

Advanced Algorithms

Introduction: Min, Max, and Sparsest Cuts

尹一通 Nanjing University, 2026 Fall

Course Info

- Instructors (授课时间顺序) :
 - 尹一通、刘景铎、栗师
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- Office hour:
 - 周一4-5pm (尹一通 804)
 - 周四4-5pm (刘景铎 516)
 - By appointment
- QQ group: 1098567018
- Course webpage: <http://tcs.nju.edu.cn/wiki/>



高级算法 (南京大学 ...)

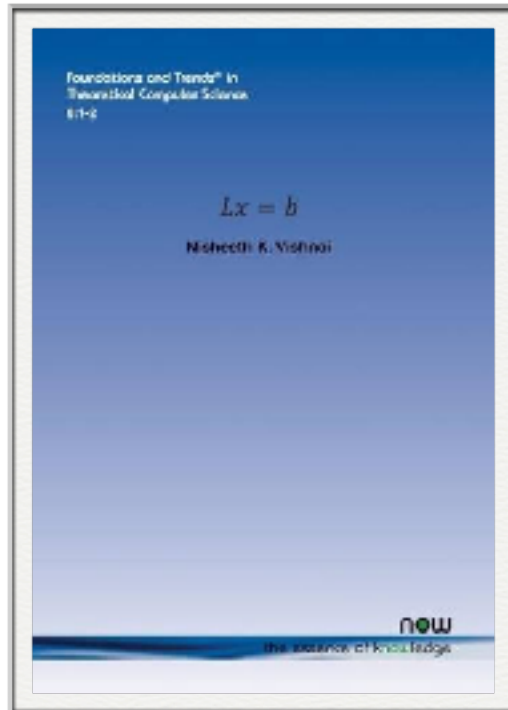
群号: 1098567018



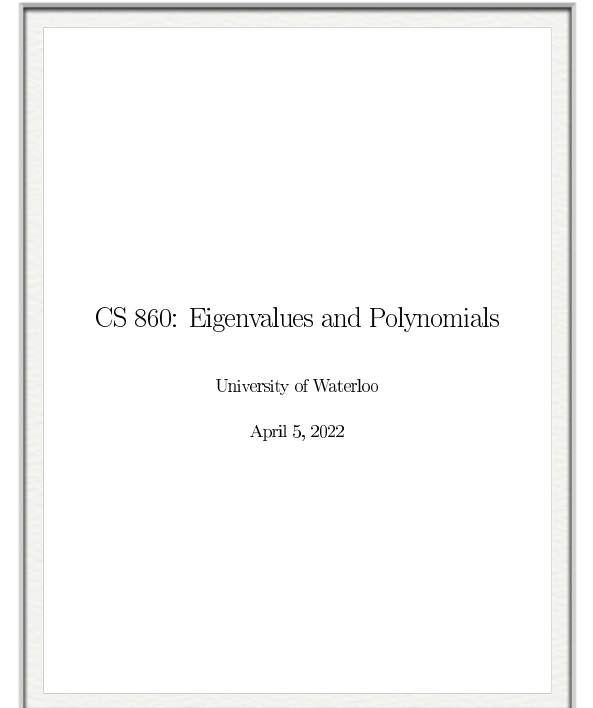
扫一扫二维码, 加入群聊



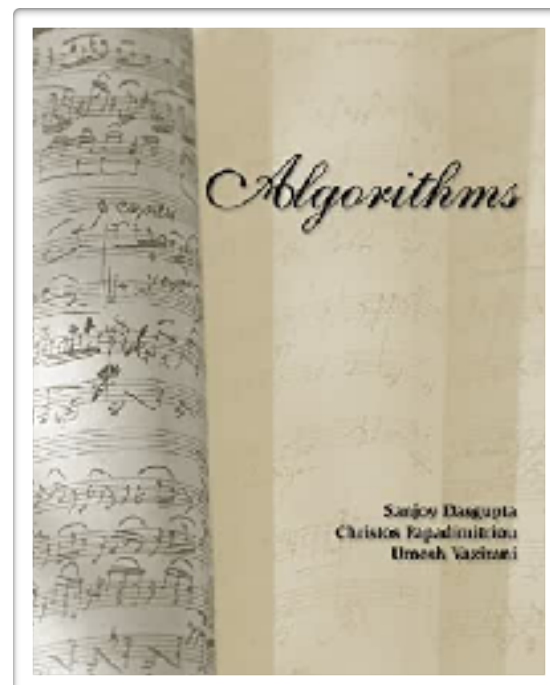
References



Nisheeth Vishnoi
 $Lx = b$



Lap Chi Lau
Eigenvalues and Polynomials



DPV
Algorithms

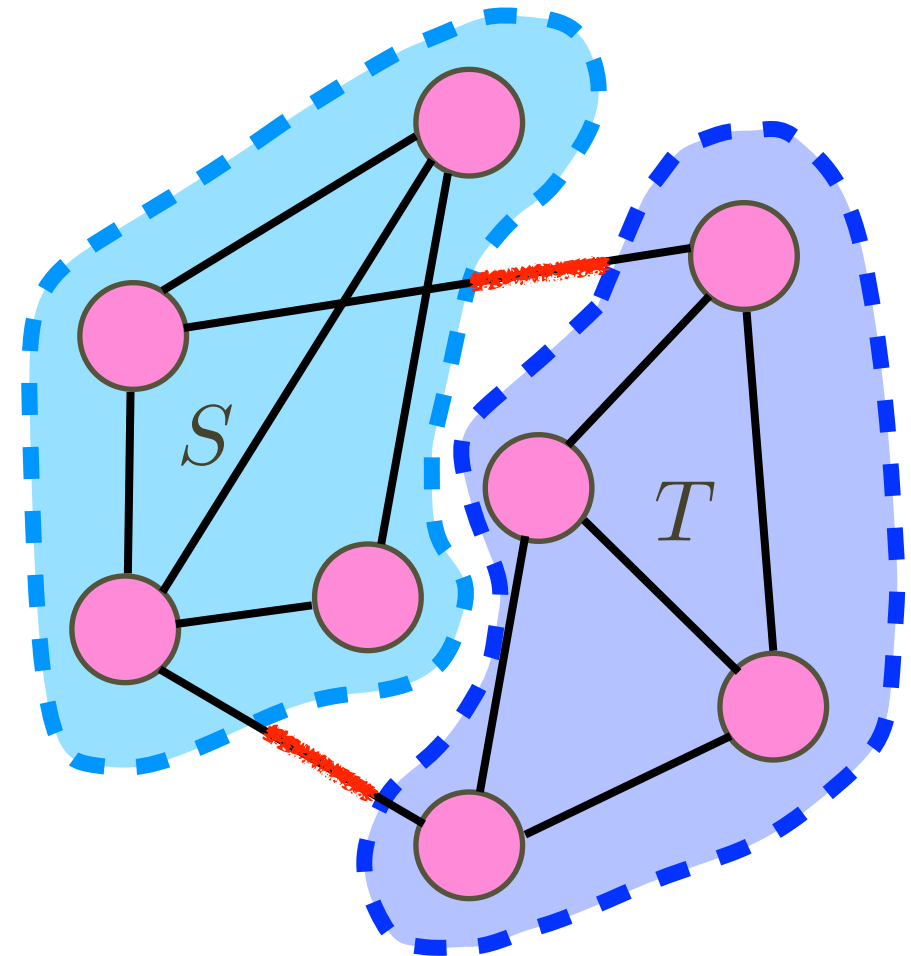
Minimum Cut

(Randomized Algorithms)

Min-Cut

- Undirected graph $G(V, E)$
- **Bi-partition** of V into nonempty S and T
- Find a **cut** $E(S, T)$ of **smallest** size (*global* min-cut)
- **Deterministic algorithms:**
 - max-flow min-cut (**duality**)
 - best upper bound (**2024**):
 $m \text{ polylog}(m)$

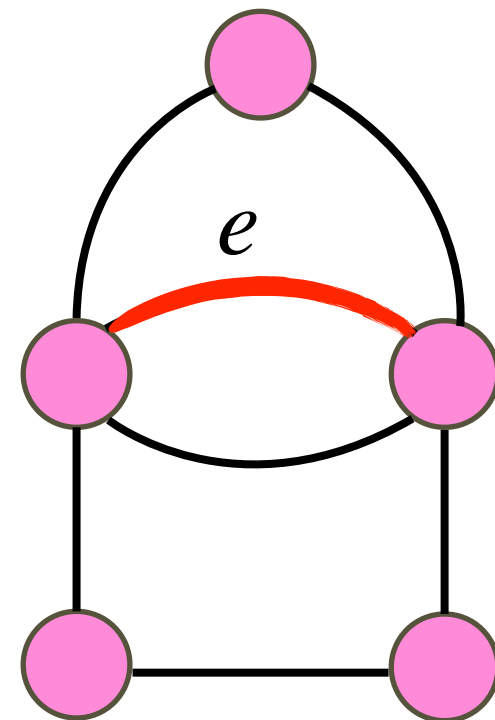
$$E(S, T) := \{uv \in E \mid u \in S, v \in T\}$$



Min-Cut

