Homework 7

Course: Algorithm Design and Analysis **Semester**: Spring 2024

Instructor: Shi Li **Due Date**: **2024/6/12**

Student Name: _________ Student ID: _________

Problems	1	2	3	4	5	Total
Max. Score	20	16	14	20	30	100
Your Score						

Problem 1. For each of the following problems, state (i) whether the problem is provably in NP, and (ii) whether the problem is provably in Co-NP. If you claim a problem is in NP, then you need to describe the certificate and the certifier for the proof.

(1a) Given a graph $G = (V, E)$ with edge weights $w \in \mathbb{Z}_{\geq 0}^E$ and an integer $W \geq 0$, the problems asks if there is a spanning tree of G with total weight at most W.

(1b) Given two boolean formulas, the problem asks whether the two boolean formulas are equivalent. For example, $(x_1 \vee x_2) \wedge (\neg x_1 \vee x_3)$ and $(\neg x_1 \wedge x_2) \vee (x_1 \wedge x_3)$ are equivalent since they give the same value for every assignment of (x_1, x_2, x_3).

(1c) An undirected graph $G = (V, E)$ is called an expander if for every $S \subseteq V$, the number of edges between S and $V \setminus S$ in G is at least $\min\{|S|, |V \setminus S|\}$. Given a graph G, the problem asks if G is an expander or not.

(1d) Given n items $[n]$ with integer weights $w_1, w_2, \cdots, w_n \geq 0$ and integer values $v_1, v_2, \cdots, v_n \geq 0$, and two integers W and V, the problem asks if there is a set $S \subseteq [n]$ of items with total weight at most W and total value at least V.

Problem 2. Indicate if each of the following statements is true or false. A true/false answer for each statement is sufficient; you do not need to give proofs/counterexamples for your answers.

(2a) If a decision problem X can be solved in polynomial time, then $X \notin$ NP.

(2b) Assume X is a NP-complete problem and X has a polynomial time algorithm. Then P = NP.

(2c) If a problem X is NP-compelte, then the circuit-satisfiability problem is polynomial-time reducible to X.

(2d) Based on our knowledge, it is possible that $\text{P} \cap \text{NP} = \emptyset$.

Problem 3. Prove that P=NP if and only if P = Co-NP.

Problem 4. In class, we proved that HP (Hamiltonian Path) $\leq_P$ HC (Hamiltonian Cycle). Prove the other direction, i.e, HC $\leq_P$ HP.

Problem 5. In the Steiner Tree problem, we are given a graph $G = (V, E)$, with edge weights $w \in \mathbb{Z}_{\geq 0}^{E}$, and a set $X \subseteq V$ of vertices. The goal of the problem is to find the minimum-weight edges to connect X in the graph G. Formally, our goal is to find a tree $T = (V_T, E_T)$ such that $X \subseteq V_T \subseteq V$ and $E_T \subseteq E$ (such a tree T is called a Steiner tree for X), so as to minimize $\sum_{e \in E_t} w_e$.

In the decision problem, we are additionally given an integer bound $W > 0$, and we need to decide if there is a Steiner tree for X with weight at most W.

Prove that Vertex-Cover $\leq_P$ Steiner-Tree.