Homework 4

Course: Algorithm Design and Analysis | **Semester**: Spring 2026

Instructor: Shi Li | **Due Date**: **June 7, 2026**

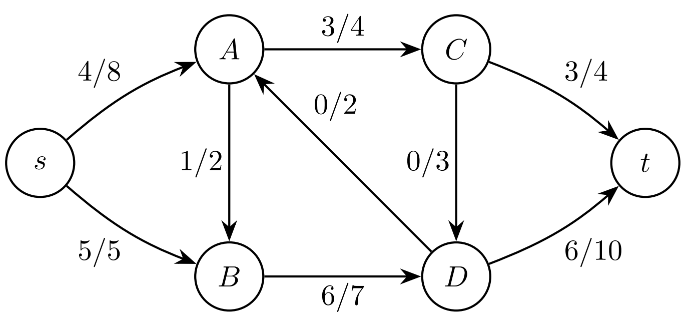
Student Name: __________ Student ID: __________

Problems	1	2	3	4	5	Total
Max. Score	20	20	20	20	25	105
Your Score						

The sum of maximum scores over all problems is 105, but your score will be capped at 100.

Network Flow

Problem 1. Consider the s-t flow network $G = (V, E)$ shown below. For each edge e, the notation $f(e)/c(e)$ represents the current flow $f(e)$ and the edge capacity $c(e)$.



Based on the given flow network, answer the following sub-problems:

(1a) Draw the corresponding residual graph G_f and state the residual capacities for all residual edges.

(1b) Find an augmenting path from s to t in the residual graph G_f and state its bottleneck capacity.

(1c) Provide the flow obtained after augmenting along the path.

(1d) Provide the final maximum flow for the problem, and give its value.

(1e) Find a minimum s-t cut (S, T) in the graph and verify that the capacity of this cut equals the value of the maximum flow.

Problem 2. In an undirected graph $G = (V, E)$, two vertices u and v are said to be k-edge-connected if there exist at least k edge-disjoint paths connecting them. Prove that the k-edge-connectivity property is transitive. That is, for any three distinct vertices $u, v, w \in V$, if u and v are k-edge-connected, and v and w are k-edge-connected, then u and w must also be k-edge-connected.

Problem 3. There are n software applications indexed as $\{1, 2, \ldots, n\}$. We are planning to move a subset of these applications to a completely new architecture. For each application i, there is a known net benefit $b_i > 0$ if application i is moved to the new system.

However, these applications interact with one another. For every pair of distinct applications $i, j \in [n]$ $(i \neq j)$, there is a separation expense $x_{ij} \geq 0$: if we decide to move one of i or j to the new system but leave the other behind on the old system, we must pay a cross-system integration expense of x_{ij}.

Due to critical legacy dependencies, application 1 cannot be moved under any circumstances (i.e., it must remain on the old system).

Design a polynomial-time algorithm to find a target set of applications $S \subseteq \{2, 3, 4, \ldots, n\}$ to migrate to the new system such that the total sum of benefits minus the total separation expenses is maximized.

NP-Completeness

Problem 4. Indicate if each of the following statements is true or false. A true/false answer for each statement is sufficient; you do not need to give proofs/counterexamples for your answers.

(4a) If a decision problem X can be solved in polynomial time, then $X \notin$ NP.

(4b) Assume X is a NP-complete problem and X has a polynomial time algorithm. Then P = NP.

(4c) If a problem X is NP-compelte, then the circuit-satisfiability problem is polynomial-time reducible to X.

(4d) Based on our knowledge, it is possible that P $\cap$ NP $= \emptyset$.

Problem 5. For each of the following decision problems, determine whether it belongs to P, NP, or co-NP (note that a problem can belong to more than one class). For the problems that you identify as being in NP, explicitly provide a valid certificate and describe a polynomial-time certifier algorithm.

(5a) Given an undirected graph $G = (V, E)$, can the vertices of G be assigned one of 2 colors such that no two adjacent vertices share the same color?

(5b) Given an undirected graph $G = (V, E)$, can the vertices of G be assigned one of 3 colors such that no two adjacent vertices share the same color?

(5c) Given a Boolean formula ϕ over a set of variables, is ϕ a tautology (i.e., does it evaluate to true under every possible truth assignment)?

(5d) An undirected graph $G = (V, E)$ is called a 1-expander if for every $S \subseteq V$, the number of edges between S and $V \setminus S$ in G is at least $\min\{|S|, |V \setminus S|\}$. Given a graph G, the problem asks if G is a 1-expander.