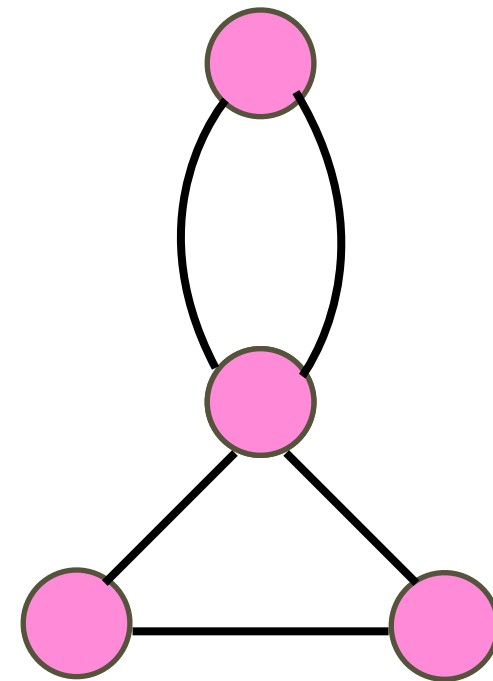
Contraction

- Undirected **multigraph** $G(V, E)$
- **multigraph**:
 - allow parallel edges
 - but no self-loop
- **contract(e)**: $e = uv$ is an edge
 - merges two endpoints u and v



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Karger's Min

Karger's Min

Karger's Min-Cut Algorithm

Karger's Min

Karger's Min

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Karger's Algorithm:</

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Sparsest Cut

(Spectral Algorithms)

Can One Hear the Shape of a Drum?

-Mark Kac

Graphs as Matrices

- Undirected graph $G(V, E)$ has **adjacency matrix** $A \in \{0,1\}^{V \times V}$:

$$A_{u,v} = 1 \text{ iff } uv \in E$$

- A is a real symmetric matrix:
 - it has real **eigenvalues** $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$
 - the **eigenvectors** $v_1, v_2, \dots, v_n$ form orthonormal basis

$$Av_i = \lambda_i v_i, \quad \forall i,$$

$$v_i^\top v_j = \begin{cases} 1, & \text{if } i = j, \\ 0, & \text{otherwise} \end{cases}$$

Maximum Cut

(Approximation Algorithms)

