Homework 8

Course: Algorithm Design and Analysis **Semester**: Spring 2024

Instructor: Shi Li **Due Date**: **2024/6/23**

Student Name: _________ Student ID: _________

Problems	1	2	3	4	Total
Max. Score	20	20	35	25	100
Your Score					

Problem 1. In a connected graph $G = (V, E)$, then number of global minimum cuts is at most $\binom{n}{2} \leq n^2$. Let $\alpha \geq 1$ be an integer. We say a cut in G is an α-approximate cut, if its cut value is at most α times the value of the global minimum cut. Prove that the number of α-approximate minimum cuts in a connected graph $G = (V, E)$ is at most $n^{2\alpha}$.

Problem 2. We are given an array A of n integers, and we are promised that some integer appears in A at least $\frac{n}{3}$ times. Design a simple Las Vegas algorithm that finds such an integer in $O(n)$ time in expectation.

Problem 3. Let $G = (V, E)$ be a graph with tree-width tw.

(3a) Prove that there is a $(\mathrm{tw} + 1)$-coloring for the vertices of G.

(3b) Suppose we are additionally given a tree-decomposition $(T, (V_t)_{t \in U})$ of G with tree-width tw. It is possible that G can be colored using k colors, for some given integer $k \leq \mathrm{tw}$. Design an $f(\mathrm{tw})\mathrm{poly}(n)$-time algorithm to check if some a k-coloring exists, where $f(\mathrm{tw})$ can be any function on tw.

Problem 4. Suppose we are given a directed acyclic graph with specified source node s and sink node t, and each arc e has an associated cost c_e and length l_e. We are also given a length bound L. Give an FPTAS for the problem of finding a minimum-cost path from s to t of total length at most L.