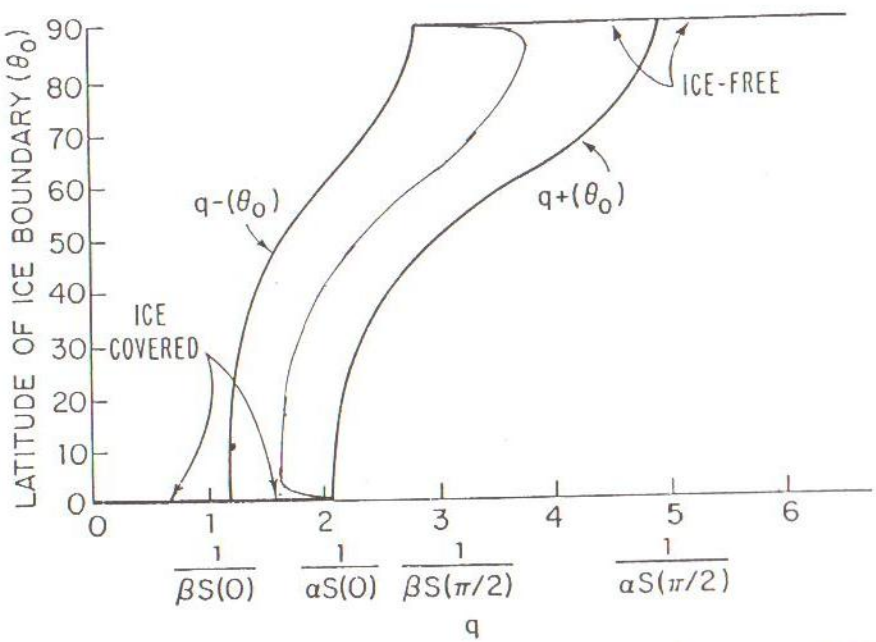

Figure 2.3: Variation of q ($q \equiv Q/I_s$) vs. θ_s ($\sin^{-1} x_s$) for radiative equilibrium. The curve q_- represents $T = T_s$ for the ice-free side of x_s, while q_+ represents $T = T_s$ for the ice-covered side of x_s. Also shown is the curve obtained with a very small amount of diffusive heat transport. From Held and Suarez (1974).
Note that θ_0 in this figure corresponds to θ_s in the text.

Evaluating (2.23) (using (2.15) and (2.17)) we get

$$\bar{\tilde{I}} = (\beta - \alpha)(1.241\,x_s - .241\,x_s^3) + \alpha. \tag{2.24}$$

Substituting (2.22) in (2.11), we get

$$\tilde{I} = \frac{\frac{C}{B}\bar{\tilde{I}}(x_s) + s(x)\,\mathcal{A}(x, x_s)}{1 + \frac{C}{B}}. \tag{2.25}$$

Equation 2.25 allows us to determine C such that $\tilde{I}(.95) = \tilde{I}_s$ for present conditions (C/B = ~~2.45~~ 1.97 is obtained). Assuming this value of C remains constant as Q varies, we then get from (2.14)

$$Q = \frac{(1 + \frac{C}{B})(A + BT_s)}{\frac{C}{B}\bar{\tilde{I}}(x_s) + s(x_s)(\frac{\alpha+\beta}{2})}. \tag{2.26}$$

Examining the denominator of (2.26) in detail we get

$$\begin{aligned} \text{den} \;=\; & \frac{\beta + \alpha}{2} \times 1.241 + \alpha\frac{C}{B} \\ & + \frac{C}{B}(\beta - \alpha) \times 1.241\,x_s \\ & - \frac{(\beta + \alpha)}{2} \times .723\,x_s^2 - \frac{C}{B}(\beta - \alpha) \times .241\,x_s^3. \end{aligned} \tag{2.27}$$

When $C = 0$, the denominator decreases as x_s increases, as already noted. But, when $C \neq 0$, there always exists some neighbourhood of $x_s = 0$ where the linear term in (2.27) dominates, and the denominator *increases* as x_s increases. This leads to the distribution of Q versus x_s shown in Figure 2.4. Two features should be noted in Figure 2.4, both being due to the existence of horizontal heat transport:

1. A much smaller value of Q is needed for ice/snow to onset at all. This represents the stabilizing effect of transport.

2. There now exists some minimum Q, below which the climate will unstably proceed to an ice/snow covered earth. This represents the destabilizing effect of transport.

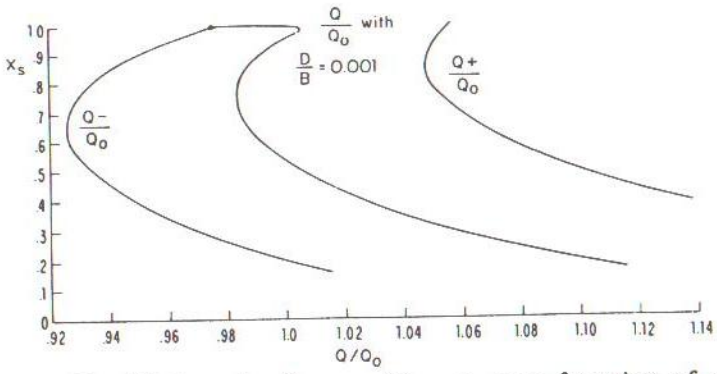

Figure 2.4: Equilibrium ice line position x_s as a function of normalized solar constant Q/Q_0 (where Q_0 is the current value of the solar constant) when Budyko-type heat transport is used. The curve Q_-/Q_0 corresponds to $T = T_s$ on the equatorward side of the ice line, while Q_+/Q_0 corresponds to $T = T_s$ on the poleward side. The single remaining curve results from adding a small amount of diffusion to the Budyko-type transport.

Both the above effects result very generally from the sharing of heat between low and high latitudes. Clearly, heat transport from low latitudes initially inhibits the onset of ice/snow at the poles. However, as the ice/snow line advances, the transport of heat out of warmer regions cools these regions to such an extent that Q must actually be increased to keep up with further advances. This situation is clearly unstable. We shall refer to the percentage Q must be reduced from its present value to reach instability as the 'global stability'. The results in Figure 2.4 correspond to a global stability of only $\sim 2\%$. To be sure, the solar constant might not vary this much. However, Q can be viewed as a general measure of global heating. Changes in Q can be simulated by changes in I and/or $\mathcal{A}$, for example.

As will be seen in the exercise, the above estimate of *global stability* is hardly firm, but our only interest at this point is in the general rôle of heat transport.

Exercise

2.1 Let $\alpha = 0.45$. Keep A and B as given in the text.

1. Determine β such that $\bar{T}$ remains unchanged.
2. Determine C for the above choices of α and β.
3. Compute $Q(x_s)$.
4. Discuss any differences between these results and those obtained for $\alpha = .4$, $\beta = .7$. In particular, how has the global stability changed and why?